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THE METHOD OF FINITE DIFFERENCES APPLIED TO DESCRIPTION OF A METALIC COMPONENT

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Abstract. *Temperature is one of the most important parameters to be monitored within the most different metallurgical processes. The temperature of process monitoring is described by the cooling curves, obtained through the instrumental of ingots with thermocouples of the most different types. The way in which metal material cooling curves occurs, describes indirectly the future mechanical properties. The process of cooling two different kinds of steel will be simulated in Matlab and C++ software, through the use of Method of Finite Differences to solve the General Equation of Heat Conduction, in transient regime. The objective of this article is to verify the differences on the cooling curves of the steel in to superficial coordinates when the material was submitted to cooling by natural convection and forced convection for ingots of the steel SAE 8620 with rectangle section*

Keywords: *Cooling curves, the Method of Finite Differences, Natural and Forced Convection*

1. INTRODUCTION

The heat transfer process is a physical phenomenon used to propagate energy from the higher to lower temperature regions. This behavior is important for the determination of the cooling parameters in a material. The Heat Conduction Equation is a mathematical expression of the conservation of energy in a solid substance (Kreith, 2014).

The Method of Finite Differences (MFD) is a numerical method based on discretization to solve complex problems involving differential equations, such as the conduction equation. It provides the solution in a finite number of discrete points within the boundary limits, providing an approximation of the exact solution. (Chapra et al., 2008). Through the approximation to the temperature values given by the General Heat Conduction Equation solved by MFD, it is possible to construct a thermal mesh for the monitoring of the cooling of the studied material.

The main objective of this work is to describe the cooling of the chosen steel through the natural convection by the atmosphere and the convection forced by ventilation, and to verify possible differences between the simulation and the experimental results. With this, it will be possible to approximate and minimize the error between experimental and simulated temperature values

2. MATERIALS AND METHODS

According to Incropera *et. al.* (2008), heat transfer involves three mechanisms and need different temperature conditions. The first is convection and occurs with the contact between a moving fluid and a surface. The conduction, the second, is characterized by the transfer of energy from the most energetic to the less energetic particles, due the interaction between them. The third mode of heat transfer is thermal radiation; its effects will be neglected in this study.

Convective transfer in fluid flows can have two distinct forms. For the case where there is presence of temperature gradient occurs forced convection, it is result of the movement of the fluid induced by a source of external impellent, such as a fan or a pump. Where there is no forced flow velocity, occurs the natural convection, this is generated by the forces of buoyancy in its exterior and it is due to the air circulation.

The Finite Differences Method is based on dividing the metal / mold system into small intervals of equal distances. In this way, a volumetric mesh composed of nodes with coordinates for a finite number of points is established, according to Fig. 1. It is used in the solidification modeling process of continuous casting of blocks, billets and plates, due to the simple geometric characteristics of these products

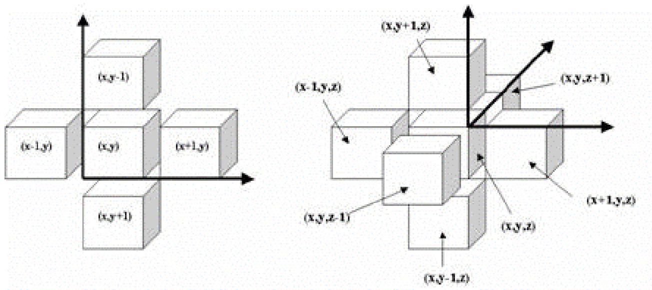


Figure 1. Reference of coordinates used in the Method of Finite Differences (MFD) (Trevisan, 2009).

The General Heat Conduction Equation, presented in Eq. (1), is used in the modeling of the heat transfer inside the ingot, from higher to lower temperature regions. It is expressed as a function of the process variables. As the object of study is a metallic plate, it is written as a function of the variables dx and dy - area differential as a function of time. According to Kreith *et. al.* (2014), the contour conditions used for resolution Eq. (1) may be specified surface temperature, where the air temperature inside the oven is attributed to edge temperatures, and surface convection, where the edge temperatures are calculated through the exchange of convective heat with air, which in this case acts as a cooling fluid.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

Equation (1) consists of the equality between the net heat conduction rate in the x and y coordinates and the rate of increase of the internal energy over time multiplied by the inverse of the thermal diffusivity of the material α.

$$\alpha = \frac{k}{\rho c} \tag{2}$$

The thermal diffusivity of the material indicates how fast the material reacts to the temperature change, is described in Eq. (2) and is determined by the thermal conductivity (k), density (ρ) and specific heat (c). These parameters depend on the quality of the studied steel and its properties, which are fundamental for obtaining values in the simulations closest to the real values. For the grades of steels studied, the values are described in Table 1.

Table 1. Thermal properties of the metal in the liquid / solid state.

Proprieties	**Value**
Thermal conductivity	29 [W/m.K]
Density	7800[kg/m³]
Specific heat	490 [J/kg.K]

According to Chapra *et. al.* (2008), the parabolic equations are used to calculate the distributions of an unknown, considering that it varies both in space and time. They are used to calculate temperature distributions on all nodes of the board. Through the MFD discretization process, where partial derivatives are replaced by finite divided differences, it is possible to solve Eq. (1).

$$\frac{\partial^2 T}{\partial d^2} = \frac{T_{(m,i+1)} - 2T_{(m,i)} + T_{(m,i-1)}}{\Delta d^2} \tag{3}$$

$$\frac{\partial T}{\partial t} = \frac{T_{(m+1,i)} - T_{(m,i)}}{\Delta t} \tag{4}$$

In equations Eq. (3) and Eq. (4) are respectively the approximations for second order temperature derivatives related to spatial variables and first order derivatives with respect to time. Applied to the General Heat Conduction Equation, they result in the equation Eq. (5) used to calculate the temperature of each node of the volumetric mesh.

$$T_{(m+1,i,j)} = T_{(m,i,j)} + \alpha * \Delta t * \left\{ \frac{T_{(m,i+1,j)} - 2T_{(m,i,j)} + T_{(m,i-1,j)}}{\Delta x^2} + \frac{T_{(m,i,j+1)} - 2T_{(m,i,j)} + T_{(m,i,j-1)}}{\Delta y^2} \right\} \quad (5)$$

To go through all nodes a coordinate system was used, where *m* represents the time elapsed after the initial time, and the letters *i* and *j* represent the *x* and *y* coordinates respectively. Eq. (5) is used to calculate the temperatures at the instant after the initial instant - *(m + 1)*, since the method uses a predetermined temperature distribution in its calculations.

To determine the temperature mesh in the numerical simulation for the specified temperature outline condition an initial temperature was assigned for all nodes, the contour temperatures are matched to the oven temperature values measured experimentally. By this the temperature distribution is calculated at the next instant, and iterations are performed until the temperature of the plate is stabilized according to the flow chart of Fig. (2).

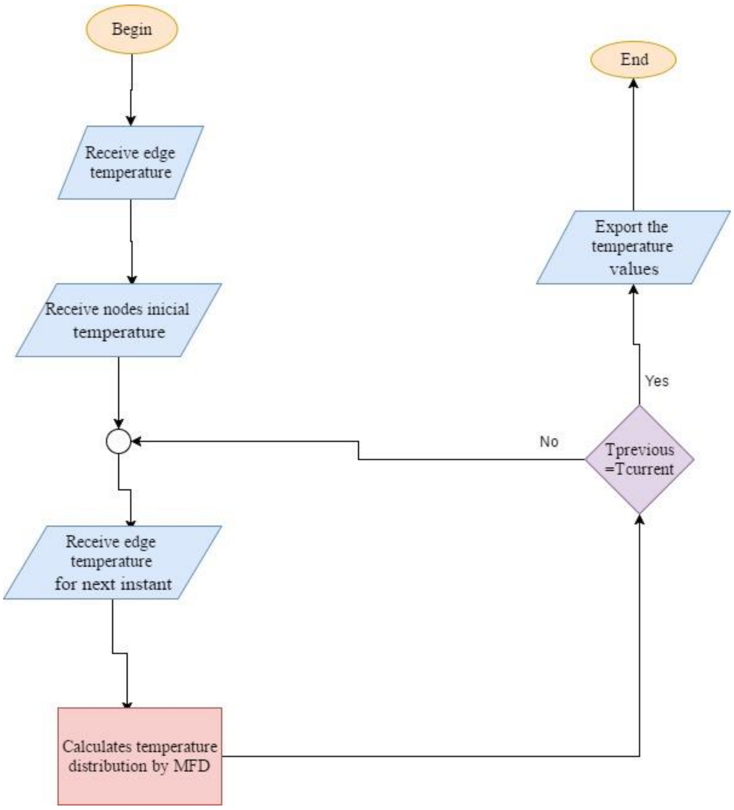


Figure 2. Resolution’s flowchart of MFD for contour condition as specified temperature.

The calculations of the edge temperatures for the surface convection contour condition are based on the adaptation of equation Eq. (5); such adaptation is carried out with the intention of including the term of the convective average heat exchange coefficient and the temperature value the air inside the oven. They need to take into account two different situations, as shown in Fig. (3).

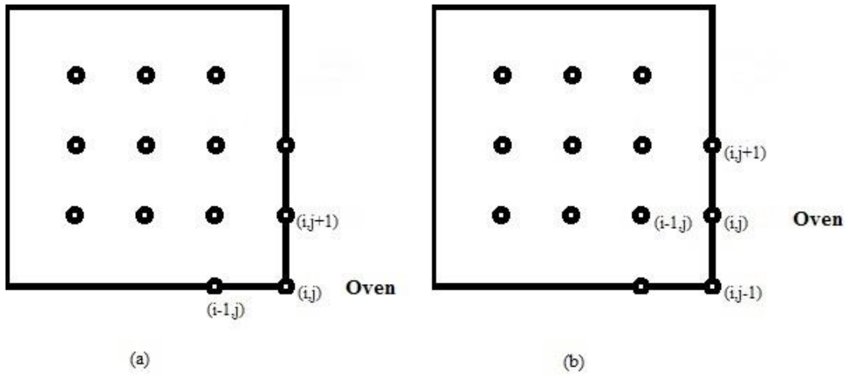


Figure 3. Contour condition of superficial convection.

The first situation, indicated by Fig (3a), is the temperature of the corners of the plate *(i, j)*, for example, the neighbors in the lower right corner are: the node immediately above *(i, j+1)*, the node immediately to the left *(i-1, j)* and the oven temperature value *(*T_∞*)*.The second one, shown in Fig. (3b), covers the temperature of the margins, in this case one of the values of the right edge *(i, j)*, which has the neighbors immediately above and below and the left, represented respectively by the coordinates *(i, j + 1)*, *(i, j-1)* and *(i-1, j)*, and the oven temperature value. Equations Eq. (6) and Eq. (7) describe these cases. Analogies are made to the other cases with the neighbors closest and it is possible to determine the temperatures for all the contour situations of the piece. The determination of the thermal mesh for this contour condition follows the same principle of resolution of the previous condition.

$$T_{(m+1,i,j)} = T_{(m,i,j)}\left(1-\frac{4\alpha\Delta t}{(\Delta x)^2}-\frac{4\bar{h}\Delta x}{k}*\frac{\alpha\Delta t}{(\Delta x)^2}\right)+\frac{2\alpha\Delta t}{(\Delta x)^2}\left\{T_{(m,i+1,j)}+T_{(m,i,j-1)}+\frac{2\bar{h}\Delta x T_\infty}{k}\right\} \tag{6}$$

$$T_{(m+1,i,j)} = T_{(m,i,j)}\left(1-\frac{4\alpha\Delta t}{(\Delta x)^2}-\frac{2\bar{h}\Delta x}{k}*\frac{\alpha\Delta t}{(\Delta x)^2}\right)+\frac{\alpha\Delta t}{(\Delta x)^2}\left\{2T_{(m,i+1,j)}+T_{(m,i,j+1)}+T_{(m,i,j-1)}+\frac{2\bar{h}\Delta x T_\infty}{k}\right\} \tag{7}$$

The convective heat exchange coefficient, found in the above equations, has different values depending on the type of convection, the properties of the fluid, the temperature at which it is subjected and the geometry of the plate. It was calculated according to Eq. (8), based on the Nusselt Number, which according to Incropera et.at. (2008), is the surface temperature gradient and can provide the amount of heat transfer between the surface and the cooling fluid.

$$\bar{h} = \frac{k_{fluid}\overline{Nu}}{\Delta x} \tag{8}$$

The flow regime of the fluid was considered Laminar, due to the low flow rates used in the experiments. The Nusselt Number was calculated from the equation Eq. (9), where the values of the properties used are described in table Tab. 2.

$$\overline{Nu} = \frac{0{,}664\overline{\Pr}V\Delta x}{\bar{\upsilon}} \tag{9}$$

Table 2. Thermal proprieties of the cooling fluid for convection coefficient calculation.

Proprieties	**Value**
Average number of Prandlt ($\overline{\Pr}$)	0.6924
Velocity to natural and forced convection (V)	10 - 20[m/s]
Average kinetic viscosity ($\bar{\upsilon}$)	50.638x10^6[m²/s]

As the MFD is based on approximations it is necessary to verify the convergence and stability of the same. According to Chapra *et.al.* (2008), convergence means that when the variations of space and time tend to zero, the results of the finite difference technique approximate the true solution and stability means that errors at any stage of resolution are attenuated as calculations progress. For both conditions to be satisfied it is necessary that the conditions expressed in Eq. (10) and Eq. (11) are respectively reached for the specified temperature and surface convection contour conditions. Through these equations maximum Δt was determined for each case, being its values approximately of 0.4949 seconds for specified temperature, 0,3773 seconds for forced convection and 0,3831 seconds for natural convection.

$$\frac{\alpha \Delta t}{(\Delta x)^2} \leq \frac{1}{2} \tag{10}$$

$$\left(2 + \frac{\bar{h}\Delta x}{k}\right) * \frac{\alpha \Delta t}{(\Delta x)^2} \leq \frac{1}{2} \tag{11}$$

With all the conditions and hypotheses established, the numerical simulations were performed in the C ++ programming language. Subsequently their results were plotted in Matlab software, to facilitate the understanding and comparison between them.

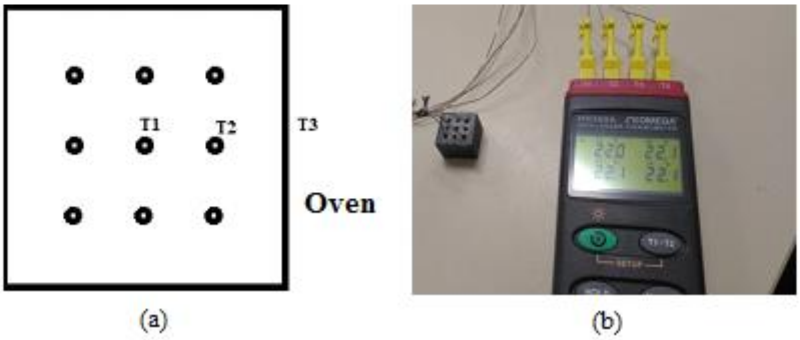


Figure 4. Model of plate used with arrangement of thermocouples

For the experimental procedure a square plate of 20 X 20 mm was used, and the depth dimension is irrelevant for the calculations. Nine holes were drilled for the insertion of the thermocouples, each approximately one millimeter in diameter. The amount and distance between perforations were based on the volumetric mesh of nodes used in the numerical simulation established by the method to maintain the stability and convergence thereof, where each perforation represents a coordinate point. The distance between nodes was Δx as five millimeters. The total area of the plaque was divided into twenty-five knots, being nine interior knots and sixteen knots in contours. Three thermocouples of type K-T1, T2 and T3 were used for the measurement of temperature, the arrangement of which is shown in Fig. (4a), and the data acquisition plate HH309A, which can be seen in Fig. Fig. (4b). Temperature readings were taken every thirty seconds.

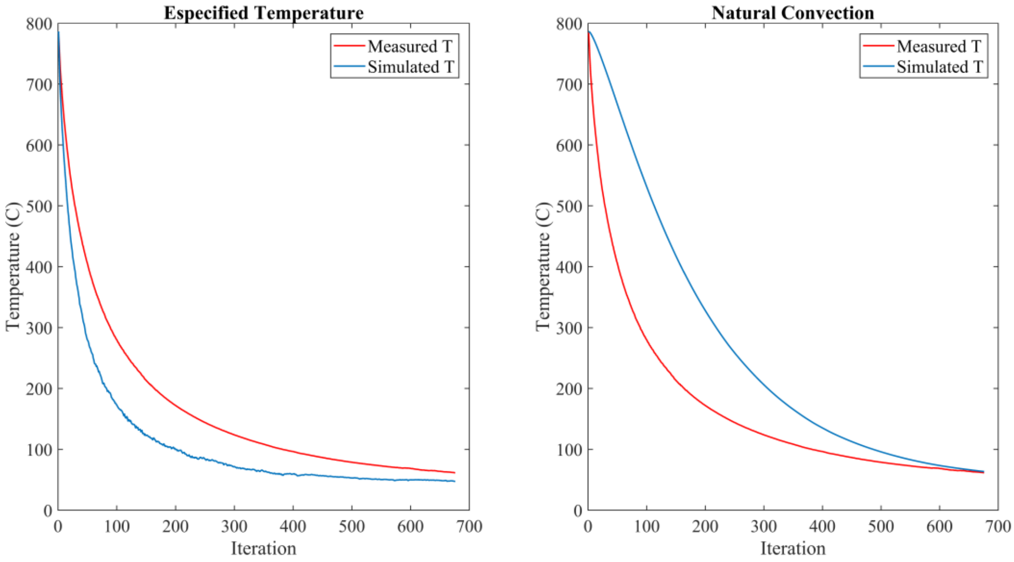
Figure 5. Experiment with the open door oven.

The furnace was heated to 800 ° C, the threshold temperature chosen because it was close to the austenitization temperature of the studied steel. The plate was subjected to this temperature for one or two hours, time required to occur to the thermal stabilization of the plate, ie all coordinates reached the threshold temperature.

The object of study was subjected to two different ways of cooling, the first consists of the natural convection, according to the figure Fig. (5), where the oven is turned off, soon after the opening of the oven door is made and begins the acquisition of the data. The second is forced convection, where the oven is switched off at the same time that the fan is driven in front of the open oven door.

3. RESULTS

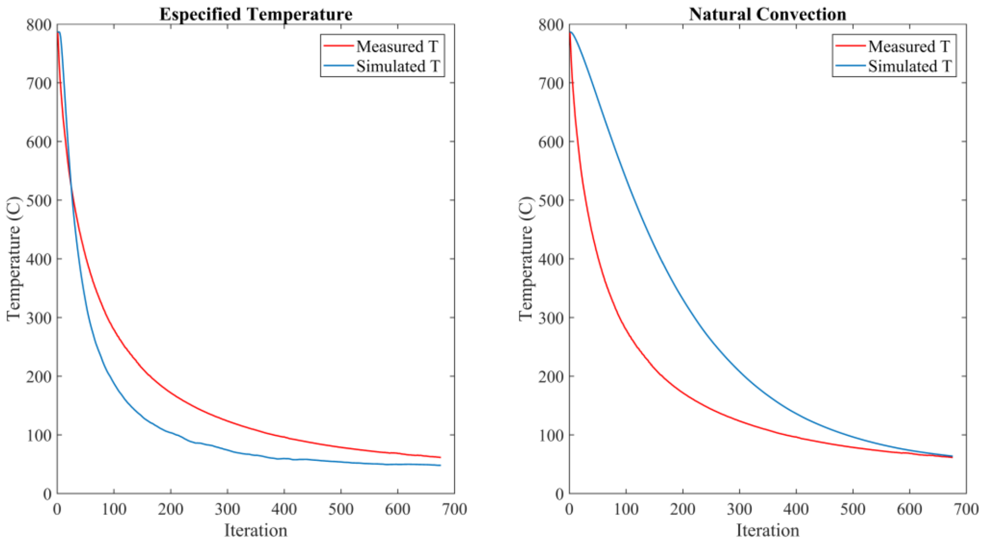
The numerical simulation was done for two distinct contour conditions, the first from the specified temperature, the second considering the heat exchange by convection. After the simulation the temperature values were compared with the experiment for the SAE 8620 ingot steel.



(a) (b)

Figure 6. Comparison between numerical simulation and experimental for the intermediate coordinate for open door

In Fig. (6) and Fig. (7), respectively for the intermediate and central coordinates of the plate, the values of temperature measured, in blue, and simulated, in red, respectively for the experiment performed with the oven door open. Fig. (6a) and Fig. (7a) show the simulations performed using the specified temperature values. It is possible to observe a smaller error in the temperature curves in the central coordinates in relation to the intermediate ones. The largest difference is approximately in the 200th iteration, and decreases as the number of iterations increases. Fig. (6b) and Fig. (7b) describe the simulation performed for natural convection as a boundary condition, which present a greater distance between the curves, when compared to the simulations of specified temperature values, but they converge more rapidly.



(a) (b)

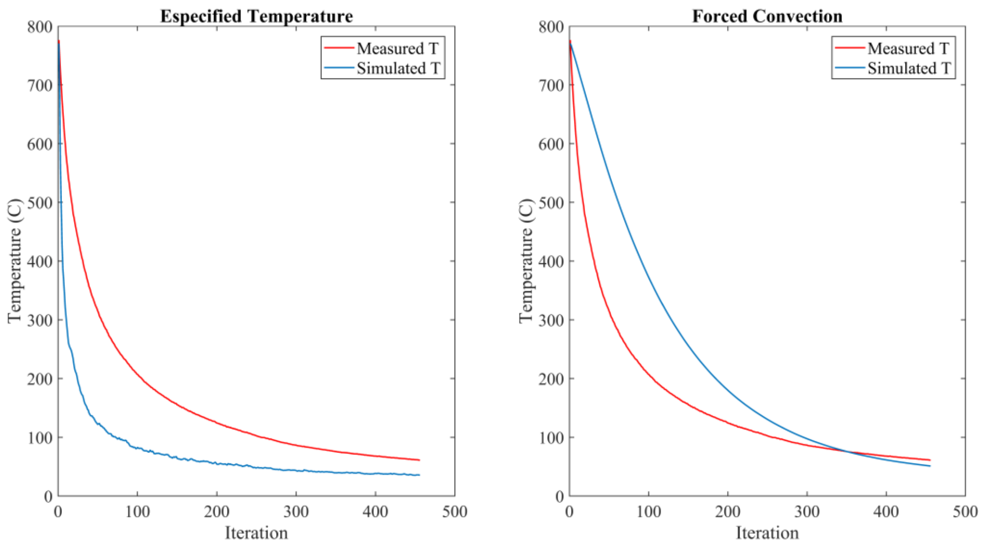
Figure 7. Comparison between numerical simulation and experimental for the central coordinate for open door

The best results obtained for the experiment with the oven door were found through the simulation with the contour conditions for the specified temperature. This occurs because the convective portion has possibly less influence than the conductive portion, which causes the contour temperatures to be very close to the cooling fluid temperatures.

Table 3. Difference between simulated and experimental values for different contour conditions for open port.

Difference between simulated and experimental values	Specified Temperature [°C]	Natural Convection [°C]
Maximun for intermediate node	122.108215	271.018524
Minimun for intermediate node	4.674988	1.993626
Maximun for central node	99.469116	277.293152
Minimun for central node	1.719421	2.890472

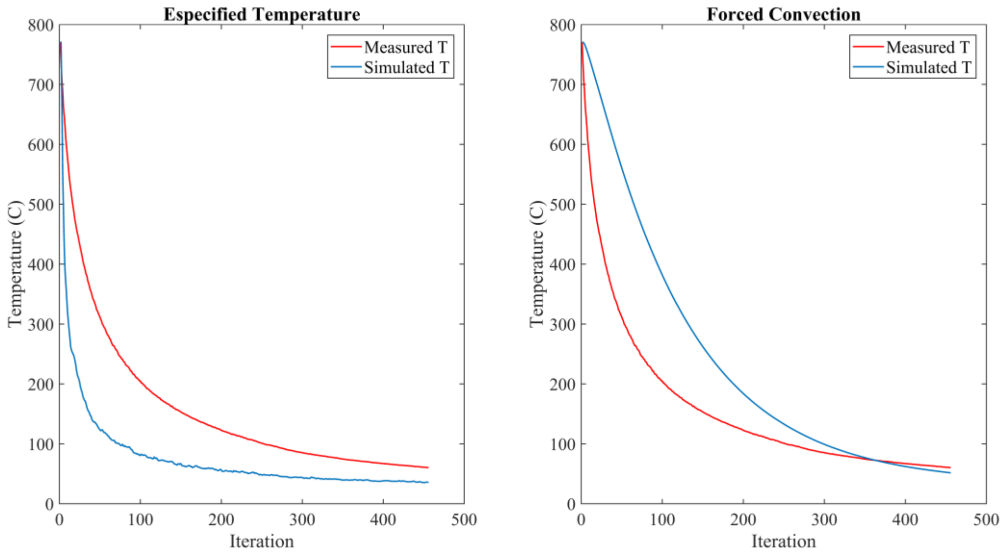
In Table 3 it is possible to check the maximum and minimum error for both types of numerical simulation. The largest error for this experimental was found through the contour condition with natural convection, more than double when compared to the other type of simulation. For the minimum error, there was no such relationship. The greatest error with the natural convection boundary condition can occur due to the low magnitude of the convective heat transfer coefficient and the low flow velocity of the cooling fluid.



(a) (b)

Figure 8. Comparison between numerical simulation and experimental for the intermediate coordinate for open door with ventilation

In Fig. (8) and Fig. (9), for the intermediate and central coordinates of the part, the values of measured and simulated temperature, in red and blue, for the experiment carried out for the door of the open kiln with fan inducing the convection. Fig. (8a) and Fig. (9a) present the simulations performed using the specified temperature values, they show a large difference between the measured and simulated values, this difference becomes even more pronounced when compared to the values obtained in the condition experimental model without fan, the graph was finalized before they reached convergence, but the two converged with time tending to infinity. Fig. (8b) and Fig. (9b) describe the simulation performed for forced convection as a boundary condition. They have a faster convergence.



(a) (b)

Figure 9. Comparison between numerical simulation and experimental for the central coordinate for open door with ventilation

For the experimental condition with the fan, the best results were calculated through the contour condition with forced convection. This is caused by the greater heat exchange between the plate and the cooling fluid, as the flow of this occurs in a forced manner, so that there is always a considerable difference in temperature between the plate and the surrounding atmosphere. The maximum and minimum values of temperature difference between simulation and experimental are described in Table 4, for both boundary conditions for the open door experiment with fan.

Table 4. Difference between simulated and experimental values for different contour conditions for open port with ventilation.

Difference between simulated and experimental values	Especified Temperature [°C]	Forced Convection [°C]
Maximun for intermediate node	271,196	243,09
Minimun for intermediate node	23,9	0
Maximun for central node	253,696	251,25
Minimun for central node	11,9	0

4. CONCLUSION

The knowledge about the cooling process through the determination of a thermal mesh in metallic components is of fundamental importance for the metalworking industry, since they allow the determination of cooling parameters in many industrial processes. With the knowledge about the cooling curves it is possible to adapt the parameters used and consequently the optimization of the properties of the components produced and their characteristics against the mechanical stresses. The mathematical description is fundamental to aid and minimize experiments, since equipment and instrumentation represent a high cost for the industry and is often unviable due to the high temperatures.

In this context, the finite difference method proved to be efficient; with the temperature condition specified for the open door and forced convection for open door with the fan. However, for cooling done in these molds, the heat transfer by convection is very high, which may have altered the simulated results and justify the considerable error values, above 200 ° C. This disrupts the scheduling and operation of the heat treatment furnaces, but few experiments were carried out for the adequacy of a numerical routine, several would be necessary in order to obtain average values. It is important to note that the heating and cooling condition with the open door is close to an annealing heat treatment widely used by the metal-mechanical industry.

In order to improve the simulation with the boundary conditions, both natural convection and forced convection, it would be necessary that the convective heat exchange coefficient was not used as an average value, but was calculated for each temperature inside the furnace, considering that this is one of the factors that influence the properties of the cooling fluid. For this, it would be necessary to call for other forms of numerical calculation, such as interpolation, since known values are limited to standard tables.

5. REFERENCES

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