24th COBEM - 2017

24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-1785
EXPERIMENTAL RESULTS ON POSE REGULATION, TRACKING AND FORMATION CONTROL OF NONHOLONOMIC ROBOTS USING POTENTIAL FUNCTIONS

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Abstract. *This work describes a control strategy that forces a group of nonholonomic mobile robots to achieve a formation while tracking some desired trajectory. The scheme uses a potential function that ensures that the formation is achieved. Then, a tracking component is added to the potential function strategy to force formation group to track a desired trajectory provided by one or two virtual leaders. If two virtual leaders are used, the pose of formation will be regulated. Experimental results confirm the efficiency of the proposed control strategy.*

Keywords: *Potential function, formation control, nonholonomic robots, trajectory tracking, pose regulation*

1. INTRODUCTION

Formation control of multi-agent systems has received significant attention due to its wide variety of applications. Recent works show its application in areas like naval engineering (Cui *et al.*, 2010) and aerospace engineering (Abdessameud and Tayebi, 2011). Among several formation control strategies used, we can mention the behavior-based (Antonelli *et al.*, 2010), consensus (Ren *et al.*, 2007; Li *et al.*, 2011), leader-following (Tanner *et al.*, 2004; Chen *et al.*, 2010), group coordination using passivity (Arcak, 2007; Franchi *et al.*, 2011), virtual structures (Beard and Hadaegh, 1998; van den Broek *et al.*, 2009) and potential function (Leonard and Fiorelli, 2001; Hengster-Movrić *et al.*, 2010).

For the formation control of mobile robots, the main objective is to control each agent using neighbor information in a decentralized control strategy. In this framework, most of the existing results deal with holonomic mobile robots (Pereira *et al.*, 2009; Xiao and Wang, 2008; Tanner *et al.*, 2007). However, in practical applications, mobile robots have to satisfy nonholonomic constraints.

The control design for nonholonomic systems is quite involved, mainly due to the Brockett's condition. Therefore, for agents with nonholonomic constraints, the formation control problem becomes more challenging.

In (Tanner *et al.*, 2004), stability properties of formation of mobile agents based on leader-following are investigated. In (Dierks and Jagannathan, 2007), a combined kinematic/torque control law is proposed using leader-follower strategy and backstepping control. In (Mastellone *et al.*, 2007), a decentralized control scheme which achieves dynamic formation control and collision avoidance for a group of nonholonomic robots with kinematic model is proposed. The collision avoidance strategy is based on locally defined potential functions which can take different shapes and only require each agent to detect other objects in its neighborhood. In (Dong and Farrell, 2009), a decentralized feedback control of a group of nonholonomic dynamic systems with uncertain parameters is considered. The control scheme is based on consensus, graph theory, and backstepping techniques. In (Consolini *et al.*, 2008, 2009) a geometric approach for the stabilization of a hierarchical formation of unicycles with velocity and curvature constraints is proposed, and a leader-following strategy is suggested. However, collision avoidance is not considered. Furthermore, a drawback of leader-following strategies is that it depends heavily on the leader for achieving the goal and over-reliance on a single agent in the formation may be undesirable, especially in adverse conditions. In (Gouvea *et al.*, 2010), an adaptive formation control for nonholonomic mobile robots with unknown dynamic parameters is proposed. The control scheme is based on a saturated artificial potential function which allows a decentralized formation control design including collision avoidance. A formation control with trajectory tracking is proposed in (Lima *et al.*, 2014), where, as well as in (Gouvea *et al.*, 2010), a saturated potential function is used. Using the Lyapunov formalism, it is shown that the a desired trajectory defined by virtual leaders is tracked while the formation is maintained with a residual error. However, only simulation results are presented and confirm the effectiveness of proposed control law.

In this paper, using three unicycle mobile robots, experimental results of the formation control law proposed in (Lima *et al.*, 2014) is presented. As the simulations results shown by authors this paper in (Lima *et al.*, 2014), the experimental results presented here confirm the effectiveness of proposed control law. Furthermore, it is shown that, if at least two virtual leaders is used, the formation and its orientation will be maintained while the desired trajectory is tracked.

It is important to stress that the formation and tracking control laws proposed in this work is decentralized, namely, there is not a central authority controlling the formation. In this case, the robots are autonomous and the control of each robot is calculated using only its own position information and of its neighbor robots.

This paper is organized as following. The Section 2 presents a description of the formation control with trajectory tracking problem. Some definitions are introduced as a requirement to understand the following sections. In section 3, the used formation control law is described. The section 4 approaches the tracking problem. In section 5 the controls laws proposed in sections 3 and 5 are used to propose a trancking and formation control law. A stability analysis using the Lyapunov formalism is presented, ensuring both formation convergence and trajectory tracking. The experimental results are shown in section 6. Finally, the conclusions are presented in section 7.

2. Problem formulation

Consider a team of N nonholonomic mobile robots. For $i = 1, \cdots N$, the kinematic model of each robot is described by

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} -\sin\theta_i & 0 \\ \cos\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_i \\ \omega_i \end{bmatrix}, \tag{1}$$

where x_i, y_i are the cartesian coordinates, θ_i is the orientation and u_i, ω_i are the linear and angular velocities respectively. The Figure 1 illustrates a robot of the team.

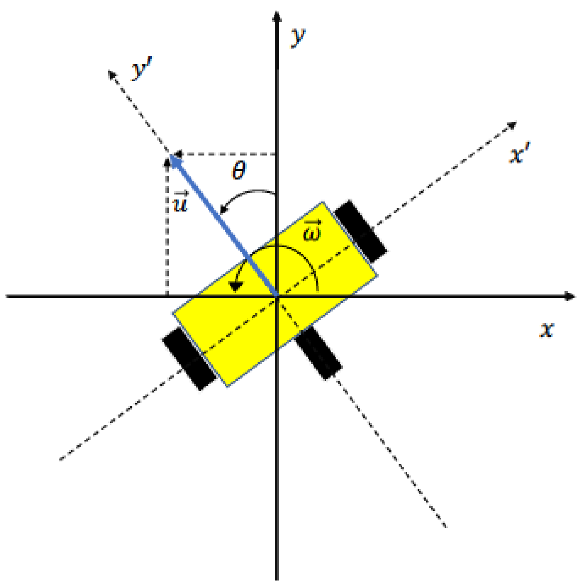

Figure 1. Nonholonomic vehicle

Since the control signal of robots are the angular velocities in the wheels, then it should be used the following transformation matrix

$$\begin{bmatrix} u_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & -\frac{r}{2} \\ -\frac{r}{L} & -\frac{r}{L} \end{bmatrix} \begin{bmatrix} \omega_{ei} \\ \omega_{di} \end{bmatrix}, \tag{2}$$

where ω_{ei} and ω_{di} are the angular velocities of the left and right wheels respectively.

The topology of information exchange among robots is described by a graph (Biggs, 1994). Then, the N mobile robots are represented as N vertices of a graph $G := \{V, E\}$, where $V := \{v_1...v_N\}$ is the set of vertices that represent the robots and $E \subseteq V \times V$ is the set of edges that define the neighborhood relationship among robots. Thus, the set of neighbors of robot i is $\mathcal{N}_i := \{j | e_{ij} = (v_i, v_j) \in E\}$. The available information for the controller of robot i is only the states of robot i and robot j for $j \in \mathcal{N}_i$. A path of length r from robot i to robot j is a sequence of $r+1$ distinct vertices starting with i and ending with j such that consecutive vertices are neighbors. If there is a path between any two vertices of a graph $\mathcal{G}$, then $\mathcal{G}$ is said to be connected. A graph is undirected if the edges have no orientation ($(i,j) = (j,i) \in \mathcal{E}$). In this paper, we assume that the formation graph is connected and undirected.

The control objective is to drive the N agents to a formation which minimizes

$$V = \sum_{i=1}^{N} V_i \tag{3}$$

with

$$V_i = \sum_{i,j \in \mathcal{N}_i}^{N} V_{ij}(\|r_{ij}\|) \tag{4}$$

where $r_i^T = [x_i, y_i]$, $r_{ij} = r_i - r_j$ and V_{ij} is defined as follows:

Definition 1 *The saturated potential function V_{ij} is a differentiable, nonnegative function of the distance $\|r_{ij}\|$ between agents i and j, such that*

1. $V_{ij}(\|r_{ij}\|) \to \infty$ *as* $\|r_{ij}\| \to c$, *where* $c > 0$.
2. V_{ij} *attains its unique minimum when agents i and j are located at a desired relative position* $r_d > c$.
3. $V_{ij}(\|r_{ij}\|) = V_{ij}(R_s)$ *if* $\|r_{ij}\| \geq R_s$, *where* $R_s > r_d$.

Furthermore, the achieved formation should be maintained while a desired trajectory defined by one or more virtual leaders is tracked. The Figure 2 shows an examples of saturated potential function. The formation control is addressed in the next section and the trajectory tracking problem will be approached in section 4.

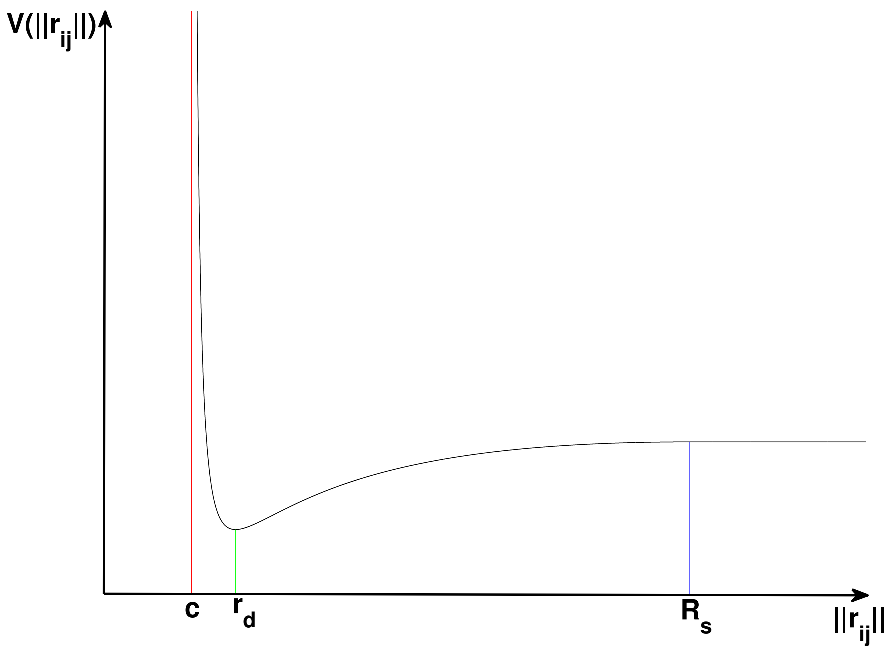

Figure 2. Saturated potential function

3. Formation control law

Initially, it will be addressed only the formation control to make understanding easier. The aims of formation control are

- achieve a formation that minimizes the potential function (3),
- avoid collision between robots while the formation evolves.

Then, are created artificial fields among robots defined as

$$f_{ij} = -\nabla_{r_i} V_{ij}(\|r_{ij}\|)$$

where $\nabla_{r_i} V_{ij}(\|r_{ij}\|)$ is the gradient vector of V_{ij}.

Note that f_{ij} is an attraction field if $\|r_{ij}\| > r_d$, or a repulsion field if $\|r_{ij}\| < r_d$. Field f_{ij} is null for $\|r_{ij}\| = r_d$ and $\|r_{ij}\| > R_s$. Thus, to reach the minimum of V_i, each robot should move in the direction of resulting artificial field

$$f_i = \sum_{j \in \mathcal{N}_i, j \neq i} f_{ij} = - \sum_{j \in \mathcal{N}_i, j \neq i} \nabla_{r_i} V_{ij}(\|r_{ij}\|) = -\nabla_{r_i} V_i.$$

Since, from Definition 1, $V_{ij}(\|r_{ij}\|) \to \infty$ as $\|r_{ij}\| \to c$, robots dimensions can be taken into account for the formation control. On the other hand, R_s defines the neighborhood region of each robot. Then, agent j belongs to $\mathcal{N}_i$ if and only if $\|r_{ij}\| < R_s$. Thus, it possible to consider a region of limited communication about each agent, which allows a decentralized control design.

It is important to stress that the saturated potential function generates a switching interconnection topology. Nevertheless, in this paper the communication graph is always assumed to be connected.

Hence, similar to Mastellone *et al.* (2007), it can be proposed the following control law for each robot

$$u_{fi} = -k_g f_i R_i = -k_g \left(-\frac{\partial V_i}{\partial x_i} \sin(\theta_i) + \frac{\partial V_i}{\partial y_i} \cos(\theta_i) \right) \tag{5}$$

$$\omega_{fi} = -k_a \left(\theta_i - \theta_{id}\right), \tag{6}$$

where $k_g > 0$, $k_a > 0$, $R_i = [-\sin(\theta_i),\ \cos(\theta_i)]^T$ and

$$\theta_{id} = \arctan 2 \left(-\frac{\partial V_i}{\partial y_i}, -\frac{\partial V_i}{\partial x_i} \right). \tag{7}$$

Note that u_{fi} is the projection of f_i in the space of directions of motion allowed by the nonholonomic constraint and w_{fi} drives the robot orientation θ_i to θ_{id}, which is the direction of resulting artificial field f_i (descendant gradient of V_i).

The following lemma shows that the closed loop system of each robot described by (1), (5) and (6) ($u_i = u_{fi}$ and $\omega_i = \omega_{fi}$) assures that the formation reaches a configuration that locally minimizes (3).

Lemma 1 *Consider a formation of N robots with kinematic model described by (1). If the communication graph is connected and undirected, then the control laws (5) and (6) assure that the formation converges to a configuration that locally minimizes (3).*

Proof: Consider the Lyapunov candidate function

$$2W_1 = \sum_{i=1}^{N} \left[V_i + \alpha \left(\theta_i - \theta_{id}\right)^2 \right] \tag{8}$$

where $\alpha > 0$. Define $r_i^T = [x_i \quad y_i]$. Due to V_i being symmetric with respect to $r_{ij} = r_i - r_j$ and the fact that $r_{ij} = -r_{ji}$ (undirected graph), $\frac{\partial V_{ij}}{\partial r_i} = -\frac{\partial V_{ij}}{\partial r_j}$. Then, the derivative of (8) with respect to time is

$$\dot{W}_1 = \sum_{i=1}^{N} \left[\nabla_{ri} V_i \dot{r}_i + \alpha \left(\theta_i - \theta_{id}\right) \left(\dot{\theta}_i - \dot{\theta}_{id} \right) \right]. \tag{9}$$

However, of (1) and (5) we have

$$\dot{r}_i = \begin{bmatrix} -\sin(\theta_i) \\ \cos(\theta_i) \end{bmatrix} \nabla_{ri} V_i \begin{bmatrix} -\sin(\theta_i) \\ \cos(\theta_i) \end{bmatrix}. \tag{10}$$

Therefore, replacing (10) and (6) in (9) and after some algebraic manipulation, we have

$$\begin{aligned}\dot{W}_1 = \sum_{i=1}^{N}\Bigg\{ &-k_g\left(-\frac{\partial V_i}{\partial x_i}\sin(\theta_i)+\frac{\partial V_i}{\partial y_i}\cos(\theta_i)\right)^2-\alpha k_a\left(\theta_i-\theta_{id}\right)^2+\\ &+\alpha L_{1_i}\sum_{j=1}^{N}L_{2_{ij}}\left(\theta_i-\theta_{id}\right)\left(-\frac{\partial V_i}{\partial x_i}\sin(\theta_i)+\frac{\partial V_i}{\partial y_i}\cos(\theta_i)\right)\Bigg\}\end{aligned} \tag{11}$$

where $\dot{\theta}_{id}$, L_{1_i} and $L_{2_{ij}}$ are described by

$$\dot{\theta}_{id} = L_{1i}\sum_{j=1}^{N}L_{2ij}u_j, \tag{12}$$

$$L_{1i} = \frac{1}{\left(\frac{\partial V_i}{\partial x_i}\right)^2+\left(\frac{\partial V_i}{\partial y_i}\right)^2}, \tag{13}$$

$$\begin{aligned}L_{2ij} = &\frac{\partial V_i}{\partial x_i}\left(\frac{\partial^2 V_i}{\partial y_i\partial y_j}\cos(\theta_j)-\frac{\partial^2 V_i}{\partial y_i\partial x_j}\sin(\theta_j)\right)+\\ &\frac{\partial V_i}{\partial y_i}\left(\frac{\partial^2 V_i}{\partial x_i\partial x_j}\sin(\theta_j)-\frac{\partial^2 V_i}{\partial x_i\partial y_j}\cos(\theta_j)\right).\end{aligned} \tag{14}$$

Then,

$$\dot{W}_1 = \sum_{i=1}^{N}\left[-k_g\bar{e}_{1i}^2-\alpha k_a\bar{e}_{2i}^2+\alpha L_{1i}\sum_{j=1}^{N}L_{2ij}\bar{e}_{2i}\bar{e}_{1j}\right]$$

where $\bar{e}_{1i} = \left(-\frac{\partial V_i}{\partial x_i}\sin(\theta_i)+\frac{\partial V_i}{\partial y_i}\cos(\theta_i)\right)$ and $\bar{e}_{2i} = (\theta_i-\theta_{id})$. Defining $e_\theta^T = [\bar{e}_{21}\cdots\bar{e}_{2N}]$, $e_{\Delta r}^T = [\bar{e}_{11}\cdots\bar{e}_{1N}]$ and

$$L_a = \begin{bmatrix} L_{11}L_{211} & L_{11}L_{212} & \cdots & L_{11}L_{21N}\\ L_{12}L_{221} & L_{12}L_{222} & \cdots & L_{12}L_{22N}\\ \vdots & & \ddots & \vdots\\ L_{1N}L_{2N1} & L_{1N}L_{2N2} & \cdots & L_{1N}L_{2NN}\end{bmatrix}, \tag{15}$$

$$\dot{W}_1 = -\left(\alpha k e_\theta^T e_\theta - \alpha k_v e_\theta^T L_a e_{\Delta r} + k_v e_{\Delta r}^T e_{\Delta r}\right).$$

Then,

$$\dot{W}_1 = -\bar{e}^T\overbrace{\begin{bmatrix}\alpha k_a I_{N\times N} & -\frac{1}{2}\alpha k_g L_a\\ -\frac{1}{2}\alpha k_g L_a^T & k_g I_{N\times N}\end{bmatrix}}^{C_1}\bar{e}$$

where $\bar{e}^T = [\;e_\theta^T \quad e_{\Delta r}^T\;]$. Hence, using the Schur's complement, it can be concluded that $\exists\ \alpha > 0$ such that $\dot{W}_1 < 0$. Therefore, $\nabla_{r_i}V_iR_i \to 0$ and $(\theta_i-\theta_{id}) \to 0$. As θ_{id} is given by (7), then $\nabla_{r_i}V_iR_i > 0\ \forall\ \frac{\partial V_i}{\partial y_i}, \frac{\partial V_i}{\partial x_i} \neq 0$. Then $\frac{\partial V_i}{\partial y_i}, \frac{\partial V_i}{\partial x_i} \to 0$ and all robots converge to a formation that locally minimizes (3).

■

4. Trajectory tracking control law

In this section, is addressed the control law that ensures the tracking of trajectories defined by a virtual leaders.

Consider the Figure 3, where a virtual leader moves on xy-plane with linear and angular velocities u_{rlv} and ω_{rlv} respectively. Define x_{rlv}, y_{rlv} and θ_{rlv} as the position and orientation of virtual leader. Thus, considering that the kinematic model of virtual leader is described by (1), $u_i = u_{rlv}$, $\omega_i = \omega_{rlv}$ and given the initial conditions $x_{rlv}(0)$, $y_{rlv}(0)$ and $\theta_{rlv}(0)$, a desired trajectory can be defined. Then, the trajectory tracking problem consists in to assure that the following agent achieves the position and orientation of its virtual leader. Therefore, as shown in Figure 4, define the position error as

$$E_p = \begin{bmatrix} x_{rlvi}-x_i\\ y_{rlvi}-y_i\end{bmatrix} \tag{16}$$

and the vector orthogonal to robot orientation R_i as $R_{ni} = [\cos(\theta_i), \sin(\theta_i)]^T$. Then, the proposed tracking control law is

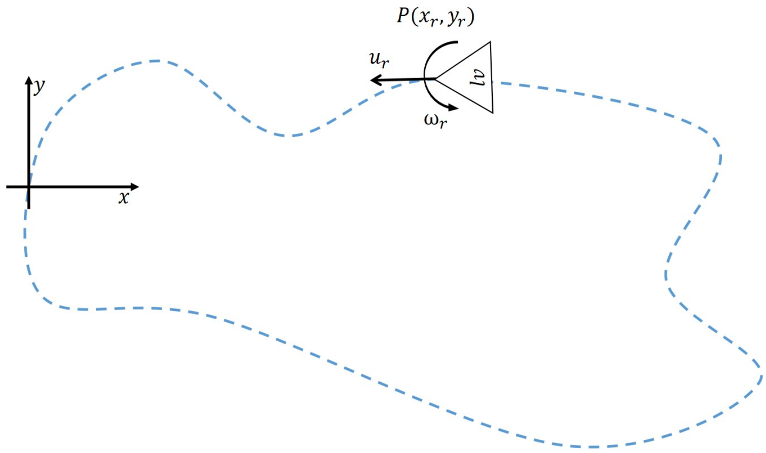

Figure 3. Trajectory defined by a virtual leader

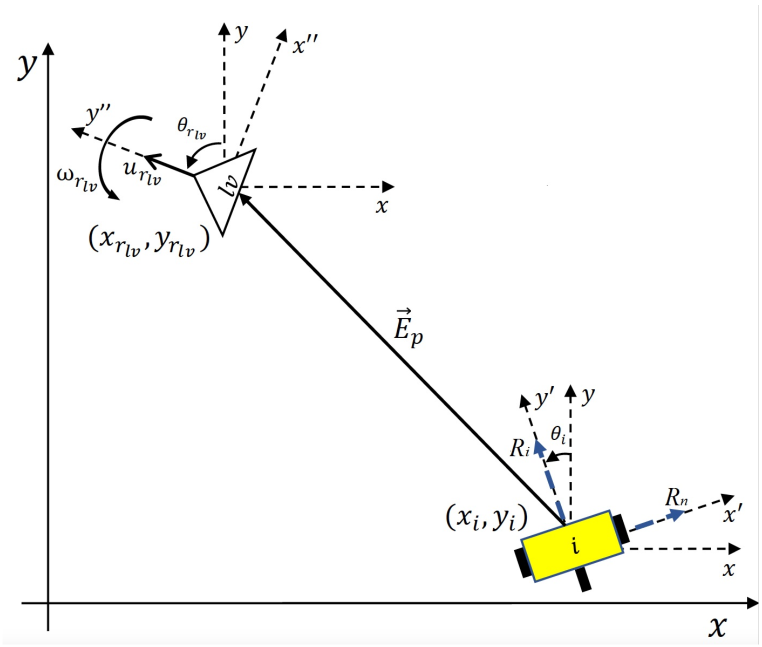

Figure 4. Vectors used to tracking trajectory.

$$u_{ti} = k_1 E_p^T R_i + u_{rlv} \cos(\theta_{rl} - \theta_i), \tag{17}$$

$$\omega_{ti} = -k_2 u_{rlv} E_p^T R_{ni} + k_3 \sin(\theta_{rlv} - \theta_i) + \omega_{rlv}. \tag{18}$$

In (Fukao *et al.*, 2000) it is shown that the control laws (17) and (18) ensures the convergence of the following robot to the position and orientation of its virtual leader.

5. Formation and tracking control law

In Sections 3 and 4 was presented the formation control law and the tracking control law respectively. Here, these control laws are used in one unique formation and tracking control law described, for each robot, by

$$u_i = u_{fi} + u_{ti} \tag{19}$$

$$\omega_i = \omega_{fi} + \omega_{ti} \tag{20}$$

for the robots tracking the m virtual leaders and $(i = m + 1, ..., N)$

$$u_i = u_{fi} \tag{21}$$

$$\omega_i = \omega_{fi} \tag{22}$$

for the others robots of formation.

The following Theorem shows that the closed loop system of each robot described by (1), (19) and (20) assures that the formation tracks a desired trajectory defined by one or more virtual leaders while maintains a configuration defined by a residual set around of minimum of potential function (3).

Theorem 1 *Consider a group of N robots modeled by (1), with formation control laws given by (19), (20), (21) and (22) and following $m \leq N$ virtual leaders. For sufficiently large k_g,k_a,k_1, k_2 and k_3 and assuming $|\omega_{rlv}|, |v_{rlv}| < l_r$, for some positive constant l_r, then there exists α such that*

- $|E_p| \to O(\alpha)$ *as* $t \to \infty$ *for* $i = 1, ...m$
- *all the closed loop signals are limited and a formation corresponding to residual value* $\nabla_V = O(\alpha)$ *is achieved asymptotically.*
- *For* $m \geq 2$ *the formation orientation converges to the orientation defined by the* m *virtual leaders.*

Proof: Consider the Lyapunov candidate function

$$W = W_1 + \alpha \sum_{i=1}^{m} \left[\frac{1}{2}(e_{1i}^2 + e_{2i}^2) + \frac{1 - \cos(e_{3i})}{k_2} \right] \tag{23}$$

where W_1 is described by (8) and (Fukao *et al.*, 2000)

$$\begin{bmatrix} e_{1i} \\ e_{2i} \\ e_{3i} \end{bmatrix} = \begin{bmatrix} -\sin(\theta_i) & \cos(\theta_i) & 0 \\ \cos(\theta_i) & \sin(\theta_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{rlvi} - x_i \\ y_{rlvi} - y_i \\ \theta_{rlvi} - \theta_i \end{bmatrix}. \tag{24}$$

Note that e_{1i}, e_{2i} and e_{3i} satisfy

$$\begin{bmatrix} \dot{e}_{1i} \\ \dot{e}_{2i} \\ \dot{e}_{3i} \end{bmatrix} = u_i \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \omega \begin{bmatrix} e_2 \\ -e_1 \\ -1 \end{bmatrix} + \begin{bmatrix} u_{rlvi}\cos(e_3) \\ u_{rlvi}\sin(e_3) \\ w_{rlvi} \end{bmatrix}. \tag{25}$$

Differentiating W, using (25) and the control laws (19), (20), (21) and (22) one can conclude that

$$\begin{aligned} \dot{W} \;=\; & \textstyle\sum_{i=1}^{N} \left[-k_g \bar{e}_{1i}^2 - \alpha k_a \bar{e}_{2i}^2 + \alpha L_{1i} \sum_{j=1}^{N} L_{2ij} \bar{e}_{2i} \bar{e}_{1j} \right] + \sum_{i=1}^{m} \left[-\bar{k}_1 e_{1i}^2 - \frac{\bar{k}_3 \sin(e_{3i})^2}{k_2} \right] + \\ & \alpha \textstyle\sum_{i=1}^{m} \left[u_{rlvi} \left(e_{1i}\cos(e_{3i}) + e_{2i}\sin(e_{3i}) \right) \right] + \\ & \alpha \textstyle\sum_{i=1}^{m} \left[L_{1i} \sum_{j=1}^{m} u_{rlvj} L_{2ij} \bar{e}_{2i} \cos(e_{3j}) + \bar{e}_{2i}\omega_{rlvi} + u_{rlvi}\bar{e}_{2i} k_2 e_{2i} \right] \end{aligned} \tag{26}$$

where $\bar{k}_1 = \alpha k_1$ and $\bar{k}_3 = \alpha k_3$. Then, defining $e_1^T = [e_{11} \cdots e_{1m}]$, $e_2^T = [\sin(e_{31}) \cdots \sin(e_{3m})]$, one can conclude that

$$\begin{aligned} \dot{W} \;=\; & -\bar{e}^T \overbrace{\begin{bmatrix} \alpha k_a I_{N\times N} & -\frac{1}{2}\alpha k_g L_a \\ -\frac{1}{2}\alpha k_g L_a^T & k_g I_{N\times N} \end{bmatrix}}^{C_1} \bar{e} - \bar{k}_1 e_1^T e_1 - \frac{\bar{k}_3}{k_2} e_3^T e_3 + \\ & \alpha \textstyle\sum_{i=1}^{m} \left[u_{rlvi} \left(e_{1i}\cos(e_{3i}) + e_{2i}\sin(e_{3i}) \right) \right] + \\ & \alpha \textstyle\sum_{i=1}^{m} \left[L_{1i} \sum_{j=1}^{m} u_{rlvj} L_{2ij} \bar{e}_{2i} \cos(e_{3j}) + \bar{e}_{2i}\omega_{rlvi} + u_{rlvi}\bar{e}_{2i} k_2 e_{2i} \right] \end{aligned} \tag{27}$$

Hence, since u_{rlvj} and ω_{rlvj} are bounded and using the Schur's complement, it can be concluded that, $\forall k_a, k_g, k_3, k_2, k_1 > 0$, $\exists\ \alpha > 0$ such that $\|\bar{e}\|$, $\|e_1\|$ and $\|e_3\|$ tend to a residual set of order $O(\alpha)$. Furthermore, this residual set can be made arbitrarily small by increasing k_a,k_g, k_3 and k_1. Note that $\|e_1\| \to 0$ and $\|e_3\| \to 0$ implies $\|E\|_p \to 0$. Then, for $m \geq 2$, the formation orientation converges to the desired orientation with a residual error of order $O(\alpha)$

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6. Experimental results

The experimental results was implemented using three nonholonomics robots with differential driver (see Figure 5). Each Robot has embedded a single-board controller of National Instruments where the trancking and formation control laws were implemented. Initially, the experimental results with only one virtual leader is presented. Soon after, the case

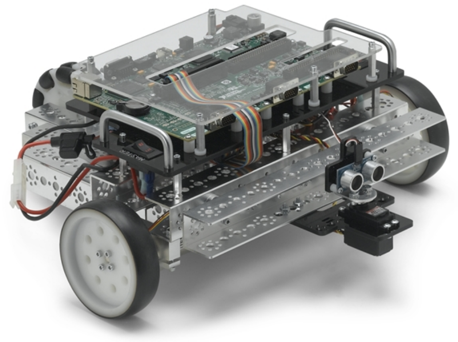

Figure 5. Robot used for the experimental results.

k_g	11000
k_a	0.25
r_d	80cm
R_s	500cm
c	35cm

Table 1. Formation control parameters: one virtual leader.

u_{rlv}	15cm/s
ω_{rlv}	-0.1rad/s
k_1	1.5
k_2	0.035
k_3	1

Table 2. Tracking control parameters: one virtual leader

with two virtual leaders is addressed. However, in both the cases, it is used the potential function described below:

$$V_{ij}(||r_{ij}||) = \begin{cases} J(||r_{ij}||), & se \quad ||r_{ij}|| < R_s \\ J(R_s), & se \quad ||r_{ij}|| \geq R_s \end{cases}, \tag{28}$$

where

$$J(||r_{ij}||) = \ln(||r_{ij}|| - c) + \frac{a_2}{||r_{ij}|| - c} - a_1(||r_{ij}|| - c), \tag{29}$$

$$a_1 = \frac{1}{r_d - 2c + R_s}, \quad a_2 = \frac{(R_s - c)(r_d - c)}{r_d - 2c + R_s}.$$

Since the formation has three robots, the configuration that minimizes the potential function has a triangular form.

6.1 Tracking and formation control: one virtual leader

Here, the experimental results are presented where it was considered only one virtual leader. The Tables 1 and 2 show the used parameters of formation and tracking controller. The inital conditions $C_i(x_i(0), y_i(0), \theta_i(0))$ of robots 1, 2 and 3 are $C_1(0,0,0)$, $C_2(125,0,0)$ and $C_3(0,-125,0)$ respectively. The inital condition $C_{lv1}(x_{rlv}, y_{rlv}, \theta_{rlv})$ of unique virtual leader linked of robot 1 is $C_{lv1}(0,0,0)$. The Figure 6 shows the robots trajectories until the formation be reached. Note that, since the formation has only one virtual leader, the formation orientation is not maintained. In this case, the formation is pulled by the virtual leader linked of robot 1. The Figure 7 shows the evolution of relative positions among robots. Note that the robots reach a configuration where the distances among them are inside of residual set around the minimum of potential function which is defined by the dashed line. This result confirms the Theorem 1.

6.2 Tracking and formation control: two virtual leader

In this experiment, two robots track his virtual leaders. The Tables 3 and 4 show the used parameters of formation and tracking controller. It is important to stress that the relative position among two virtual leaders $r_{lvij}^T = [x_{rlvi} - x_{rlvj}, y_{rlvi} - y_{rlvj}]$ should satisfy the condition that minimize the potential function. In other words, $\|r_{lvij}\| = r_d$. Hence, the linear and angular velocities of virtual leaders will be the same. The inital conditions of robots are $C_1(0,125,0)$, $C_2(125,125,0)$ and $C_3(0,0,0)$. The inital condition of two virtual leader linked to robots 1 and 2 are $C_{lv1}(0,125,0)$ and $C_{lv2}(80,125,0)$ respectively. The Figure 8 shows the robots trajectories while the formation is reached. Note that the orientation is approximately maintained while the formation tracks the virtual leaders. The Figure 7 shows the evolution of relative positions among robots. Note once more that the robots reach a configuration where the distances among them are inside of residual set around the minimum of potential function which is defined by the dashed line. Once more, the Theorem 1 is confirmed.

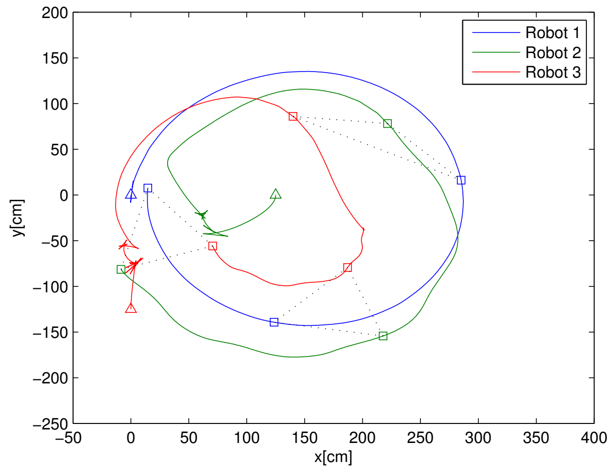

Figure 6. Trajectory of formation with one virtual leader.

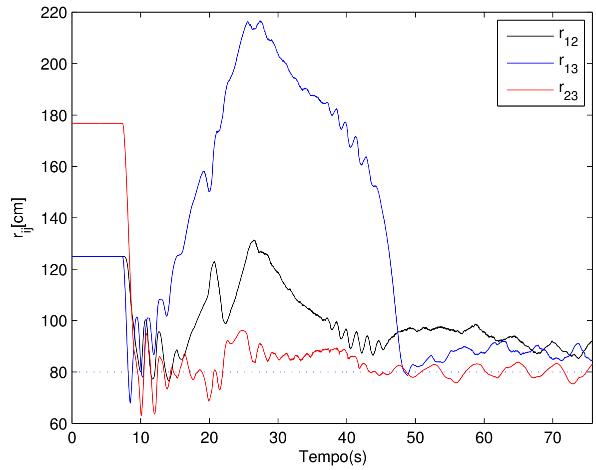

Figure 7. Distance among robots with one virtual leader.

k_g	6000
k_a	0.75
r_d	80cm
R_s	500cm
c	35cm

Table 3. Formation control parameters: two virtual leaders

u_{rlv}	15cm/s
ω_{rlv}	-0.1rad/s
k_1	1.5
k_2	0.035
k_3	1

Table 4. Tracking control parameters: two virtual leaders

7. Conclusions

This work described a control strategy that forces a group of nonholonomic mobile robots to achieve a formation while tracking some desired trajectory. The scheme used a formation control component, based on potential function, and a tracking component which assured the tracking of a trajectory defined by virtual leaders. A stability analysis based on Lyapunov formalism shown that the proposed control law achieves the formation and tracking objectives. Experimental results using a formation with three virtual leaders confirmed the results of stability analysis.

8. ACKNOWLEDGEMENTS

The authors would like thanks resource agents FAPERJ(E-26/200.820/2017), CNPQ and CAPES for the partial financial supports of the present work.

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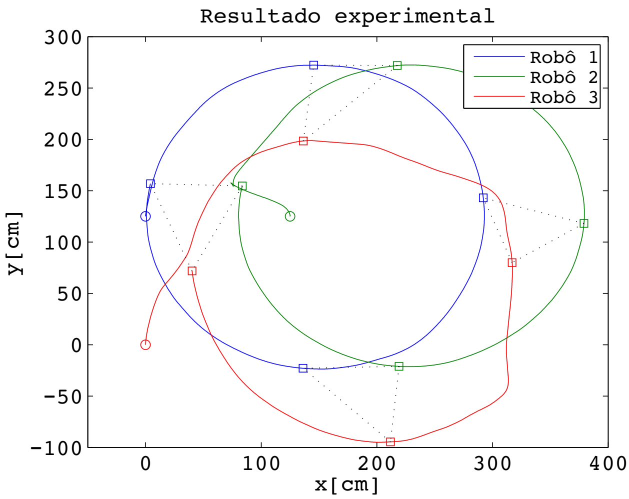

Figure 8. Trajectory of formation with two virtual leader.

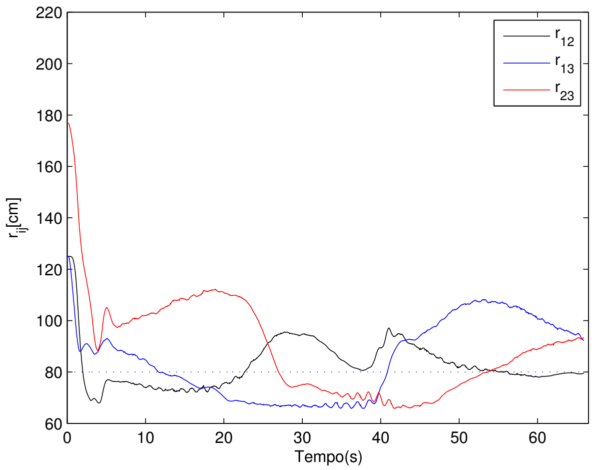

Figure 9. Distance among robots with two virtual leader.

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