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ROBUST OUTPUT FEEDBACK CONTROL OF AN ELECTRO-HYDRAULIC ACTUATOR WITH UNCERTAIN PARAMETERS

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Abstract. *Electro-hydraulic servo-systems are widely employed in industrial applications such as robotic manipulators, active suspensions, precision machine tools, and aerospace systems. They provide many advantages over electric motors, including high force to weight ratio, fast response time, and compact size. One major difficult in that precise control of electro-hydraulic systems cannot be easily obtained with conventional linear controllers, because of intrinsic non-linearities. This work describes the development of an output feedback controller for an electro-hydraulic system that compensates the effects of nonlinearities and uncertain parameters. Numerical results are presented in order to demonstrate the control system performance.*

Keywords: *Robust control, ADRC, Electro-hydraulic actuator, uncertain parameters.*

1. INTRODUCTION

Electro-hydraulic actuators play an important role in many industrial applications. They are often considered as the most appropriate choice of actuation when large loads and high speeds are required. From the operation point of view, one of the main advantages of this type of actuator is the ability to maintain the load indefinitely, which can hardly be obtained by using electric actuators, without achieving overheating. Because they present a highly non-linear dynamic behavior, the efficient control of electro-hydraulic devices can not be easily obtained by means of conventional linear control techniques. In addition to the common nonlinearities generated by the compressibility of the hydraulic fluid, flow properties, and valve pressure, many of the electro-hydraulic systems are also subjected to large nonlinearities such as dead-zone, which occurs when the valve spool overlaps the fluid through-hole, preventing its flow even for a small displacement of the spool. In this context, the increase in the number of papers, proposing new control strategies for this class of systems, demonstrates the great interest of the industrial sector and the academic community for the theme. The most common approaches are the adaptive (Ahn *et al.*, 2014) and variable structure (Bessa *et al.*, 2010) methodologies, but nonlinear controllers based on, optimal tuning PID (Ye *et al.*, 2017), adaptive neural network and adaptive fuzzy system (Liem *et al.*, 2016) were also presented recently.

An emerging interest from both academia and industry in the *Active Disturbance Rejection Control* (ADRC) has been observed in recent years (Gao, 2006). The potential of the ADRC method as a viable solution to industrial control has become increasingly evident after the recent adoptions by major industrial concerns, mainly because of its ability of disturbance rejection and its lack of requirement on the detailed mathematical model of the plant (Madoński *et al.*, 2015).

2. PROBLEM STATEMENT

This paper considers the problem of controlling the piston displacement of an electro-hydraulic actuator to reach a desired set-point position with respect to a fixed coordinate system. The plant is assumed to possess nonlinearities and parametric uncertainties that can degrade the system performance and then needs to be regarded in the controller design. Figure 1 shows a schematic diagram of the electro-hydraulic actuator considered in this work.

3. METHODOLOGY

To solve the problem of controlling the actuator, in the presence of nonlinearities and parametric uncertainties, we developed a modified version of the Active Disturbance Rejection Method (ADRC) (Han, 1998; Gao *et al.*, 2001; Han, 2009; Madoński *et al.*, 2015). It consists basically of an extended observer that estimates the system states and non-measurable signals that act together with a state feedback control law. The key idea of the proposed method is to perform slight modifications both on the input/output dynamics of the plant and on the observer equations that brings some mathematics advantages to the closed loop control design. The real contributions that are achieved after applying such modifications are: (i) the exact knowledge of system parameters are not previously required in the control design; (ii) the control synthesis is simplified due to the reduced set of design constants; (iii) complexity reduction due to the use of linear components both on the observer and on the control law design.

4. SYSTEM MODEL

The electro-hydraulic system considered in this work consists of a four-way proportional valve, a hydraulic cylinder and variable load force. The variable load force is represented by a mass-spring-damper system. The schematic block diagram of the system under study is presented in Fig. 1.

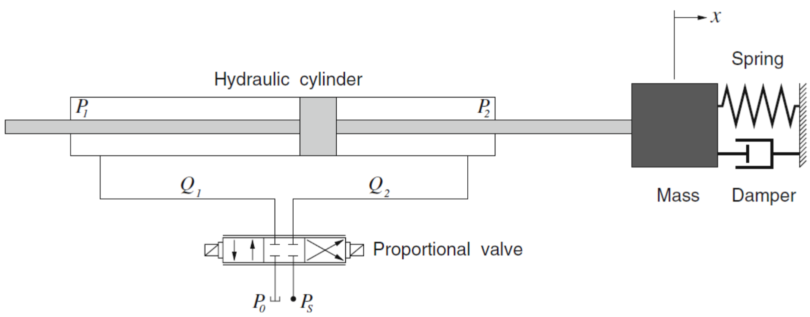

Figure 1: Schematic block diagram of the electro-hydraulic servo-system.

The balance of forces on the piston leads to the following equation of motion:

$$F_g = A_1 P_1 - A_2 P_2 = M_t \ddot{x} + B_t \dot{x} + K_s x_a \tag{1}$$

where F_g is the force generated by the piston, P_1 and P_2 are the pressures at each side of cylinder chamber, A_1 and A_2 are the ram areas of the two chambers, M_t is the total mass of piston and load referred to piston, B_t is the viscous damping coefficient of piston and load, K_s is the load spring constant and $x_a \in \mathbb{R}$ is the piston displacement. Defining the pressure drop across the load as $P_l = P_1 - P_2$ and considering that for a symmetrical cylinder $A_p = A_1 = A_2$, Eq. (1) can be rewritten as

$$M_t \ddot{x}_a + B_t \dot{x}_a + K_s x_a = A_p P_l \tag{2}$$

Applying continuity equation to the fluid flow, the following equation is obtained:

$$Q_l = A_p \dot{x}_a + C_{tp} + \frac{V_t}{4\beta_e} \dot{P}_l \tag{3}$$

where $Q_l = (Q_1 + Q_2)/2$ is the load flow, C_{tp} the total leakage coefficient of piston, V_t the total volume under compression in both chambers and β_e the effective bulk modulus.

Considering that the return line pressure is usually much smaller than the other pressures involved ($P0 \approx 0$) and assuming a closed center spool valve with matched and symmetrical orifices, the relationship between load pressure P_l and load flow Q_l can be described as follows

$$Q_l = C_d w \bar{x}_{sp} \sqrt{\frac{1}{\rho}[P_s - sign(\bar{x}_{sp}) P_l]} \tag{4}$$

where C_d is the discharge coefficient, w the valve orifice area gradient, $\bar{x}_{sp}$ the effective spool displacement from neutral, ρ the hydraulic fluid density, P_s the supply pressure and $sign(.)$ is defined by

$$sign(z) = \begin{cases} -1 & \text{if} \quad z < 0 \\ 0 & \text{if} \quad z = 0 \\ +1 & \text{if} \quad z > 0 \end{cases} \tag{5}$$

Assuming that the dynamics of the valve are fast enough to be neglected, the valve spool displacement can be considered as proportional to the control voltage (u). For closed center valves, or even in the case of the so-called critical valves, the spool presents some overlap. This overlap prevents from leakage losses but leads to a dead-zone nonlinearity within the control voltage. The adopted dead-zone model is a slightly modified version of that proposed by (Zhang and Ge, 2007), which can be mathematically described by

$$\bar{x}_{sp} = \begin{cases} g_l(u) & \text{if} \quad u \leq \delta_l \\ 0 & \text{if} \quad \delta_l < u < \delta_r \\ g_r(u) & \text{if} \quad u \geq \delta_r \end{cases} \tag{6}$$

where g_l and g_r are functions of the control voltage and the dead-band parameters δ_l and δ_r depends on the size of the overlap region. In respect of the dead-zone model presented in Eq. (6), the following assumptions can be assumed:

Assumption 1 *The dead-zone output $\bar{x}_{sp}$ is not available to be measured.*

Assumption 2 *The dead-band parameters d_l and d_r are unknown but bounded and with known signs, i.e., $d_{l_{min}} \leq d_l \leq d_{l_{max}} < 0$ and $0 < d_{r_{min}} \leq d_r \leq d_{r_{max}}$.*

Assumption 3 *The functions $g_l : (-\infty, d_l]$ and $g_r : [dr, +\infty)$ are C^1 and with bounded positive-valued derivatives, i.e.,*

$$0 < k_{l_{min}} \leq g_l^{'}(u) \leq k_{l_{max}}\,, \quad \forall u \in (-\infty, \delta_l]\,,$$
$$0 < k_{r_{min}} \leq g_r^{'}(u) \leq k_{r_{max}}\,, \quad \forall u \in [\delta_r, +\infty)\,,$$

where $g_l^{'}(u) = dg_l(z)/dz|_{z=u}$ and $g_r^{'}(u)(u) = dgr(z)/dz|_{z=u}$.

Remark 1 *Assumption 3 means that both g_l and g_r are Lipschitz functions.*

From the mean value theorem and noting that $g_l(\delta_l) = g_r(\delta_r) = 0$, it follows that there exist $\xi_l : \mathbb{R} \to (-8, dl)$ and $\xi_r : \mathbb{R} \to (dr, +8)$ such that

$$g_l(u) = g_l^{'}(\xi_l(u))[u - \delta_l]$$
$$g_r(u) = g_r^{'}(\xi_r(u))[u - \delta_r]$$

In this way, Eq. 5 can be rewritten as follows:

$$\bar{x}_{sp} = \begin{cases} g_l^{'}(\xi_l(u))[u - \delta_l] & \text{if} \quad u \leq \delta_l \\ 0 & \text{if} \quad \delta_l < u < \delta_r \\ g_r^{'}(\xi_r(u))[u - \delta_r] & \text{if} \quad u \geq \delta_r \end{cases} \tag{7}$$

or, in a more appropriate form, as:

$$\bar{x}_{sp} = k_v(u)[u - d(u)] \tag{8}$$

in which

$$k_v(u) = \begin{cases} g_l^{'}(\xi_l(u)) & \text{if} \quad u \leq 0 \\ g_r^{'}(\xi_r(u)) & \text{if} \quad u > 0 \end{cases} \tag{9}$$

$$d(u) = \begin{cases} \delta_l & \text{if} \quad u \leq \delta_l \\ u & \text{if} \quad \delta_r < u < \delta_l \\ \delta_r & \text{if} \quad u \geq \delta_r \end{cases} \tag{10}$$

Remark 2 *Considering Assumption 4 and Eq. (10), it can be easily verified that $d(u)$ is bounded: $|d(u)| \leq \delta$, where $\delta = max\{-\delta_{l_{min}}, \delta_{r_{max}}\}$.*

Now, combining Eqs. (2), (3), (4) and (8) leads to a third-order differential equation that represents the dynamical behavior of the electro-hydraulic system:

$$\dddot{x}_a = -a^T \mathbf{X_a} + b(\mathbf{X_a}, u)u - b(\mathbf{X_a}, u)d(u) \tag{11}$$

where $\mathbf{X_a} = [\ddot{x}_a, \dot{x}_a, x_a]^T$ is the state vector and $\mathbf{a} = [a_2, a_1, a_0]^T$ is the coefficient vector which values are defined by:

$$a_0 = \frac{4\beta_e C_{tp} K_s}{V_t M_t}\,; \quad a_1 = \frac{K_s}{M_t} + \frac{4\beta_e A_p^2}{V_t M_t} + \frac{4\beta_e C_{tp} B_t}{V_t M_t}\,; \quad a_2 = \frac{B_t}{M_t} + \frac{4\beta_e C_{tp}}{V_t} \tag{12}$$

and

$$b(\mathbf{X_a}, u) = \frac{4\beta_e A_p}{V_t M_t} C_d w k_v \sqrt{\frac{1}{\rho}\left[P_s - \frac{sign(u)(M_t \ddot{x}_a + B_t \dot{x}_a + K_s x_a)}{A_p}\right]} \tag{13}$$

With respect to the dynamical system presented in Eq. (11), the following assumptions will also be assumed:

Assumption 4 *The coefficients a_0, a_1 and a_2 are real, uncertain and constants.*

Assumption 5 *The input gain $b(\mathbf{X_a}, u)$ is uncertain but positive and bounded away from zero, i.e., $0 < b_m \leq b(\mathbf{X_a}, u) \leq b_0$, for some $b_m, b_0 \in \mathbb{R}$.*

5. CONTROL DESIGN PRELIMINARIES

In this section, the position control of the electro-hydraulic actuator plant described by Eq. (11) is addressed. To deal with model uncertainties and nonlinearities, a modified version of the *Active Disturbance Rejection Control* approach will be introduced. As will be shown, the proposed scheme will perform slight modifications on the input/output plant description that will result on some mathematics advantages for the design of the closed loop controller.

5.1 ADRC design - a motivating example

Consider a linear time-invariant version of the plant on Eq. (11) with $\mathbf{X} = [\ddot{x}, \dot{x}, x]^T$, $x \in \mathbb{R}$, and a constant $b(\mathbf{X}, u) = b_0$, i.e.:

$$\begin{aligned} \dddot{x} &= f(t) + b_0 u \\ f(t) &= -a^T \mathbf{X} - b_0 d(u)\,. \end{aligned} \tag{14}$$

Notice that in the ADRC formulation of Eq. (14), the original plant was reduced to a simple third order integrator with an input $u(t) \in \mathbb{R}$, an output $x(t) \in \mathbb{R}$ and subjected to an input disturbance given by $f(t) \in \mathbb{R}$. In (Han, 1998; Gao *et al.*, 2001; Han, 2009), the function $f(t)$ is denoted by *generalized disturbance* and represents a combination of the non measurable signals of the system. According to the model of Eq. (10), inside the dead-zone region the term $[u - d(u)]$ is identically zero. In such situation, no control effort is applied to the system of Eq. (14). However, outside that region, the term $[u - d(u)]$ can be given by $[u - \delta_l]$ or $[u - \delta_r]$. Once we are interested to control the actuator outside the dead zone region, we assume the general case $d(u) = \delta_0$, in which $\delta_0 \in \mathbb{R}$ represents δ_l or δ_r. Then, by rewritting Eq. (14), we obtain:

$$\begin{aligned} \dddot{x} &= f(t) + b_0 u \\ f(t) &= -a^T \mathbf{X} - b_0 \delta_0\,. \end{aligned} \tag{15}$$

5.1.1 Control law synthesis

In the tradicional ADRC scheme discussed in (Han, 1998; Gao *et al.*, 2001; Han, 2009), the control input to be applied to Eq. (15) is designed as:

$$u(t) = u^*(t) = \left(\frac{1}{b_0}\right)\Big[-f(t) - \lambda_2 \ddot{x} - \lambda_1 \dot{x} - \lambda_0 (x - x^*)\Big]\,, \tag{16}$$

in which $u^*(t) \in \mathbb{R}$ denotes the ideal expression, $x^* \in \mathbb{R}$ is a constant reference position for the actuator and $\lambda_2, \lambda_1, \lambda_0 \in \mathbb{R}$ are the constant coefficients of a stable and monic polynomial of order 3. In general case, $f(t)$ and the higher order derivatives of $x(t)$ are non available signals. Then, in this case, the ADRC methodology proposes a control law that is computed by using estimates of such quantities, namely,

$$u(t) = \left(\frac{1}{b_0}\right)\Big[-\hat{f}(t) - \lambda_2 \ddot{\hat{x}} - \lambda_1 \dot{\hat{x}} - \lambda_0 (\hat{x} - x^*)\Big]\,, \tag{17}$$

that are all generated by an extended state observer (ESO). In Eq. (15), choosing the extended state vector as $Z = [x, \dot{x}, \ddot{x}, f(t)]^T$ and supposing initially that $f(t)$ is differentiable, the state space representation will be given by:

$$\begin{aligned}\dot{\mathbf{Z}} &= \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{A}} \mathbf{Z} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ b_0 \\ 0 \end{bmatrix}}_{\mathbf{B}} u + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{\mathbf{\Lambda}} \dot{f}(t)\,, \\ y &= \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{C}} \mathbf{Z}\,. \end{aligned} \tag{18}$$

The ESO will assume the following format:

$$\begin{aligned}\dot{\hat{\mathbf{Z}}} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \hat{\mathbf{Z}} + \begin{bmatrix} 0 \\ 0 \\ b_0 \\ 0 \end{bmatrix} u + \underbrace{\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{bmatrix}}_{\mathbf{L}} e_y\,, \\ \hat{y} &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \hat{\mathbf{Z}}\,, \end{aligned} \tag{19}$$

in which $e_y = y - \hat{y}$ is the observer output error and $L_1,\ L_2,\ L_3,\ L_4$ are the observer gains obtained after choosing the observer poles as $-w_0\,,\ w_0 > 0 \in \mathbb{R}$, i.e,

$$\begin{aligned} &(s + w_0)^4 = s^4 + L_1 s^3 + L_2 s^2 + L_3 s + L_4\,. \\ &L_1 = 4w_0\,, \quad L_2 = 6w_0^2\,, \quad L_3 = 4w_0^3\,, \quad L_4 = w_0^4\,. \end{aligned} \tag{20}$$

After computing the time derivative of the observer error vector $\mathbf{e_x} = \mathbf{Z} - \hat{\mathbf{Z}}$, from Eqs. (18) and (19), we ended up with

$$\dot{\mathbf{e}}_x = \underbrace{\begin{bmatrix} -L_1 & 1 & 0 & 0 \\ -L_2 & 0 & 1 & 0 \\ -L_3 & 0 & 0 & 1 \\ -L_4 & 0 & 0 & 0 \end{bmatrix}}_{(\mathbf{A}-\mathbf{LC})} \mathbf{e}_x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{\mathbf{\Lambda}} \dot{f}(t)\,, \qquad e_y = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}}_{C_1} \mathbf{e}_x\,. \tag{21}$$

5.1.2 Observer convergence

In order to study the influence of the generalized disturbance $f(t)$ in the convergence of the observer estimates $\hat{\mathbf{Z}}$ in Eq. (21), let us analyse the input/output relationships regarding $\dot{f}(t)$ as a system input and the state variables $\hat{Z}_1, \hat{Z}_2, \hat{Z}_3, \hat{Z}_4$, one at a time, as the system output, i.e.,

$$\begin{aligned} &\frac{E_{xi}(s)}{sF(s)} = \mathbf{C}_i[sI - (\mathbf{A} - \mathbf{LC})]^{-1}\mathbf{\Lambda}\,, \quad sF(s) = \mathcal{L}\left\{\dot{f}(t)\right\}\,, \quad E_{xi}(s) = \mathcal{L}\left\{e_{xi}(t)\right\} \\ &\qquad e_{xi} = \mathbf{C_i}\mathbf{e_x}\,, \quad (i = 1, \cdots, 4)\,. \end{aligned} \tag{22}$$

In Eq. (22), $\mathbf{C_i}$ accounts for the choice of the error variable, for instance, for e_{x1} one may assign $\mathbf{C}_i = \mathbf{C}_1 = [1\,0\,0\,0]$, for e_{x2} one may assign $\mathbf{C}_i = \mathbf{C}_2 = [0\,1\,0\,0]$ and so on. Then, by using Eq. (22), we obtain the following relationships:

$$\begin{aligned} &\frac{E_{x1}(s)}{sF(s)} = \frac{1}{s^4 + L_1 s^3 + L_2 s^2 + L_3 s + L_4}\,, \qquad \frac{E_{x2}(s)}{sF(s)} = \frac{s + L_2}{s^4 + L_1 s^3 + L_2 s^2 + L_3 s + L_4}\,, \\ &\frac{E_{x3}(s)}{sF(s)} = \frac{s^2 + L_1 s + L_2}{s^4 + L_1 s^3 + L_2 s^2 + L_3 s + L_4}\,, \qquad \frac{E_{x4}(s)}{sF(s)} = \frac{s^3 + L_1 s^2 + L_2 s + L_3}{s^4 + L_1 s^3 + L_2 s^2 + L_3 s + L_4}\,, \end{aligned} \tag{23}$$

Then, as can be seen from the expressions in Eqs. (23) and (20), for sufficiently large choices of w_0, we can force the observer state error variables to reach values that are sufficiently close to zero. Such particular characteristic, becomes more clear when taking a closed look at $E_{x4}(s)$ expression in Eq. (23):

$$\frac{E_{x4}(s)}{sF(s)} = \frac{F(s) - \hat{F}(s)}{sF(s)} \quad \longrightarrow \quad \hat{F}(s) = \frac{L_4}{s^4 + L_1 s^3 + L_2 s^2 + L_3 s + L_4} F(s)\,. \tag{24}$$

That is, the estimate $\hat{f}(t)$ of the generalized disturbance $f(t)$ can be made arbitrary precise by choosing sufficiently large values $w_0 > 0$ for the observer poles in Eq. (20).

Remark 3 *Provided that the term $\dot{f}$, in Eq. (21), is not eliminated by the observer design, the separation principle that is usually applied to prove stability of the overall plant-observer system can not be used in the demonstrations. Although the choice of sufficiently large value for the observer poles at w_0 increases estimation precision, this fact alone does not guarantee boundedness properties of the closed loop system of Eqs. (15), (17) and (19).*

Remark 4 *As can be verified from the previous analysis, the design and performance of both the control law of Eq. (17) and the extended state observer of Eq. (19) are dependent on the exact knowledgement of the control gain b_0. Such design difficulty has been avoided in several ADRC schemes by assuming that b_0 is completely or partially known a priori (Han, 1998; Gao* et al.*, 2001; Han, 2009; Madoński and Herman, 2011; Zhu* et al.*, 2014; Xue* et al.*, 2015; Xia* et al.*, 2016). Contrasting with the strategies proposed on the cited works, in the present paper we introduce a modified ADRC framework in order to deal with uncertainties within the parameter b_0.*

6. ELECTRO-HYDRAULIC ACTUATOR CONTROL VIA ADRC

In order to deal with the uncertainties and nonliearities that appear in the control gain $b(\mathbf{X}, t)$ of Eqs. (11) and (13), a modified framework is proposed for the ADRC scheme of Section 5. The main idea of the proposal is to introduce a structural transformation in the input/output description of the original system, in order to obtain a new dynamical equation with known control gain. Once it has been achieved, then the ADRC method will be applied without restrictions. As will be shown later, such dynamical transformation does not affect the control objectives outlined for the original plant.

6.1 Modified ADRC framework

The methodology adopted here to produce the proposed transformation consists in the introduction of a constant gain β in series with the plant output and a third order linear (stable) filter $Q_0(s)$ in parallel with the overall input/output model, as depicted in Fig. 2.

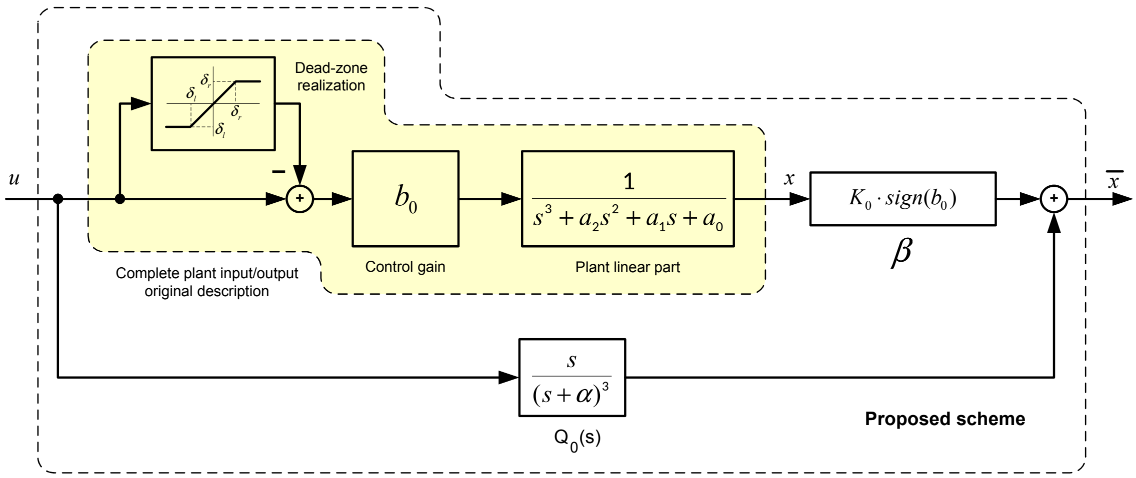

Figure 2: Block diagram of the proposed (modified) ADRC framework.

In time domain, the output equation of the transformed plant can be written as:

$$\bar{x} \;=\; \beta\, x + u_f\,, \qquad Q_0(s)\;:\; \dddot{u}_f \;=\; -\gamma_2 \ddot{u}_f - \gamma_1 \dot{u}_f - \gamma_0 u_f + \dot{u}\,, \quad \gamma_2, \gamma_1, \gamma_0 > 0\,, \tag{25}$$

in which $u_f(t) \in \mathbb{R}$ is the filtered version of the control signal u, generated in the output of the filter $Q_0(s)$ in Fig. 2. Differentiating the signal $\bar{x}$ of Eq. (25) three times, the dynamics of the transformed system will be given by:

$$\dddot{\bar{x}} = -\beta a^T \mathbf{X} + K_0 |b_0| u - K_0 |b_0| \delta_0 - \gamma_2 \ddot{u}_f - \gamma_1 \dot{u}_f - \gamma_0 u_f + \dot{u}\,. \tag{26}$$

Now, adopting the ADRC formalism to rewrite Eq. (26), as in Eq. (15), we obtain:

$$\begin{cases} \dddot{\bar{x}} = \psi(t) + \dot{u}\,, \\ \psi(t) = -\beta a^T \mathbf{X} + b_p[u - \delta_0] - \gamma_2 \ddot{u}_f - \gamma_1 \dot{u}_f - \gamma_0 u_f\,, \\ b_p = K_0 |b_0| \;>\; 0\,. \end{cases} \tag{27}$$

Remark 5 *In the proposed framework, the control problem is redefined for the plant of Eq. (27) which is subjected to a new control signal represented by $\dot{u}$. Noting that the redefined output $\bar{x}$ is measurable and the control gain is unitary, then the ADRC method can be applied with no restriction. The key advantage is that the only information needed to be known a priori from the plant is the sign of the control gain b_0.*

6.2 Control law definition

Following the design procedures presented on Section 5.1, also making analogy to Eq. (17), the control law for the system (27) can be chosen as

$$\dot{u} = -\hat{\psi}(t) - \lambda_2 \ddot{\hat{\bar{x}}} - \lambda_1 \dot{\hat{\bar{x}}} - \lambda_0 (\hat{\bar{x}} - \bar{x}^*) \,, \tag{28}$$

being $\bar{x}^* = \beta x^*$ the modified set-point value and $\hat{\bar{\mathbf{Z}}} = [\ddot{\hat{\bar{x}}},\ \dot{\hat{\bar{x}}},\ \hat{\bar{x}}, \hat{\psi}(t)]^T$ the state vector estimate governed by the ESO:

$$\dot{\hat{\bar{\mathbf{Z}}}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \hat{\bar{\mathbf{Z}}} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \dot{u} + \underbrace{\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{bmatrix}}_{\mathbf{L}} e_{\bar{y}} \,, \qquad \hat{\bar{y}} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \hat{\bar{\mathbf{Z}}} \,, \qquad \mathbf{e}_{\bar{\mathbf{y}}} = \bar{\mathbf{y}} - \hat{\bar{\mathbf{y}}} \,. \tag{29}$$

For $\bar{\mathbf{e}}_x := \bar{\mathbf{Z}} - \hat{\bar{\mathbf{Z}}}$ and supposing that $\psi(t)$ is differentiable, then the observer error dynamics will be given by:

$$\dot{\bar{e}}_x = \underbrace{\begin{bmatrix} -L_1 & 1 & 0 & 0 \\ -L_2 & 0 & 1 & 0 \\ -L_3 & 0 & 0 & 1 \\ -L_4 & 0 & 0 & 0 \end{bmatrix}}_{(\mathbf{A}-\mathbf{LC})} \bar{\mathbf{e}}_x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \dot{\psi}(t) \,, \qquad \bar{e}_y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \bar{\mathbf{e}}_x \,. \tag{30}$$

6.3 Observer dynamics - revisited

Comparing the observers' error dynamics of Eqs. (21) and (30), from input/output point of view, it is possible to verify that they are the same. Thus, the analysis carried out on Section 5.1.2 can be fully utilized here. Indeed, the system of Eq. (30) has the same convergence properties of the one in Eq. (21). The major differences are in the definitions of the control law $\dot{u}$ (28) and in the definition of the generalized disturbance $\psi(t)$ (27). That is the main reason why the analysis of the overall closed loop system must be conveniently accomplished in this section.

From analogy to the relations in Eq. (23), we can write:

$$\begin{aligned} \frac{\bar{E}_{x1}(s)}{s\Psi(s)} &= \frac{1}{s^4 + L_1 s^3 + L_2 s^2 + L_3 s + L_4} \,, \qquad & \frac{\bar{E}_{x2}(s)}{s\Psi(s)} &= \frac{s + L_2}{s^4 + L_1 s^3 + L_2 s^2 + L_3 s + L_4} \,, \\ \frac{\bar{E}_{x3}(s)}{s\Psi(s)} &= \frac{s^2 + L_1 s + L_2}{s^4 + L_1 s^3 + L_2 s^2 + L_3 s + L_4} \,, \qquad & \frac{\bar{E}_{x4}(s)}{s\Psi(s)} &= \frac{s^3 + L_1 s^2 + L_2 s + L_3}{s^4 + L_1 s^3 + L_2 s^2 + L_3 s + L_4} \,. \end{aligned} \tag{31}$$

From Eq. (31), it is also possible to set the following relantions:

$$\bar{E}_{x2}(s) = (s + L_2)\bar{E}_{x1}(s) \,, \quad \bar{E}_{x3}(s) = (s^2 + L_1 s + L_2)\bar{E}_{x1}(s) \,, \quad \bar{E}_{x4}(s) = (s^3 + L_1 s^2 + L_2 s + L_3)\bar{E}_{x1}(s) \,, \tag{32}$$

which will be very useful in the future developments.

6.4 Stability and convergence analysis

Replacing the control law defined in Eq. (28) into the open-loop dynamics of Eq. (27), the following closed-loop system is obtained:

$$\dddot{\bar{x}} + \lambda_2 \ddot{\bar{x}} + \lambda_1 \dot{\bar{x}} + \lambda_0 \bar{x} = \lambda_0 \bar{x}^* + \bar{e}_{x4} + \lambda_2 \bar{e}_{x3} + \lambda_1 \bar{e}_{x2} + \lambda_0 \bar{e}_{x1} \,. \tag{33}$$

In frequency domain, according to the expressions in Eq. (32), the closed-loop system of Eq. (33) becomes:

$$\begin{aligned} &\Lambda(s)\bar{X}(s) = P(s)\bar{E}_{x1}(s) + \lambda_0 \bar{X}^*(s) \,, \qquad \Lambda(s) = (s^3 + \lambda_2 s^2 + \lambda_1 s + \lambda_0) \,, \\ &P(s) = [s^3 + (L_1 + \lambda_2)s^2 + (L_2 + L_1\lambda_2 + \lambda_1)s + (L_3 + \lambda_2 L_2 + \lambda_1 L_2 + \lambda_0)] \,. \end{aligned} \tag{34}$$

By highlighting the expression of $\bar{E}_{x1}(s)$ from Eq. (31) and replacing it into Eq. (34), we obtain

$$\begin{aligned} &\Lambda(s)\bar{X}(s) = \frac{s\,P(s)}{L(s)}\Psi(s) + \lambda_0 \bar{X}^*(s) \,, \\ &L(s) \;=\; (s + w_0)^4 \;=\; s^4 + L_1 s^3 + L_2 s^2 + L_3 s + L_4 \,, \\ &\bar{X}(s) = \underbrace{\frac{s\,P(s)}{L(s)\Lambda(s)}\Psi(s)}_{T(s)} + \frac{\lambda_0}{\Lambda(s)}\bar{X}^*(s) \,. \end{aligned} \tag{35}$$

The expression for $\Psi(s)$ can be obtained from Eqs. (25) and (27) after some few manipulations, i.e.,

$$\begin{aligned}&\Psi(s) = -\beta\bar{A}(s)X(s) + \beta b_0 U(s) - \frac{\beta b_0 \delta_0}{s} - \frac{s\bar{\Gamma}(s)}{\Gamma(s)}U(s)\,, \quad A(s) \;=\; s^3 + a_2 s^2 + a_1 s + a_0\,,\\ &\bar{A}(s) \;=\; a_2 s^2 + a_1 s + a_0\,, \quad \Gamma(s) = \; s^3 + \gamma_2 s^2 + \gamma_1 s + \gamma_0\,, \quad \bar{\Gamma}(s) \;=\; \gamma_2 s^2 + \gamma_1 s + \gamma_0\,,\end{aligned} \tag{36}$$

At this point, we need to simplify the function $T(s)$ by manipulating Eq. (36). Since the constant design parameter β introduced in Eq. (25) can be chosen sufficiently large, then

$$\bar{X}(s) = \beta X(s) + U_f(s) \;\approx \beta X(s)\,, \tag{37}$$

$$\begin{aligned}T(s) &= \frac{s\,P(s)}{L(s)\Lambda(s)}\left[-\beta\bar{A}(s)X(s) + \beta b_0 U(s) - \frac{\beta b_0\delta_0}{s} - \frac{s\bar{\Gamma}(s)}{\Gamma(s)}U(s)\right] = \\ &= \frac{s\,P(s)}{L(s)\Lambda(s)}\left[-\beta\bar{A}(s)X(s) + \underbrace{\left(\beta b_0 - \frac{s\bar{\Gamma}(s)}{\Gamma(s)}\right)}_{\approx\,\beta b_0}U(s) - \frac{\beta b_0\delta_0}{s}\right]\\ T(s) &\approx \frac{s\,P(s)}{L(s)\Lambda(s)}\left[-\beta\bar{A}(s)X(s) + \beta b_0 U(s) - \frac{\beta b_0\delta_0}{s}\right].\end{aligned} \tag{38}$$

From the plant description in Fig. (2), we can write:

$$X(s) = \frac{b_0}{A(s)}\left[U(s) - \frac{\delta_0}{s}\right], \quad \text{or} \quad U(s) = \frac{A(s)}{b_0}X(s) + \frac{\delta_0}{s}\,. \tag{39}$$

Replacing the last expression of Eq. (39) into Eq. (38), $T(s)$ assumes the following simplified format:

$$T(s) \approx \beta\,\frac{s^4\,P(s)}{L(s)\Lambda(s)}X(s)\,. \tag{40}$$

Thus, Eq. (35) reduces to

$$\bar{X}(s) \approx \frac{s^4\,P(s)}{L(s)\Lambda(s)}\bar{X}(s) + \frac{\lambda_0}{\Lambda(s)}\bar{X}^*(s) \quad \longrightarrow \quad \bar{X}(s) = \left[\frac{\lambda_0 L(s)}{L(s)\Lambda(s) - s^4\,P(s)}\right]\bar{X}^*(s)\,. \tag{41}$$

Manipulating the expressions in Eqs. (34) and (35), we obtain

$$[L(s)\Lambda(s) - s^4\,P(s)] \;=\; \lambda_1 L_1 s^4 + L_4\,(s^3 + \lambda_2 s^2 + \lambda_1 s + \lambda_0)\,. \tag{42}$$

Under the assumption that the poles of the observer (or, the roots of $L(s)$) in Eq. (35) were chosen conveniently to have sufficiently large values, then it is not difficult to demonstrate by *Routh's criterion* that (43) is a stable polynomial. Moreover, since $\bar{x}^* = \beta x^*$ is a constant set-point value, the steady-state behavior of the signal $\bar{x}(t)$ can be computed by applying the *Final value Theorem* in Eq. (41):

$$\lim_{t\to\infty}\bar{x}(t) \longrightarrow \lim_{s\to 0}\left[\left(\frac{s\,\lambda_0(s^4 + L_1 s^3 + L_2 s^2 + L_3 s + L_4)}{\lambda_1 L_1 s^4 + L_4\,(s^3 + \lambda_2 s^2 + \lambda_1 s + \lambda_0)}\right)\left(\frac{\bar{x}^*}{s}\right)\right] = \frac{L_4\lambda_0\bar{x}^*}{L_4\lambda_0} \;= \bar{x}^*\,, \tag{43}$$

Base on the last conclusion, in Eq. (34) we can assert that, as $t\to\infty$,

$$\Lambda(s)\bar{X}(s) = \lambda_0\bar{X}^*(s)\,, \tag{44}$$

which implies that the observer error

$$\bar{e}_{x1}(t) \to 0 \quad \text{as} \quad t\to\infty \quad \longrightarrow \quad \bar{e}_{x2}(t),\ \bar{e}_{x3}(t),\ \bar{e}_{x4}(t) \to 0 \quad \text{as} \quad t\to\infty\,. \tag{45}$$

Yet based on the previous conclusions, from ESO dynamics in Eq. (29) we can also verify that $\dot{u}(t) \to 0,\ t\to\infty$, which implies that the generalized disturbance $\psi(t) \to 0,\ t\to\infty$. As $\dot{u}(t)$ is the input of filter $Q_0(s)$ (25), then $u_f(t),\ \dot{u}_f(t),\ \ddot{u}_f(t) \to 0$, which are conditions that guarantee that $x(t) \to x^*,\ t\to\infty$. An interesting conclusion that arises from the previous demonstrations, can be noticed from the definition of $\psi(t)$ in Eq. (27). According to Eqs. (25) and (27), $\beta = K_0\,sign(b_0)$ and $b_p = K_0\,|b_0|$, then in Eq. (27),

$$\begin{aligned}&-\beta a^T\mathbf{X} + b_p[u - \delta_0] \;= K_0 sign(b_0)[-a_0 x^* + b_0(u - \delta_0)] = 0\,,\\ &u(t) = \frac{a_0 x^*}{b_0} + \delta_0\,, \quad \text{as} \quad t\to\infty\,.\end{aligned} \tag{46}$$

As the output signal $x(t)$ of the original plant approaches the desired constant set-point $x^*(t)$, $u(t)$ approaches to a constant value. It is important to remember that δ_0 can represent either δ_r or δ_l. In the particular case $x^*(t) = 0$ then the control signal amplitude will achieve the boundaries of the dead-zone, as expected in the present regulation control problem. Although the analysis presented in the previous section has succeeded regarding the nonlinear control gain $b(\mathbf{X_a}, u)$ fixed in its maximum value, the stability and convergence properties demonstrated can be extended to the general case of non fixed $b(\mathbf{X_a}, u)$. Reproducing the demonstration mechanism, replacing the control gain b_0 by the minimum value b_m of the real gain $b(\mathbf{X_a}, u)$, then the stability results will remain the same except for few changes in the convergence values of the steady-state response.

7. SIMULATION RESULTS

In this section, we present and discuss simulation of the proposed modified ADRC scheme applied to an electro-hydraulic actuator. For the simulation, the dynamics of the overall control system composed by: (i) the actuator in Eqs. (10), (11), (12), (13), (ii) the extended observer (ESO) in Eq. (29), (iii) the filter Q_0 in Eq. (25), and the control law in Eq. (28) , were all coded using Simulink^{TM} block programing. The parameters and design constants used in the simulation are described as follows. *For the plant*: $P_s = 7\ MPa$; $\rho = 850\ kg/cm^3$; $C_d = 0.6$; $w = 2.5^5$; $A_p = 3 \times 10^{-4}\ m^2$; $C_{tp} = 2 \times 10^{-12}\ m^3/(sPA)$; $\beta_e = 700\ MPa$; $V_t = 6 \times 10^{-5}\ m^3$; $M_t = 250\ kg$; $B_t = 0\ Ns/m$; $K_s = 0\ N/m$; $\delta_l = -1.5\ V$ and $\delta_r = 1.5\ V$; *For the observer*: $w_0 = 100$; $L_1 = 400$; $L_2 = 6 \times 10^4$; $L_3 = 4 \times 10^6$; and $L_4 = 10^8$; *For the control law*: $K_0 = \beta = 2.7$; $\lambda_2 = 45$; $\lambda_1 = 675$ and $\lambda_0 = 3375$; *For the filter Q_0*: $\gamma_2 = 45$; $\gamma_1 = 675$ and $\gamma_0 = 3375$. The simulation sample time was $0.001\ s$.

The curves in Fig. 3 illustrate the performance of the proposed ADRC strategy applied in a fixed set-point control problem in which the reference signal is a square wave with period $(T = 100\ s)$ and an amplitude of $x^* = 0.005\ m$. As predicted by the stability analysis, the observer output error converges to zero as fast as we choose larger values for its poles at $s = -w_0$. Also, as the actuator position $x_a(t)$ tends to the desired set-points $x^* = 0.005\ m$ and $x^* = -0.005\ m$, represented by the square signal, the control input converges to constant voltages. In the detailed curves of Fig. 3, on the bottom, it is possible to visualize the control gain variation according to the set-point changes. In this simulation, the reason of choosing the design parameter $K_0 = \beta = 2.7$ was due the values assumed by the control gain $b(\mathbf{X_a}, u)$, i.e., approximately between 134 and 139. In other words, recalling to Eq. (38), it was chosen large enough to guarantee the results predicted by the analysis.

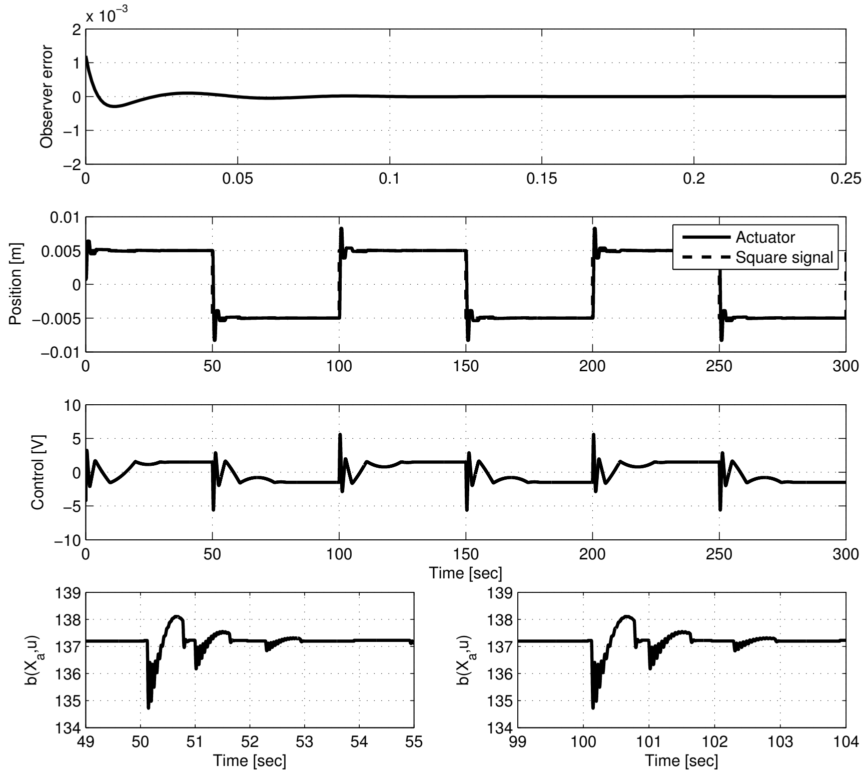

Figure 3: Simulation results. Square wave tracking case.

8. CONCLUSION

This work proposed a mathematical solution for the position control of an electro-hydraulic actuator with uncertain parameters, based on the Active Rejection Control method. The idea of the work was to develop an extension of the ADRC controller for uncertain systems in which the uncertainty in the control gain of the plant is also considered. To solve the problem of uncertainty over the original control gain of the plant, a modification in the original ADRC control scheme was proposed that consisted in the introduction of a compensator in parallel with the plant. The central objective was to produce an input/output system equivalent to the original one but with a known control gain, without, however, changing the original control objective. An interesting feature of the developed strategy was the relaxation of the requirement of exact knowledge of the value of the original control gain of the plant that. In this work, such requirement was reduced to the one of knowing its sign. The stability and convergence properties of the closed loop system were proved by the theoretical demonstrations and the application of the method in a numerical simulation.

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10. REFERENCES

Ahn, K.K., Nam, D.N.C. and Jin, M., 2014. “Adaptive backstepping control of an electrohydraulic actuator”. *IEEE/ASME transactions on mechatronics*, Vol. 19, No. 3, pp. 987–995.

Bessa, W.M., Dutra, M.S. and Kreuzer, E., 2010. “Sliding mode control with adaptive fuzzy dead-zone compensation of an electro-hydraulic servo-system”. *Journal of Intelligent & Robotic Systems*, Vol. 58, No. 1, pp. 3–16.

Gao, Z., 2006. “On active disturbance rejection control”. In *American Control Conference*.

Gao, Z., Huang, Y. and Han, J., 2001. “An alternative paradigm for control system design”. In *Proceedings of the 40th IEEE Conference on Decision and Control*. IEEE, Vol. 5, pp. 4578–4585.

Han, J., 1998. “Auto-disturbance rejection control and its applications”. *Control and Decision*, Vol. 13, No. 1, pp. 19–23.

Han, J., 2009. “From pid to active disturbance rejection control”. *IEEE transactions on Industrial Electronics*, Vol. 56, No. 3, pp. 900–906.

Liem, D.T., Truong, D.Q., Park, H.G. and Ahn, K.K., 2016. “A feedforward neural network fuzzy grey predictor-based controller for force control of an electro-hydraulic actuator”. *International Journal of Precision Engineering and Manufacturing*, Vol. 17, No. 1, pp. 309–321.

Madoński, R., Gao, Z. and Łakomy, K., 2015. “Towards a turnkey solution of industrial control under the active disturbance rejection paradigm”. In *54th Annual Conference of the Society of Instrument and Control Engineers of Japan (SICE)*. IEEE, pp. 616–621.

Madoński, R. and Herman, P., 2011. “An experimental verification of adrc robustness on a cross-coupled aerodynamical system”. In *EEE International Symposium on Industrial Electronics (ISIE)*. IEEE, pp. 859–863.

Xia, Y., Pu, F., Li, S. and Gao, Y., 2016. “Lateral path tracking control of autonomous land vehicle based on adrc and differential flatness”. *IEEE Transactions on Industrial Electronics*, Vol. 63, No. 5, pp. 3091–3099.

Xue, W., Bai, W., Yang, S., Song, K., Huang, Y. and Xie, H., 2015. “Adrc with adaptive extended state observer and its application to air-fuel ratio control in gasoline engines”. *IEEE Transactions on Industrial Electronics*, Vol. 62, No. 9, pp. 5847–5857.

Ye, Y., Yin, C.B., Gong, Y. and Zhou, J.j., 2017. “Position control of nonlinear hydraulic system using an improved pso based pid controller”. *Mechanical Systems and Signal Processing*, Vol. 83, pp. 241–259.

Zhang, T.P. and Ge, S.S., 2007. “Adaptive neural control of mimo nonlinear state time-varying delay systems with unknown dead-zones and gain signs”. *Automatica*, Vol. 43, No. 6, pp. 1021–1033.

Zhu, E., Pang, J., Sun, N., Gao, H., Sun, Q. and Chen, Z., 2014. “Airship horizontal trajectory tracking control based on active disturbance rejection control (adrc)”. *Nonlinear Dynamics*, Vol. 75, No. 4, pp. 725–734.

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