# COBEM-2017-5909
# UNCERTAINTIES IN APPLYING THE INDEPENDENCE PRINCIPLE TO THE VIV RESPONSES OF YAWED CYLINDERS

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*Abstract. The evaluation of Vortex-Induced Vibrations (VIV) in slender bodies that have some yaw regarding the flow direction has become a major matter of research, since as solution it is a common practice to apply the Independence Principle (IP), a theoretical principle that is robust for a wide range of flow conditions but that starts disagreeing with the experimental results when the yaw angle increases. Thereby, an error propagation is herein carried out to access the influence of each parameter uncertainty in the reduced velocity calculation. As a result, the differences observed in the IP might be more than just a physical matter, but also an uncertainty issue outcome.*

*Keywords: VIV, independence principle, uncertainty analysis.*

## 1. INTRODUCTION

The VIV phenomenon negatively affects several slender systems in engineering, most of them circular cylinders not perpendicular to the flow direction. In these cases, the Independence Principle (IP), see Eq. (1), ensures that forces and frequencies of vortex shedding in yawed cylinders are the same of those measured in cylinders perpendicular to the flow direction, considering that only the normal component of flow velocity acts on the yawed cylinders.

$$V_r = \frac{U cos\theta}{D f_n} \tag{1}$$

According to experiments as those found in King (1977) and Ramberg (1983), by applying the IP to fixed cylinders it is possible to predict reasonably well the forces induced by the near vortex shedding for yaw angles up to 45 degrees, value that depends on the experimental setup, particularly due to the boundary conditions in the cylinder tips. The IP, however, says nothing about the VIV responses, although according to Franzini et.al. (2013) it has a good capability to predict the lock-in range of velocities for the yawed cylinders, provided that the related physical quantities are well measured, including respective uncertainties.

Based on the theory for propagation of uncertainties, for instance found in Preston and Dietz (2001), this work aims to analyze the uncertainties in the calculation of the reduced velocities by inspecting each of the quantities involved and their relative importance, additionally, taking into account, that the most significant of them are considered under a feasible range of values.

## 2. METHODOLOGY

The methodology consists of verifying the influence of the involved errors of the input variables in the mathematical formulation of the IP, $V_r = f\left(U, \theta, L, \rho_{mat}, C_a, \rho_{H_2O}, k\right)$, using the propagation of uncertainties for independent variables:

$$\delta^2 V_r = \sum_{i=1}^{n} \left(\frac{\partial V_r}{\partial x_i}\right)^2 \delta^2 x_i, \tag{2}$$

where the reduced velocity is considered in its explicit form:

$$V_r = \pi U cos\theta \sqrt{\frac{\pi L\left(\rho_{mat} + C_a \rho_{H_2O}\right)}{k}}, \tag{3}$$

where $U$ is the flow velocity, $\theta$ the yaw angle, $L$ and $\rho_{mat}$ are respectively the length and the material density of the cylinder, $C_a$ the added mass coefficient in still water, $\rho_{H_2O}$ the water density and $k$ is the stiffness constant.

In order to determine the variables with greater influence in the reduced velocity uncertainty, a comparative analysis between the contribution of each variable to the error propagation was carried out. For the comparison an initial value of uncertainty was applied for a preliminary evaluation of the reduced velocity uncertainty, equal for every variable, with the preliminary results the more important ones were obtained. The value used for the uncertainty of the variable with more influence was then analyzed to verify if it agreed with typical values found in experiments. The evaluation was made based on the experiment of VIV in yawed cylinders found in Franzini (2013), with a range of feasible values for the variable of interest.

## 3. RESULTS AND DISCUSSION

After calculations of partial derivatives of the function in relation to the independent variables, Eq. (2), the uncertainty for Vr is given by:

$$\left(\frac{\delta V_r}{V_r}\right)^2 = C_1 \left(\frac{\delta U}{U}\right)^2 + C_2 \left(\frac{\delta\theta}{\theta}\right)^2 + C_3 \left(\frac{\delta L}{L}\right)^2 + C_4 \left(\frac{\delta\rho_{mat}}{\rho_{mat}}\right)^2 + C_5 \left(\frac{\delta C_a}{C_a}\right)^2 + C_6 \left(\frac{\delta\rho_{H_2O}}{\rho_{H_2O}}\right)^2 + C_7 \left(\frac{\delta k}{k}\right)^2, \tag{4}$$

where the estimated errors are $\delta$ from the variables defined in Eq. (2), and Ci are constants defined as:

$$C_1 = 1; C_2 = \theta^2 tan^2\theta; C_3 = \frac{1}{4}; C_4 = C_3 \left(\frac{\rho_{mat}}{\rho_{mat}+C_a\rho_{H_2O}}\right)^2; C_5 = C_3 \left(\frac{C_a\rho_{H_2O}}{\rho_{mat}+C_a\rho_{H_2O}}\right)^2; C_6 = C_5; C_7 = C_3 \tag{5}$$

A first analysis is carried out by considering typical quantities of water density $\rho_{H_2O} = 1000\ kg/m^3$, material density of the model $\rho_{mat} = 2700\ kg/m^3$, added mass coefficient at still water $C_a = 1$, velocities up to $U = 1\ m/s$ and yaw angles in the range $0° \leq \theta \leq 60°$. Also, a maximum value of 2% is adopted for all the relative uncertainties.

The graphs on the left of Fig.1 directly compare the relative weight of each portion in Eq. (3).

Clearly, the reduced velocity calculation is quite sensitive to the uncertainty of dimensional velocity for yaw angles up to approximately 50 degrees, above which the uncertainty of measuring this angular quantity becomes gradually more important.

Curiously, according to many experiments found in literature the validity of the IP is lost for angles a little higher than 45 degrees, which is a behavior mostly assigned to the changes in the vortex shedding pattern.

Also, as explicit in Eq. (1), the model diameter and its uncertainty have no influence for the calculation of the reduced velocity uncertainty. Moreover, the uncertainties related to the elastically supported model are less important (namely its length, the material density and the structural stiffness), as well as those related to the fluid (its density and the added mass). In fact, the added mass cannot be measured directly which justify a deeper investigation about its influence on the reduced velocity determination.

Figure 1 on the right presents a visual comparison of the uncertainties in calculations of the reduced velocities as function of the uncertainties in measuring dimensional velocities and yaw angles. According to it, for the higher uncertainties related to velocities and angles, when calculating the reduced velocity, the uncertainties reach values up to $\partial V_r / V_r = 0.045$, which considered together with the changes in the flow pattern can explain the poor validity of the IP for higher angles of inclination.
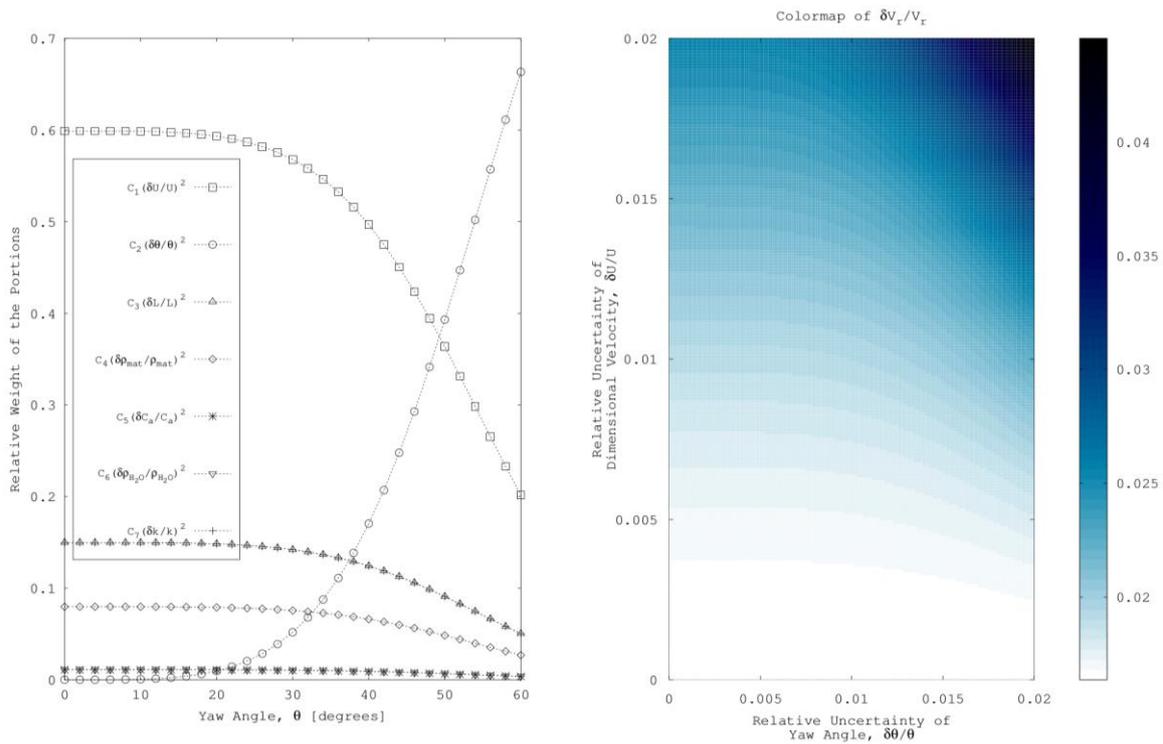
Figure 1. On the left, comparison between relative weight of each portion in Eq. (3). On the right, mapping of the relative uncertainty of the reduced velocity, $\partial V_r/V_r$, as function of combination of relative uncertainties of dimensional velocity, $\partial U/U$, and yaw angle, $\partial \theta/\theta$.

Since the velocity plays an important role in the determination of $\partial V_r/V_r$, for angles up to 20°, a proper evaluation was conducted to determine whether, or not, 2% is a good representation of the relative uncertainty of velocity.

### 3.1. Evaluation of the Relative Uncertainty of Velocity

The data was obtained from the work of Franzini (2013), an experiment with yawed cylinders in a wide range of flow velocities in a water channel. The cases evaluated are shown on Tab. 1 below, where seven different arrangements (conditions) were examined, due to the fact that the time histories of velocity were similar in magnitude in each of those arrangements. The conditions also consider variations in the yawed angle, heading of the cylinder, number of degrees of freedom (1DOF and 2DOF), resulting a total of 126 time histories of velocity analyzed.

Table 1. Details on the arrangements for which the velocities were evaluated in this paper.

| Yaw Angle | Heading | Degrees of Freedom |
|---|---|---|
| 0° | - | 1 and 2 |
| 10° | Downstream | 1 and 2 |
| | Upstream | 1 and 2 |
| 20° | Downstream | 1 and 2 |
| | Upstream | 1 and 2 |
| 30° | Downstream | 1 and 2 |
| | Upstream | 1 and 2 |
| 45° | Downstream | 1 and 2 |
| | Upstream | 1 and 2 |

From the experiment, for each time history, the mean value of velocity and its standard deviation were obtained, as seen in Tab. 2. These values were used to obtain the mean value of velocity for each of the seven velocities conditions (VC), as well as the standard deviation was a mean value from each record standard deviation. With the velocities for the

seven conditions and the standard deviation the relative uncertainty of dimensional velocity was attained, by dividing the standard deviation by the mean velocity, as seen in Tab. 2 and in the Fig. 2 below.

Table 2. Data used to determine the mean velocities and the standard deviations, to obtain the uncertainties.

| | VC 1 | | VC 2 | | VC 3 | | VC 4 | | VC 5 | | VC 6 | | VC 7 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean Velocity | Standard Deviation | Mean Velocity | Standard Deviation | Mean Velocity | Standard Deviation | Mean Velocity | Standard Deviation | Mean Velocity | Standard Deviation | Mean Velocity | Standard Deviation | Mean Velocity | Standard Deviation |
| 0° | 0.10729 | 0.0011494 | 0.13165 | 0.0022551 | 0.15737 | 0.0015516 | 0.1842 | 0.002115 | 0.21192 | 0.001949 | 0.24284 | 0.0043488 | 0.28125 | 0.0070501 |
| | 0.10779 | 0.0012089 | 0.1334 | 0.001065 | 0.16072 | 0.0018366 | 0.18506 | 0.0016018 | 0.21221 | 0.0015338 | 0.24003 | 0.0027004 | 0.26833 | 0.0028546 |
| 10° | 0.10624 | 0.0013936 | 0.13214 | 0.0012383 | 0.15957 | 0.0022498 | 0.18475 | 0.0026779 | 0.21716 | 0.0047106 | 0.24888 | 0.0068523 | 0.27118 | 0.0051945 |
| | 0.10661 | 0.0013393 | 0.13261 | 0.001323 | 0.161 | 0.001766 | 0.18662 | 0.0020259 | 0.21836 | 0.0067627 | 0.23852 | 0.0014147 | 0.27378 | 0.0040924 |
| | 0.10517 | 0.0016913 | 0.13226 | 0.0017518 | 0.15765 | 0.0022759 | 0.18439 | 0.0015313 | 0.20847 | 0.0014729 | 0.23856 | 0.0018385 | 0.26636 | 0.0046061 |
| | 0.10727 | 0.0013673 | 0.13348 | 0.0019594 | 0.15997 | 0.0023203 | 0.18541 | 0.0025102 | 0.21366 | 0.0018835 | 0.24011 | 0.0016174 | 0.26689 | 0.0019283 |
| 20° | 0.1082 | 0.0015231 | 0.13576 | 0.001571 | 0.16012 | 0.002669 | 0.18524 | 0.0019809 | 0.23102 | 0.0050775 | 0.25841 | 0.0070963 | 0.27401 | 0.0028483 |
| | 0.1109 | 0.0016424 | 0.1371 | 0.0019857 | 0.1709 | 0.0045862 | 0.1992 | 0.0041027 | 0.22804 | 0.0064706 | 0.25119 | 0.0037045 | 0.27734 | 0.0026538 |
| | 0.11007 | 0.0010603 | 0.13833 | 0.0018163 | 0.16693 | 0.001303 | 0.19483 | 0.0025818 | 0.22109 | 0.0027999 | 0.25053 | 0.0021732 | 0.28179 | 0.0052899 |
| | 0.11115 | 0.0012851 | 0.14014 | 0.0013645 | 0.16845 | 0.0015215 | 0.19464 | 0.0018413 | 0.22431 | 0.0024225 | 0.25152 | 0.0020721 | 0.27945 | 0.0028253 |
| 30° | 0.11491 | 0.0015246 | 0.14475 | 0.0013919 | 0.17338 | 0.0021177 | 0.21392 | 0.006802 | 0.23556 | 0.0032561 | 0.26203 | 0.0030645 | 0.29603 | 0.0026145 |
| | 0.11974 | 0.001929 | 0.14699 | 0.0017865 | 0.17611 | 0.0033735 | 0.20588 | 0.0023444 | 0.23883 | 0.0046528 | 0.26766 | 0.0033645 | 0.2944 | 0.0039367 |
| | 0.11674 | 0.0014661 | 0.14575 | 0.0020938 | 0.17987 | 0.0049721 | 0.2015 | 0.0038024 | 0.23629 | 0.004851 | 0.27118 | 0.0078516 | 0.29925 | 0.0093589 |
| | 0.12063 | 0.0014632 | 0.14738 | 0.0026326 | 0.18037 | 0.0026966 | 0.20879 | 0.0037358 | 0.23891 | 0.0051759 | 0.26608 | 0.0033242 | 0.29656 | 0.0041913 |
| 45° | 0.13862 | 0.001661 | 0.17128 | 0.002188 | 0.207 | 0.0027933 | 0.24144 | 0.0030088 | 0.27645 | 0.0033061 | 0.30972 | 0.0047612 | 0.3446 | 0.0052414 |
| | 0.14136 | 0.0026698 | 0.17607 | 0.0026717 | 0.21241 | 0.003001 | 0.25142 | 0.0042885 | 0.28833 | 0.0063131 | 0.32908 | 0.0066494 | 0.36507 | 0.0054951 |
| | 0.13953 | 0.0031077 | 0.17527 | 0.002835 | 0.21133 | 0.0055889 | 0.24479 | 0.0046271 | 0.28317 | 0.0048073 | 0.32428 | 0.012264 | 0.35045 | 0.0063923 |
| | 0.13754 | 0.0024104 | 0.17351 | 0.0030513 | 0.22409 | 0.0066335 | 0.25071 | 0.0069732 | 0.29972 | 0.0068176 | 0.34533 | 0.0065618 | 0.37864 | 0.007853 |
| Final Mean Veloctiy | 0.111025 | | 0.139235 | | 0.169675 | | 0.197015 | | 0.22953 | | 0.254965 | | 0.28152 | |
| Final Standar Deviation | 0.0014946 | | 0.00188785 | | 0.00249465 | | 0.00262985 | | 0.0046817 | | 0.0035345 | | 0.0043987 | |
| Uncertainty | 0.013461833 | | 0.013558732 | | 0.01470252 | | 0.013348476 | | 0.020396898 | | 0.013862687 | | 0.015624822 | |

As can be observed in the graphic on Fig. 2, the relative uncertainty follows an approximate linear progression with the velocity, being more affected by the fluctuations of the phenomenon, and less by the experimental setup. As one can see, the relative uncertainty of velocity varies in a range from 0.013 up to 0.019, then a variation of 0.02, or 2%, is plausible for the matter of the previous analysis herein conducted.
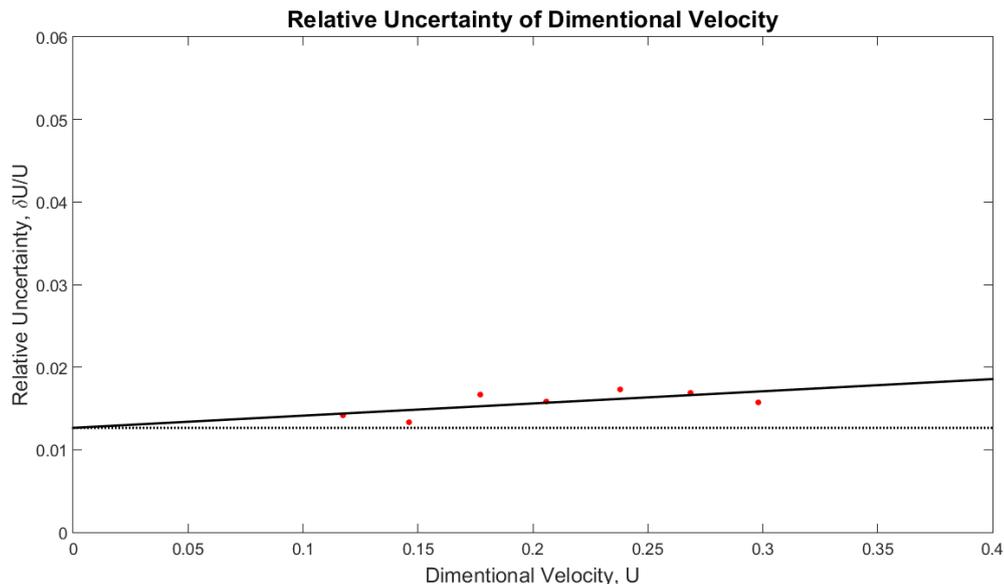


Figure 2. Relative uncertainty as function of the dimensional velocity, used as a base for defining a plausible value for evaluations under IP.

The next step is to evaluate influence of the yaw angle, since as shown in Fig. 1 its importance increases considerably for angles bigger than 20°.

The first approach was with a relative uncertainty of 2%, as all other variables, resulting an error equivalent to 1° for a desired yaw angle equal to 45°, which is a very acceptable and possible condition for carefully assembled arrangements in VIV investigation of yawed cylinders.

However, considering all the difficulties for experimental angular assemblies, it is not impossible an error about 5%, or even 10%, being that, during an experimental setup the support of the model, or even the installation of the model itself, can show some undesired inclination. In Fig. 3 the influence of this proposed uncertainties in the reduced velocity is presented.
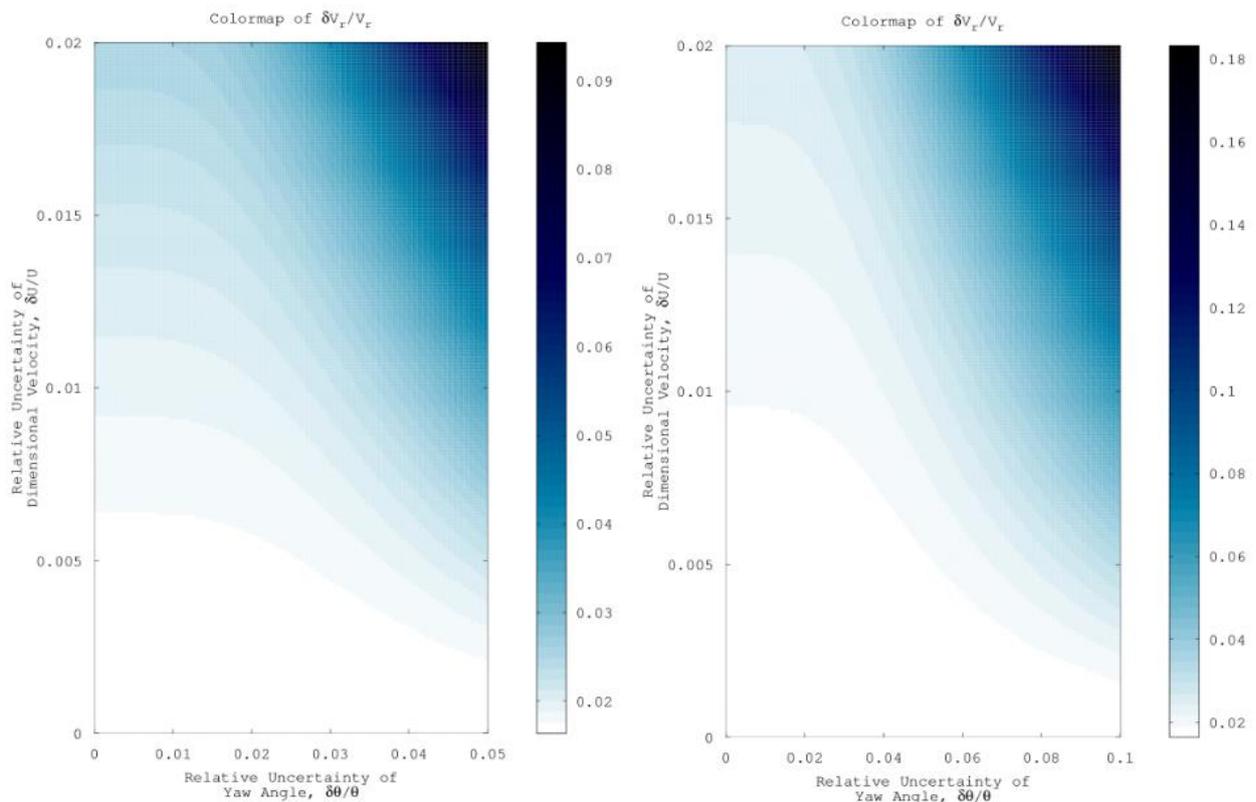
Figure 3. Relative Uncertainty of Reduced Velocity, for $\delta\theta/\theta$ of 5% (left), and 10% (right).

For a relative uncertainty of 5% in the yaw angle, the relative uncertainty in the reduced velocity can reach values close to 10%, and for an angle uncertainty of 10%, the reduced velocity uncertainty can get up to 18%, values that result in uncertainties of approximately +/-1 unit in the magnitude of reduced velocity. Accordingly, by considering a reduced velocity of 8 for instance, this desired value in fact would correspond to any reduced velocity varying from 6.56 to 9.44 for an error of 10% in the yaw angle (4.5°), what can invalidate the prescribed result by IP and even the principle itself.

## 4. CONCLUSIONS

Uncertainty analysis is applied to the equation for calculation of the reduced velocity in experiments of VIV on yawed cylinders. The results show that the uncertainties related to the dimensional velocity plays the most important role for cylinders inclined up to 20 degrees. For higher angles the importance relies in the uncertainty of the yaw angle, since an error of 5% in the angle, can add up to a 10% error in the reduced velocity, compromising the results of the experiment.

## 5. ACKNOWLEDGMENTS

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