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BAYESIAN DYNAMIC MODEL UPDATING USING MARKOV CHAIN MONTE CARLO METHODS

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Abstract. *The application of Bayesian model updating techniques such as Markov Chain Monte Carlo to structural dynamic finite element models is a growing interest of the industry, including applications requiring the determination of damping. Viscoelastic materials, such as the skin damping that is typically used on aircraft fuselages to attenuate the propagation of noise and vibration to the cabin, present non-linear properties that are difficult to model, as well as high damping values that are not always possible to determine with classical methods. Using the dynamic model of a beam with viscoelastic material, correlating model data to experimental results, this paper implements an adaptive Metropolis-Hastings (AMH) method to the determination of equivalent elastic properties and damping at the frequency domain. The method is shown to be capable of determining the parameters adequately, allowing the determination of equivalent elastic modulus and damping loss factor of the beam with viscoelastic constrained-layer damping composites.*

Keywords: *Bayesian model updating, Markov Chain Monte Carlo, viscoelastic, non-linear.*

1. INTRODUCTION

Finite element models have become standard practice for the industry. However, the correlation between numerical and experimental data is not always satisfactory, especially when complex or nonlinear systems are being modeled. The reasons for the discrepancy between finite-element model data and measured data include the difficulty to model certain geometries and boundary conditions, as well as variations of material properties, among others (Friswell and Mottershead, 1993). In order to overcome those difficulties, model updating techniques are being developed and have been successfully applied. Most literature on structural dynamics model updating have used the Maximum-likelihood framework, however recent works have shown the advantages of the Bayesian framework over the former, such as the ability to incorporate prior information to the updating variables (Marwala, 2010).

In order to reduce vibration and noise propagated to the cabin of passenger aircraft, viscoelastic materials such as skin damping are typically used. However, these materials present non-linear properties that are difficult to model, as well as high damping values that are not always possible to determine with classical methods such as circle-fit or half-power bandwidth. An implementation of the adaptive Metropolis-Hastings proposed by Beck and Au (2002) was used to update the dynamic model of a beam with viscoelastic material. The modal damping values of the model were updated using frequency-domain data.

2. THE AMH ALGORITHM

The Metropolis–Hastings algorithm is a procedure to simulate samples according to an arbitrary target PDF. This technique is named after the works of Metropolis *et al.* (1953) and Hastings (1970). The idea behind the MH algorithm is to define a Markov chain over possible θ values, so that the stationary distribution of the chain is the target PDF. A problem with the MH algorithm arises from the fact that the updated PDF $p(\theta, D)$ is concentrated on a small region of the proposal PDF. In order to overcome this limitation, a series of intermediate PDFs can be used to gradually populate the updated PDF (Beck and Au, 2002).

The adaptive method starts with an estimation of values for the parameters θ and simulation level s=1. Then, a proposed value ξ is sampled from the proposal distribution, and the model is run with θ=ξ. The updated PDF for each simulation level is given by:

$$p(\theta|\mathcal{D}) = \kappa \exp\left[-\frac{J_g'(\theta)}{2\varphi^2}\right] \tag{1}$$

where $J_g'(\theta)$ is the goodness-of-fit function normalized by the prediction error covariance, and φ is a measure of the size of the prediction error, which is used to thin each simulation level, given by $\varphi^2 = 1/2^s$.

For each proposed sample, acceptance is determined by:

$$\mathcal{Q}(\xi, \theta_n) = \frac{q(\xi)p^*(\theta_n|\xi)}{q(\theta_n)\mathrm{p}^*(\xi|\theta_n)} \tag{2}$$

where q represents the normalized target distribution and p^* is a proposal distribution.

These new values are accepted or not depending on the goodness-of-fit of the data and the generation of a random number. After N samples are draw, the proposal distribution is updated for the next simulation level, repeating until the total number of levels are simulated. The procedure, known as Adaptive Metropolis-Hastings (AMH) was applied in this paper to the dynamic model of a beam with skin damping viscoelastic material, correlating numerical model to experimental data.

3. EXPERIMENTAL SETUP

A methodology is here proposed in order to determine equivalent properties of constrained-layer damping composites attached a substrate structure, focusing on the primary application of damping of aircraft fuselages. Firstly, the experimental setup is presented, followed by the application of the AMH method to updated the properties of the finite-element model. The experiment consisted in using a small electrodynamic shaker was used to excite a beam, as per Figure 1. In order to determine the frequency response function at the driving point, and impedance head was used, which is capable of measuring both acceleration and force simultaneously. The beam has 38.1 mm of width, 508 mm of length, and the driving force was applied at its geometric center.

The setup is similar to the one proposed by Wojtowicki *et al.* (2004), who proposed a method to determine the properties of viscoelastic materials as an alternative to ASTM E756 (2005). The objective of this work, however, is not to be an alternative method to determine the properties of the viscoelastic material itself, but to determine the equivalent properties of the damped beam.

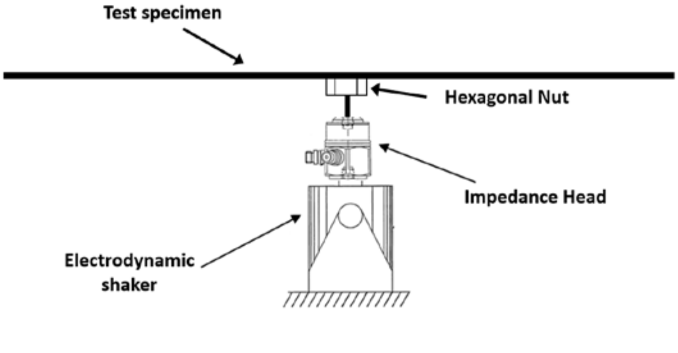

Figure 1 – Constrained layer damping experiment setup

The tests were conducted at room temperature, around 25ºC. The properties of viscoelastic materials vary greatly with temperature, so the results of these tests are valid only for that condition. Random vibration input was used to excite the structure, so that the actual fuselage vibration environment, which is mainly controlled by the turbulent boundary layer excitation is properly represented. Additionally, since the random vibration input excites all frequencies simultaneously, the temperature is stabilized over the whole frequency range, as opposed to what could happen if a sinusoidal sweep was used, where temperature would need to be carefully controlled so that there are no significant differences of temperature between frequencies that could have an influence on the frequency response curve behavior over the spectrum.

The chosen thickness of the base (or substrate) beam (1.25 mm) was made in order to simulate a real aircraft fuselage, so the equivalent properties resulted from the model updating can later be inputted directly into the periodic substructures finite element model in order to determine the properties of the fuselage.

4. TEST AND UPDATING PROCEDURE

The tested is intended to be used as a fuselage skin damping, so it is designed to work at low temperatures and presents peak performance at a range of -20 to -30ºC. At the tested temperature of around 25ºC, its damping performance is not so high, making it a suitable example for the updating method. The damping composite has a total thickness of 1.70 mm, and surface density of 3 kg/m². Instead of modeling each layer of the composite separately, an equivalent single layered model was created, considering a variable Young's Modulus along the frequency range to account for the non-linear behavior of the viscoelastic material. Adding the damping composite properties to those of the plain aluminum beam, a total thickness of 2.95 mm and volumetric density of 2161kg/m³ was considered.

Firstly, the AMH procedure was used to determine the equivalent elastic modulus of the beam for each of the natural frequencies that are detectable on the FRF. The updating process is done using the goodness-of-fit equation based on the natural frequencies, given by Equation (3).

$$J_g'(\theta) = \sum_{m=1}^{N} \left[\frac{\widehat{\Omega}^{(m)2} - \Omega^{(m)}(\theta)^2}{\widehat{\Omega}^{(m)2}} \right]^2 \tag{3}$$

Where $\widehat{\Omega}^{(m)}$ and $\Omega^{(m)}(\theta)$ are the measured modal frequency and the model modal frequency of the m^{th} mode, which depend on the parameters in θ. The values for each frequency are shown in Table 1. This first updating step allows the determination of a curve of Equivalent Elastic Modulus versus frequency, that is presented in Figure 2.

Table 1 – Odd numbered natural frequencies and estimated equivalent elastic modulus values for case b

Mode	Frequency (Hz)	Equivalent Elastic Modulus (GPa)
1	38	25.9
3	131	10.9
5	296	8.88
7	520	7.71
9	827	7.53
11	1191	7.16
13	1636	6.98
17	2137	6.71

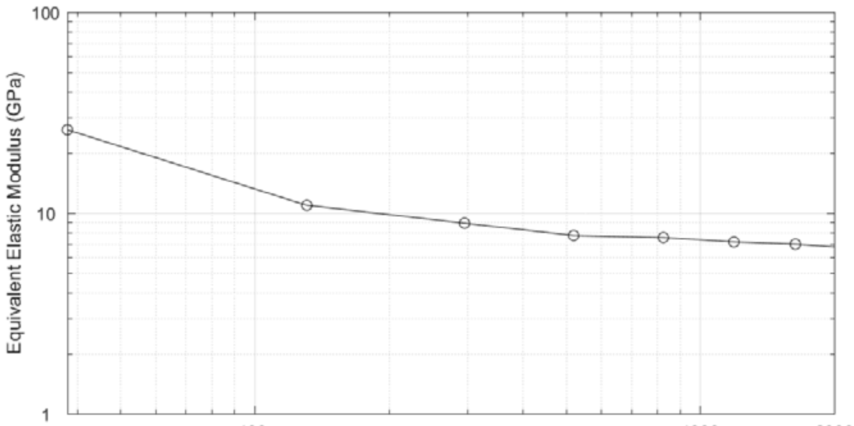

Figure 2 – Estimated equivalent elastic modulus vs. frequency – Case b

Following that, the updated procedure continued by updating simultaneously the Elastic Modulus and damping loss factor for each one-third octave band. The inertance frequency response function was used based on the modal properties (Ewins, 2000):

$$A_{jk}(\omega) = \sum_{r=1}^{N} -\omega^2 \frac{\phi_{jr}\phi_{kr}}{\omega_r^2-\omega^2+i\eta_r\omega_r^2} \tag{4}$$

where j and k are the excitation and response points, and r is the number of the mode. The modal structural damping is given by η_r, ϕ represents the eigenvectors, ω_r are the natural frequencies. The goodness-of-fit function in this case is given by the root mean square error (RMS) between experimental and predicted frequency response functions, based the proposed by Mares *et al.* (2006):

$$J_g'(\theta) = \sum_{k=1}^{Nfreq} \left(A_{jk}^{exp}(\omega_k) - A_{\mathrm{jk}}(\omega_k;\theta)\right)^2 \tag{5}$$

The priors for each parameter were taken to be independent normal PDFs. The mean value for the E was taken as the interpolated equivalent elastic modulus from the curve from Figure 2 with unit variance, while for the damping value η, the mean was considered as 0.20 with a 0.05 variance for the first band. For each subsequent band, the loss factor mean was considered as the result from the previous band.

For each TOB, the simulation was run for 15 levels, with a sampling size of 500 for each level and a burn-in period of 10 samples. The updated values for equivalent elastic modulus and damping are shown in Table 2.

Table 2 – Final updated Equivalent Elastic Modulus and damping for each TOB of case b

TOB Center Frequency (Hz)	Equivalent Elastic Modulus (GPa)	η (%)
31.5	25.0	29.9
40	24.9	28.3
50	21.1	27.5
63	17.8	27.4
80	12.3	28.1
100	11.2	28.5
125	10.2	25.6
160	9.7	25.9
200	9.5	24.6
250	8.7	23.8
315	8.2	20.2
400	7.8	23.4
500	7.5	13.2
630	7.4	18.5
800	7.2	11.5
1000	7.1	9.2
1250	7.1	8.2
1600	6.9	7.3
2000	6.5	6.4

Figure 3 shows an example of the evolution of the Equivalent Elastic Modulus and damping loss factor for the 125 Hz TOB. Figure 4 show the comparison between the experimental and the final updated curves for case b.

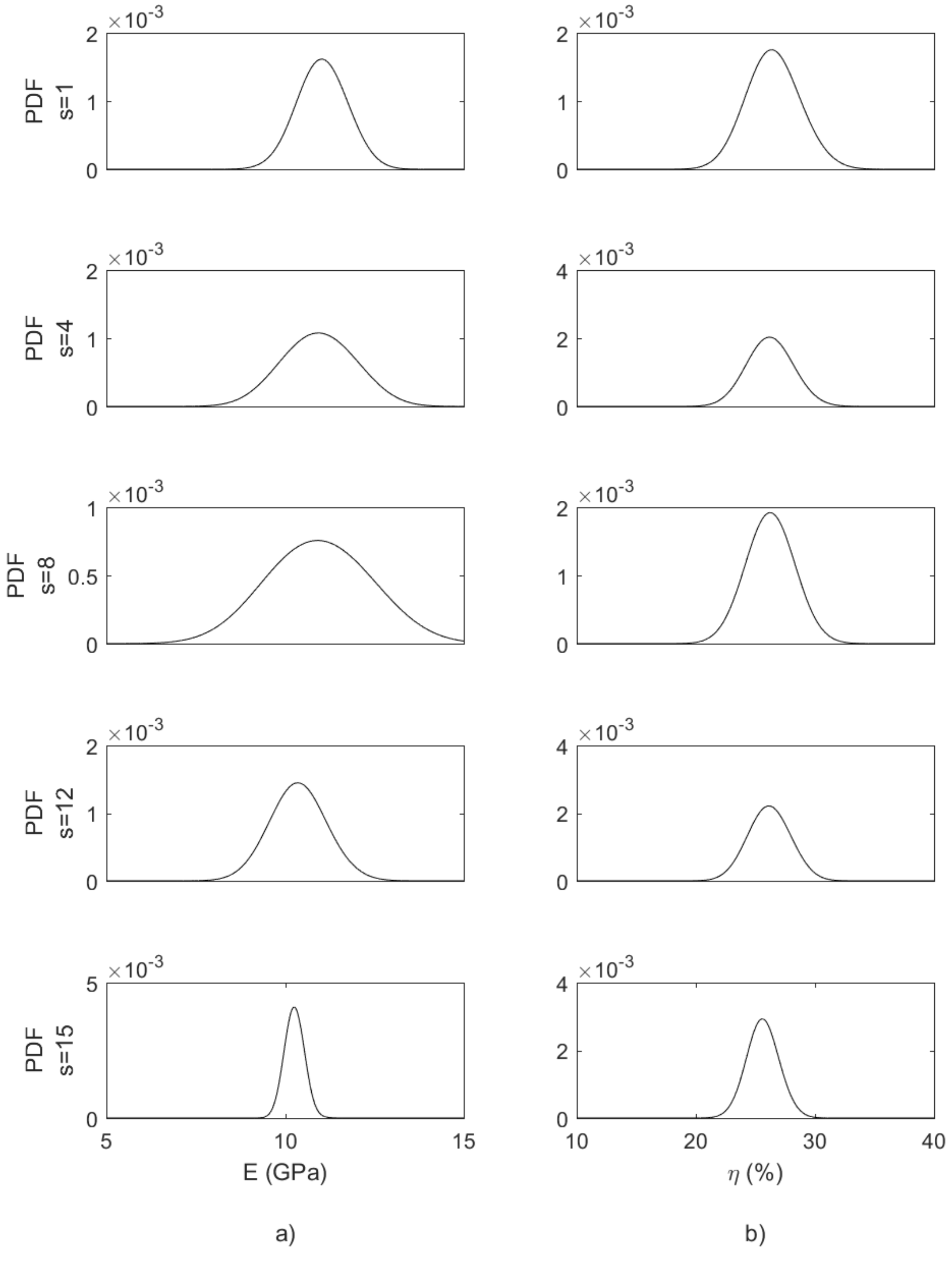

Figure 3 – Example of the updating PDFs for case b at the 125 Hz TOB; a) Elastic Modulus; b) Loss factor

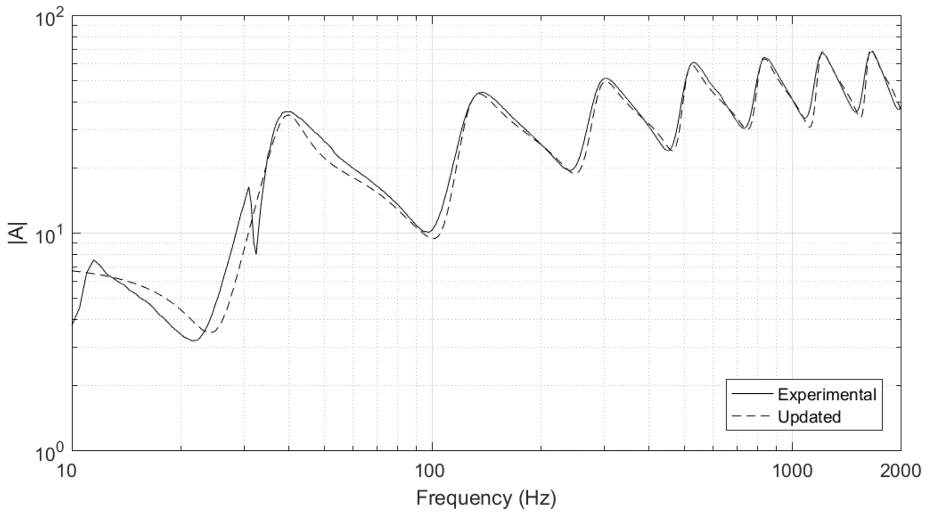

Figure 4 – Experimental vs final updated model – Case b

5. CONCLUSIONS

This paper showed a detailed explanation of the application of an adaptive Metropolis-Hastings method on dynamic model updating of the damping of a beam with viscoelastic material, correlating experimental and numerical data on the frequency domain. The results showed good correlation between experimental results and numerical model, allowing the model updating of damping of viscoelastic materials that are typically hard to model due to non-linear properties.

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7. RESPONSIBILITY NOTICE

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