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COUPLED INTEGRAL EQUATIONS APPROACH ON THE TRANSIENT HEAT TRANSFER PROBLEM IN SUPERCRITICAL FLUIDS

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Abstract. *This work aims to use the Coupled Integral Equations Approach (CIEA) to simplify the transient heat transfer problem in supercritical fluids, and to compare these solutions with results obtained by the Generalized Integral Transform Technique (GITT). It is shown that for certain combinations of boundary conditions there exists a transient overheating in the fluid, which raises the bulk temperature above the steady state temperature. The CIEA formulations with low order of approximations present good results in the absence of overheating, but are unable to capture the bulk temperature profile that exists on its presence. On the other hand, the solutions using higher orders of approximation show the transient overheating and present good results with respect to the the solutions obtained by the GITT. A non-orthogonality analysis is done for the simplified equations obtained by the CIEA formulation in an attempt to justify why this ability of capturing the overheating is only present in the higher order approximations.*

Keywords: *Coupled Integral Equations Approach, CIEA, supercritical fluids, piston effect*

1. INTRODUCTION

The study of supercritical fluids is of high interest to both Physics and Engineering due to its wide range of applications. Because of this, it is necessary to understand its properties and behaviors. It is known that many physical properties of fluids diverge near their critical points, which is the so called critical phenomenon. This rapid change on its properties has great impact on its heat transfer.

Because of this divergence, the thermal diffusivity of a fluid suffers a large drop near the critical point (Alves, 2012). For this reason, it is expected that it takes a long time for the thermal equilibrium to be achieved. However, Dahl and Moldover (1972) show that the heat transfer happens a lot faster than what it was expected due to the low thermal diffusivity. At first, the appearing of natural convection was considered responsible for that, until Nitsche and Straub (1987) showed with experiments in low gravity that this wasn't the case.

Today it is known that the reason for this increase on the heat transfer is the high compressibility of fluids near their critical point. In a closed cavity, a small increase in temperature makes the thermal boundary layer to expand, compressing the rest of the fluid like a piston, generating thermal-acoustical waves. The propagation and reflection of these waves is called piston effect, and causes the fluid to increases its temperature rapidly (Pinheiro and Alves, 2015).

Boukari *et al.* (1990) and Onuki *et al.* (1990) initially developed a thermodynamical model for the heat transfer in supercritical fluids. Nikoyalev *et al.* (2003) showed that this model agrees well with the heat transfer in the presence of the piston effect, by comparing its results with those obtained by direct numerical simulation of the compressible Navier-Stokes equations with the energy conservation equation. Teixeira and Alves (2014) utilize the Generalized Integral Transform Technique (GITT) to obtain an analytical solution for the problem, and shows that, for some combination of boundary conditions, there is an increase in the bulk temperature which goes above the steady state temperature.

The main objective of this work is to reproduce the results obtained with the GITT by using the Coupled Integral Equations Approach (CIEA) to simplify this thermodynamical model. In addition to that, a non-orthogonality analysis is made in an attempt to explain the capacity of these simplified equations to describe the increase in the bulk temperature.

2. MATHEMATICAL MODEL

The thermodynamical model proposed by Boukari *et al.* (1990) e Onuki *et al.* (1990) can be described by the adimensionalized equation

$$\frac{\partial T}{\partial \tau} - \left(1 - \frac{1}{\gamma}\right)\frac{dT_b}{d\tau} = \frac{\partial^2 T}{\partial \xi^2}. \tag{1}$$

The parameter γ determines how important is the piston effect on the specific problem. Notice that when $\gamma = 1$, on the incompressible limit, the classic thermal diffusion equation is recovered. The bulk temperature Tb is defined as

$$T_b(\tau) = \int_0^1 T(\xi,\tau)d\xi. \tag{2}$$

The initial conditions and boundary conditions for both cavity walls are given by

$$T(\xi, 0) = 0, \tag{3}$$

$$-\left.\frac{\partial T}{\partial \xi}\right|_{\xi=0} + Bi_L T(0,\tau) = 1 + Bi_L, \tag{4}$$

$$\left.\frac{\partial T}{\partial \xi}\right|_{\xi=l} + Bi_R T(1,\tau) = 0. \tag{5}$$

The parameters Bi_L and Bi_R are the Biot number on the left and right wall, respectively. Tables 1 and 2 show how this parameter can be used to define the different possible combinations of boundary conditions.

Table 1: Boundary conditions for the left wall

General boundary condition (Robin's)	Dirichlet's condition	Neumann's condition
-	$Bi_L \to \infty$	$Bi_L = 0$
$-\left.\frac{\partial T}{\partial \xi}\right\|_{\xi=0} + Bi_L T(0,\tau) = 1 + Bi_L$	$T(0,\tau) = 1$	$-\left.\frac{\partial T}{\partial \xi}\right\|_{\xi=0} = 1$

Table 2: Boundary conditions for the right wall

General boundary condition (Robin's)	Dirichlet's condition	Neumann's condition
-	$Bi_R \to \infty$	$Bi_R = 0$
$\left.\frac{\partial T}{\partial \xi}\right\|_{\xi=1} + Bi_R T(1,\tau) = 0$	$T(1,\tau) = 0$	$\left.\frac{\partial T}{\partial \xi}\right\|_{\xi=1} = 0$

The CIEA proposed by Cotta and Corrêa (1998) will be used to simplify this problem by using an approximation for the bulk temperature and the heat flux. These approximations are made using an improved lumped parameters formulation as proposed by Mennig *et al.* (1983), to estimate the value of an integral of a function by using the function's values on the extremes of the integration interval as shown in eq. 6.

$$\int_{x_0}^{x_1} y(x)dx \approx \sum_{\nu=0}^{\alpha} c_\nu(\alpha,\beta)h^{\nu+1}y^{(\nu)}(x_0) + \sum_{\nu=0}^{\beta} c_\nu(\beta,\alpha)(-1)^\nu h^{\nu+1}y^{(\nu)}(x_1), \tag{6}$$

where $h = x_1 - x_0$, and $\alpha; \beta$ determine the order of approximation, with

$$c_\nu(\alpha,\beta) = \frac{(\alpha+1)!(-\nu+\alpha+\beta+1)!}{(\nu+1)!(\alpha-\nu)!(\alpha+\beta+2)!}. \tag{7}$$

3. RESULTS

The combination of boundary conditions for which the overheating appears is Dirichlet's condition in one of the cavity walls and Robin's condition on the second one. All the results shown have prescribed temperature on the left wall ($Bi_L \to \infty$). Figure 1 shows that, for $Bi_R = 1$, the overheating exists for certain values of γ.

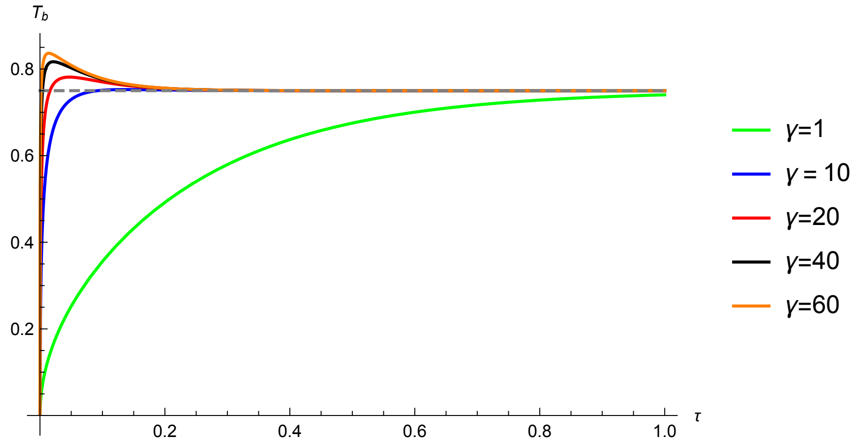
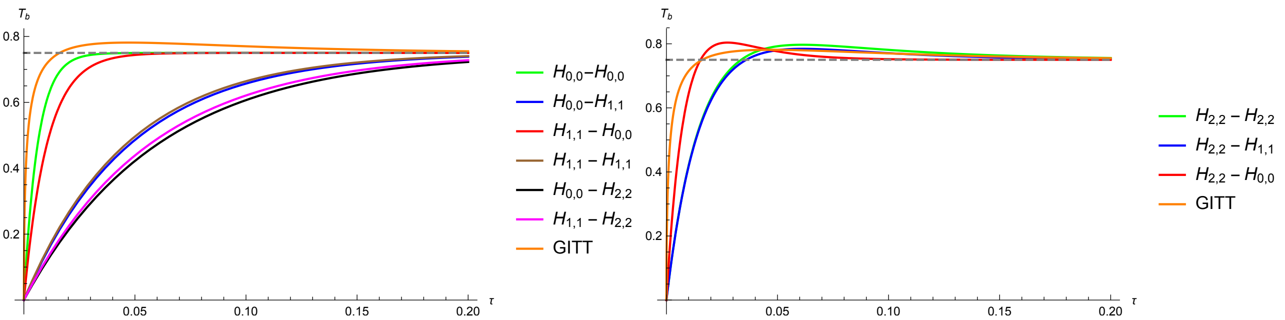


Figure 1: GITT solutions for different values of γ. $Bi_R = 1$.

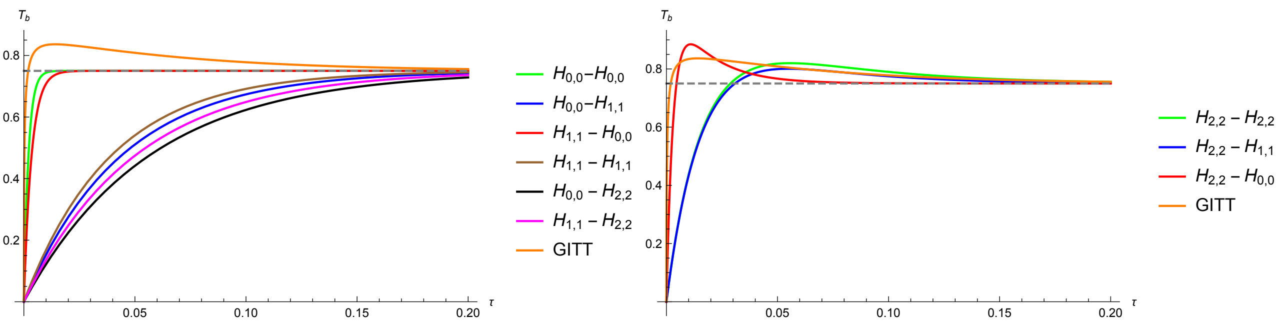
On the cases where the overheating is present, the CIEA using approximations of order 0 and 1 were not able to capture the expected behavior. However, when using 2nd order of approximation on the formulations for the bulk temperature, the overheating was observed on the CIEA results, agreeing with those obtained by the GITT. This can be observed for different values of γ. Figures 2a and 2b show the results when $\gamma = 20$. Similarly, for $\gamma = 60$, figures 3a and 3b also show that only the higher order approximations were able to capture the expected behavior.



(a) Order 0 and 1

(b) Order 2

Figure 2: CIEA solutions with different orders of approximation for the bulk temperature. $\gamma = 20$ and $Bi_R = 1$.

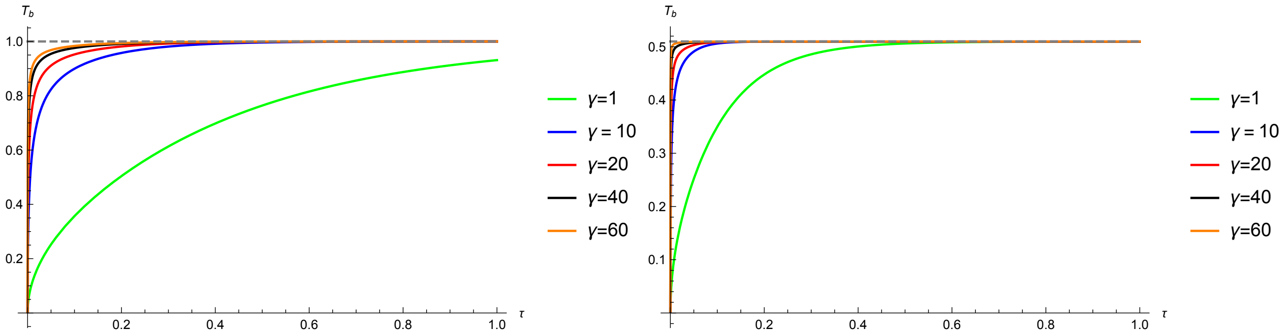


(a) Order 0 and 1

(b) Order 2

Figure 3: CIEA solutions with different orders of approximation for the bulk temperature. $\gamma = 60$ and $Bi_R = 1$.

The non-orthogonality of the equations obtained by the CIEA can be quantified by the condition number, defined as $\|\mathbf{S}\|^2\|\mathbf{S}^{-1}\|^2$, where $\mathbf{S}$ is the eigenvector matrix of the system. The GITT results for the problem shows that the overheating does not happen when the values of Bi_R goes to 0 (Neumann's condition) or to infinity (Dirichlet's condition), but only for intermediate values. This can be observed on figures 4a and 4b.



(a) $Bi_R = 0$. (b) $Bi_R = 50$.

Figure 4: GITT solutions for different values of γ.

The figure 5 shows that, for the 2nd order approximations for the bulk temperature, the condition number has the same behavior, which sugests that the capture of the overheating by the simplified equations is indeed caused by a non-orthogonality. Notice that the lumped parameter approximation doesn't present good results for high values of Bi_R, which could explain why the condition number doesn't decays for large values of Bi_R. It is important to note that the eigenvalues of the problem are all negative, and therefore the classic modal stability analysis doesn't indicate any reason for the overheating. When using 3rd order of approximations, the correlation between the condition number and the appearing of the overheating is even stronger, as shown in figure 6. This approach offers yet the possibility to estimate the parameter Bi_R which leads to the maximum overheating.

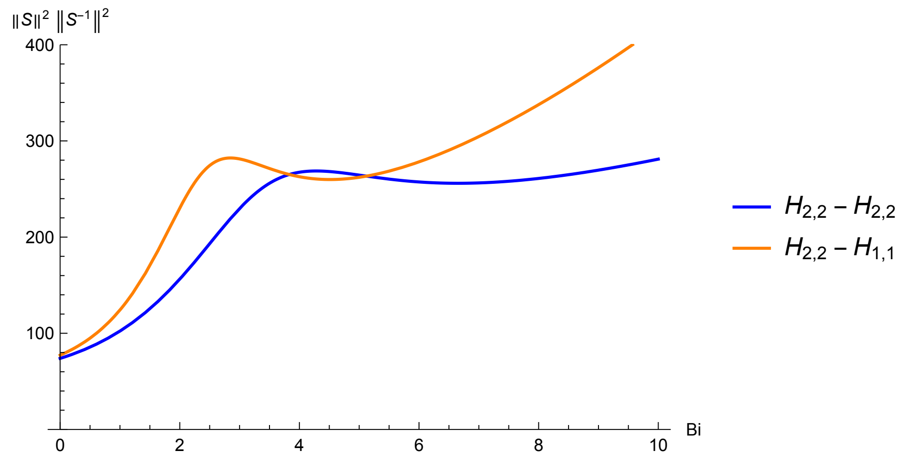


Figure 5: Condition number as a function of Bi_R for 2nd order approximations. $\gamma = 60$.

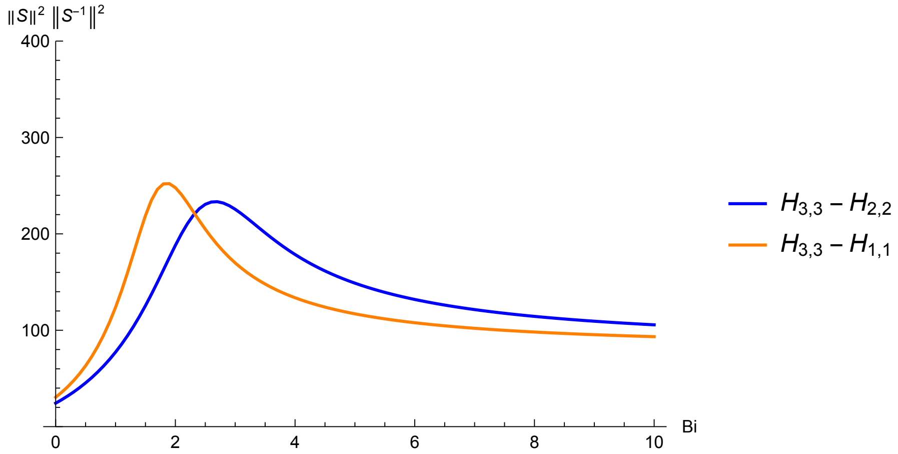


Figure 6: Condition as a function of Bi_R for 3rd order approximations. $\gamma = 60$.

4. CONCLUSIONS

This paper analyzed the possibility of using the CIEA method to solve a transient heat transfer problem in supercritical fluids, when the combination of boundary conditions induce an overheating of the bulk temperature.

It has been observed that this method, with order 0 and 1 of approximation for the bulk temperature, is not effective in capturing the correct behavior for the system over time. However, the CIEA with 2nd order approximations for the bulk temperature is shown to be able to represent the overheating present on the cases studied.

Also, the non-orthogonality analysis showed that the ability of the model to capture this overheating is related with the presence of high condition numbers for the problem. On these models, this parameter peaks at the values of Bi_R for which the GITT solutions show the presence of overheating. This correlation is even stronger for higher orders of approximation, and present a possibility to estimate a range of values for Bi_R for which the overheating is at maximum amplitude.

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