

COBEM-2017-0749

MATHEMATICAL MODELING OF HEAT TRANSFER IN RICE VERTICAL SILO

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Abstract. *The thermal state of the rice mass in steel vertical silo with an aeration system, located in a rice storage facility in Rio Grande do Sul State, Brazil, was studied. The experimental data obtained by thermometry system were compared with the predicted data obtained by the mathematical model proposed. The model presents the system of two partial differential equations, describing the heat transfer and conservation of energy for air and for grain mass. The proposed model satisfactorily describes the thermal state of the studied silo. The use of the last data reading as a new initial condition increases the simulation accuracy. In the case of a significant transverse temperature inhomogeneity, a hypothetical division of the silo into independent vertical cylinders centered on the respective cables permits to increase the accuracy.*

Keywords: *Mathematical Modeling, Silo, Aeration, Heat Transfer.*

1. INTRODUCTION

In order to preserve the rice grain quality, it is important to have a storage system capable to provide a safe environment. The temperature condition inside the grain silo with aeration depends on several factors such as the ambient air temperature, air flow rate, the temperature of the grain in neighboring layers and the amount of aeration time employed (Navarro and Noyes, 2001). It is also important to note that temperature uneven distribution occurs not only along the height of the silo, but also along its cross-section. This happens because of uneven external temperature conditions associated with exposure to solar radiation on silo metallic surface, another factor is the non-uniformity of aeration air distribution along the grain mass as well as other factors such as the presence of impurities in the grain mass. To measure the humidity and temperature inside the silo it is recommended to use an automatic system with sensors and computer control (Lopes *et al.*, 2008).

The grain sub-domains in the silo where the temperature is too high are considered risk areas and must have its temperature decreased by the use of aeration. These areas can evolve to catastrophic conditions and could compromise the quality of the whole grain mass stored in the silo. In theory, the use of a large number of temperature sensors inside the silo could allow a precise detection of such areas, their evolution, allowing the selection of a suitable aeration mode. But in practice, the amount of temperature sensors that can be installed in a cost effective fashion is heavily reduced. The modeling the dynamics of temperature in the grain mass could allow the prediction of the thermal state in each point of the silo. However, the difference between the measured and the calculated temperature fields can become too large with time and with changes in boundary conditions. Therefore, a combination of grain mass thermal state modeling with the use of the sensors to obtain new initial data for simulation appears to be a suitable strategy for the temperature control of the grain mass during the storage period.

There are models in the literature that satisfactorily describe airflow in silos under isothermal conditions Khatchatourian and Savicki (2004). The silo thermal state simulation in the presence non-isothermal conditions presents difficulties related to the absence of reliable data for heat transfer coefficients between grain and air, the coefficients of thermal diffusivity of the grain and of the grain layer. Taken this into consideration, drying models Brooker *et al.* (1974); Sander-son *et al.* (1988); Montross and Maier (2000); Lamrani *et al.* (2001); Kurpaska *et al.* (2004); Khatchatourian *et al.* (2013) can be used for the thermal state of the grain mass calculation.

Even though the drying models concern the moisture dynamics, the grain moisture variation of grains stored in silos

is negligible when compared to the variation of grain temperature during the same period. Considering these conditions, to calculate the temperature field, it is possible to use a model that neglects moisture transfer. Some of these models were presented by Bakker-Arkema and Bickert (1966); Foster (1967); Khatchatourian and Oliveira (2006); Oliveira *et al.* (2007).

The main objectives of the present work are: 1. To experimentally study the dynamics of temperature variation of the grain mass inside a real silo with aeration system; 2. To develop a mathematical model to calculate the thermal state of the grain mass inside the silo. 3. To compare simulation results with experimental data to validate the model.

2. THE OBJECT STUDY DESCRIPTION

A vertical steel silo, from a rice storage facility located in Rio Grande do Sul State, Brazil was used to obtain the experimental data. The silo was equipped with a thermometry system, composed of 6 thermometry cables with 10 temperature sensors in each lateral cable and 11 sensors in the central cable one, and an aeration system. The silo dimensions were 18.19 m of diameter and 23.61 m at its central height, with a storage capacity of $3.1 \times 10^3 t$. Even though the silo was not completely loaded, Sensor 11 of the central cable was not considered in this work, since the silo was not completely loaded. The temperature sensors were T type thermocouples and their layout is shown in Fig 1a. The thermometry cables were located in the centres of equal areas of cross-section of the silo (Fig 1b). The sensors were connected to a computer automation system. Also connected to the system was a weather station measuring the air temperature, humidity and rain outside of the silo. For this work, only the air temperature was considered.

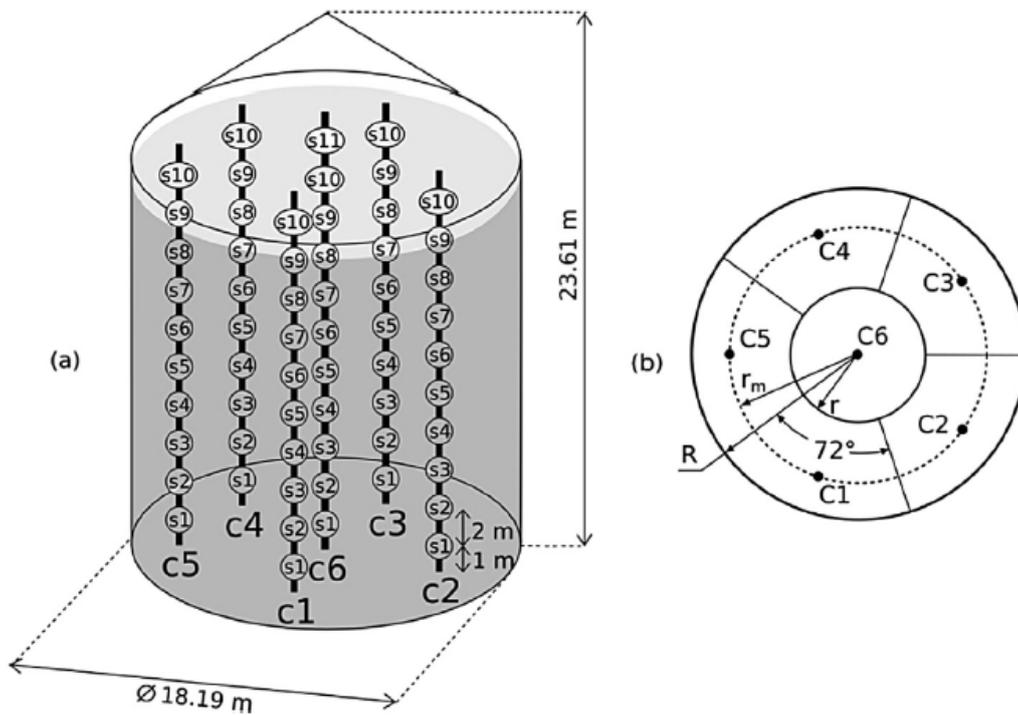


Figure 1. Outline sketch of studied silo (a) and (b): division of cross-section of the silo to locate the thermometry cables in the centres of equal areas: R is radius of silo; $r_m = \sqrt{21}R/6$ is radial location of peripheral cables; $r = R/\sqrt{6}$ is radius of central area for cable C6.

The automation system registered, in a database, weather data at 10 min intervals, and a complete report of the thermometry system twice a day, at 07:00 and 14:00. An important aspect taken into account was the low resolution (1°C) of the sensors and their error ($\pm 1^\circ\text{C}$). Despite the relatively low accuracy of the thermometers used, the results of the analysis showed that the resolution of the sensors was adequate sufficient characterise the temperature behaviour of the grain subjected to the aeration system.

The aeration system was composed of two ventilators powered by 23 kW electric motors. The specific volume airflow rate was calculated from the fan performance for known values of pressure and rotor speed and maintained in all experiments of $Q = 9 \text{ m}^3 \text{ h}^{-1} \text{ t}^{-1}$ ($2.5 \times 10^{-6} \text{ m}^3 \text{ s}^{-1} \text{ kg}^{-1}$), which is the most recommended values in aerated grain storage. At the silo inlet, a constant pressure of 600 Pa was maintained, measured with a manometer accurate to 0.5%.

The automation system registered in the database all turning on and turning off events of the ventilators, with this information it was possible to know the time intervals when the aeration system was working and what were the weather

conditions during these intervals. The data used in this work were obtained from 13 November 2015 to 15 November 2015.

The system was turned on in 13 November at 08:30 and turned off at 17:55. Later, in the same day, it was turned on at 22:42 and stayed working until the next day 14 November at 14:00. Total experiment time can be divided into 8 periods. Figure 2 shows the periods in accordance with their duration (min). The period boundaries were determined by the time on and off the aeration or by the moment of data reading. Numbers of period boundaries, enclosed in a rectangle, correspond to the moments of the recording the temperature distribution in a silo.

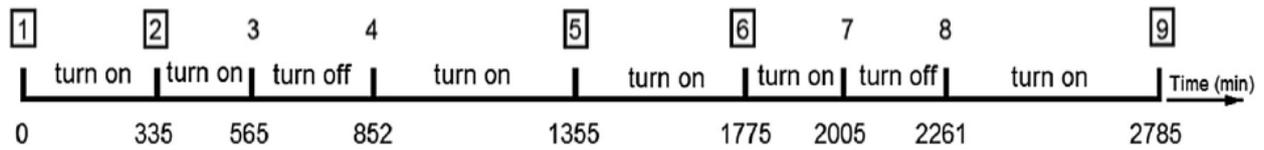


Figure 2. The scheme of the experiment: period boundaries with numbers 1, 2, 5, 6 and 9 correspond to the moments of recording the temperature in the silo.

3. MATERIALS AND METHODS

The experimental data were obtained from a steel vertical silo, equipped with an aeration system and a thermometry system. The silo was located in a rice storage facility in Rio Grande do Sul State, Brazil. In the present study a mathematical model for calculating the thermal state of the silo is offered and compared with the experimental data.

The proposed model was obtained by simplifying the system of equations used by Khatchatourian *et al.* (2013) to calculate the drying process of grain. Since the grain mass before filling the silo is exposed to artificial drying, during storage the grain moisture varies very slowly if compared with the changes in temperature of grain and ventilation air. Therefore, the equations related to changes in grain moisture and humidity were eliminated from the system. The resulting model presents a system of two partial differential equations (PDE) describing the energy conservation of the air and the grain mass:

$$\frac{\partial T_a}{\partial t} + V \frac{\partial T_a}{\partial y} = \frac{-a(1 - \varepsilon)\phi_m c_{pv}(T_g - T_a) + \phi_h}{\rho_a \varepsilon (c_{pa} + c_{pv}W)} \quad (1)$$

$$\frac{\partial T_g}{\partial t} = \frac{a \{(\phi_h - \phi_m) [H_v + (c_{pv} - c_{pw}) T_g]\}}{\rho_g (c_g + M c_{pw})}$$

where W is the air humidity; M is the grain moisture content, d.b.; a is the grain surface area/volume ratio in m^{-1} ; H_v is the latent heat of the water vaporization in $J kg^{-1}$; c_g is the specific heat of the grain in $J kg^{-1} K^{-1}$; c_{pv} is the specific heat of the waters vapour in $J kg^{-1} K^{-1}$; c_{pw} is the specific heat of the water in $J kg^{-1} K^{-1}$; ρ_g is the specific mass of the grain in $kg m^{-3}$; ρ_a is the specific mass of the air in $kg m^{-3}$; ε is the porosity; v is the vertical velocity in $m s^{-1}$; T_a is the air temperature in $^{\circ}C$; T_g is the grain temperature in $^{\circ}C$; ϕ_h is the heat flux in $W m^{-2}$ and ϕ_m is the mass flux in $kg m^{-2} s^{-1}$. For the calculation of the thermal state of the silo filled with pre-dried grain mass, the mass flux was considered equal to zero, i.e., $\phi_m = 0$.

The calculations were performed by the method of finite differences using the Crank-Nicholson scheme which has absolute convergence (Anderson *et al.*, 1984; Crank and Nicholson, 1947). To determine the grain surface area/volume ratio a , the average geometric characteristics of the grain were determined using digital image processing (Khatchatourian *et al.*, 2008). The heat flux ϕ_h was calculated in accordance with the work of Khatchatourian and Oliveira (2006).

4. RESULTS AND DISCUSSION

Figure 3 shows the experimental distribution of grain temperature along the height of the silo (points), obtained by 6 thermometry cables for the 5 time points.

The variance analysis (Tab. 1) showed that the variation of temperature with radial position (influence of the thermometry cables) is not significant. At the same time, this analysis showed a strong difference between time points (an influence of time) and between heights. These results justified the application of one-dimensional non-stationary models to simulate the thermal state of studied silo.

Applying the presented models and the programs developed by the authors with standard Matlab language, numerical simulations of the thermal state of the studied silo with ventilated grain mass were performed, using *MathWorks Matlab*

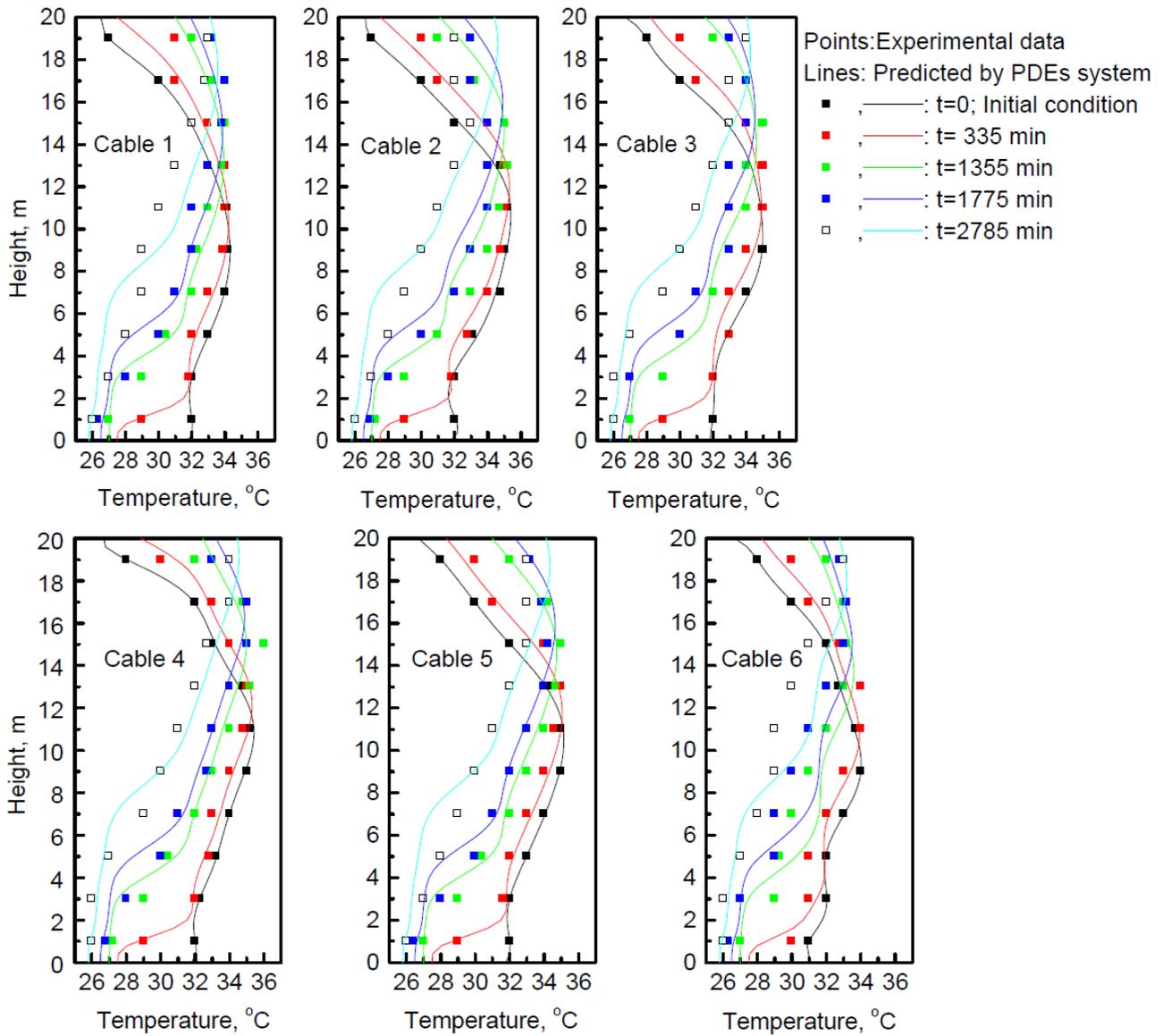


Figure 3. Thermal state variation of rice mass in the studied silo: the experimental (points) and predicted by the PDE system data.

Table 1. Analysis of variances of the experimental factors for the thermal state of studied silo.

Variation source	df	SQ	MQ	F	Pp value	F critical
Between time points	4	246.6	61.66	25.81	0	2.404**
Between cables	5	9.470	1.894	0.793	0.556	2.246 ^{n.s.}
Between heights	9	878.5	97.61	40.86	0	1.913**
Residual	281	671.2	2.389			

n.s., means not significant.

** p value < 0.01

2009a software package under Windows 8 64 bits operation system on an Intel Core i7-4000 at 2.40 GHz with 8 GB RAM computer. Figure 5 present the results of calculations and experimental data. The data points in Fig. 5 represent the arithmetic average temperature in the cross sections for each silo height.

As shown in Tab. 2, the proposed model is able to predict the temperature behavior of the silo with reasonable accuracy considering the sensors limited resolution (1°C) and limited precision ($\pm 1^\circ\text{C}$). We can see that the distance in time increases the results discrepancy. To mitigate this problem we can use the successive measurements as new initial

conditions for the model, as shown in Fig. 4.

Table 2. The sum of squared errors (sse) of prediction and the coefficient of determination (R^2) for studied models EDP.

Ordinal number of measurement	sse	R^2
1st measurement	0	1
2nd measurement	0.9724	0.9836
3rd measurement	2.6308	0.9637
4th measurement	2.7061	0.9742
5th measurement	7.1627	0.9551

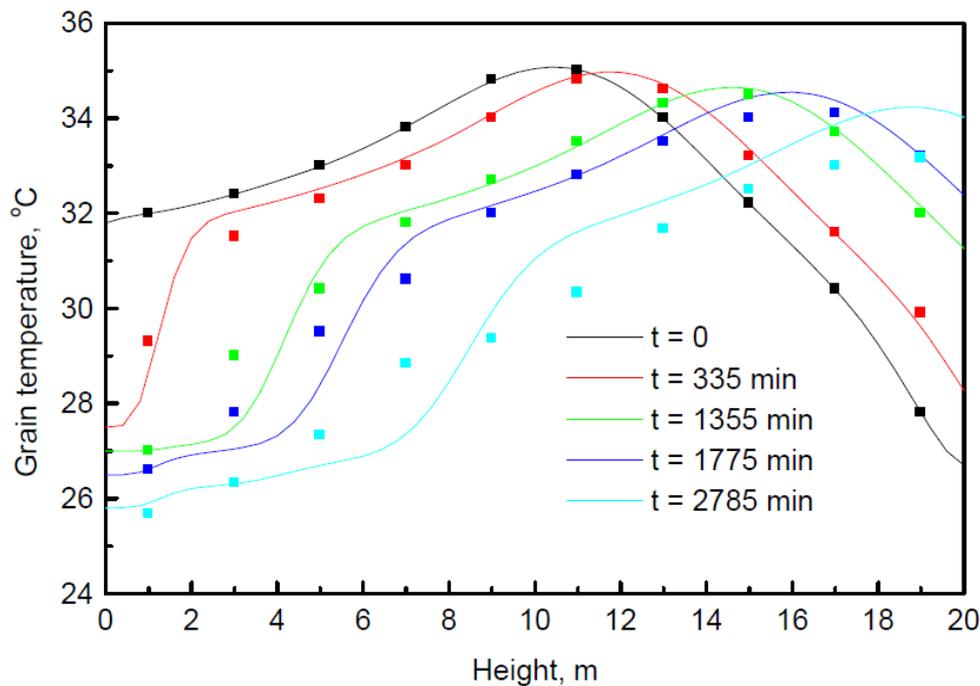


Figure 4. The grain temperature distribution along the height of the silo by time; comparison between the experimental (points) and predicted (lines) data: the points are average experimental temperatures by cross-section and the solid lines are the solution of the PDE system.

Figure 5 shows the results of these calculations performed for two time intervals $t \in [0; 335]$ and $t \in [1335; 1775]$ in minutes. The solid blue line corresponding to the local initial condition, much better describes the experimental points ($sse = 0.3520$), than the dotted line corresponding to the calculation with global initial condition ($sse = 2.7061$). These results show that the proposed models predict the grain mass temperature with satisfactory accuracy over a long period of time and with good accuracy if the chosen data reading intervals are less or equal to 6 h. Obviously, the combination of thermal state modelling with reasonable use of a thermometry system represents a good tool for continuous grain mass temperature control at each silo point.

Despite the some nonuniformity of distribution of air entering the silo, calculations show that the current lines are practically parallel to each other throughout the height of the silo. This allows the use of the proposed one-dimensional models, even in the presence of a substantial transverse inhomogeneity in temperature in the silo. To substantially reduce the influence of the nonuniformity of transverse temperature, when a one-dimensional model is used, the studied silo can be conventionally divided into several cylindrical elements (according to the number of thermometry cables).

The obtained cylindrical elements are centred around the respective cable and have the same height, equal bases (Fig. 1b), and hence have equal volumes. Calculation of the temperature distribution can be made separately for each of the cylinders, using as the initial condition the first temperature measurement by the corresponding cable. The results of such calculations obtained by solution PDE system for each of six cylinders of the same volume, centred around the thermometry cables, are shown in Fig. 3 (solid lines). Divergences between the measured and predicted data, estimated by the sse (and R^2), were about the same both for all six subdomains calculated separately, and for the whole cylinder when averaging the experimental temperature in a cross section. Also, division of the total volume into subdomains results in smaller local errors in temperature estimation than the temperature calculation for the whole cylinder produces by averaging the experimental temperature across the crosssection. This is due to the individual (i.e. not averaged) choice

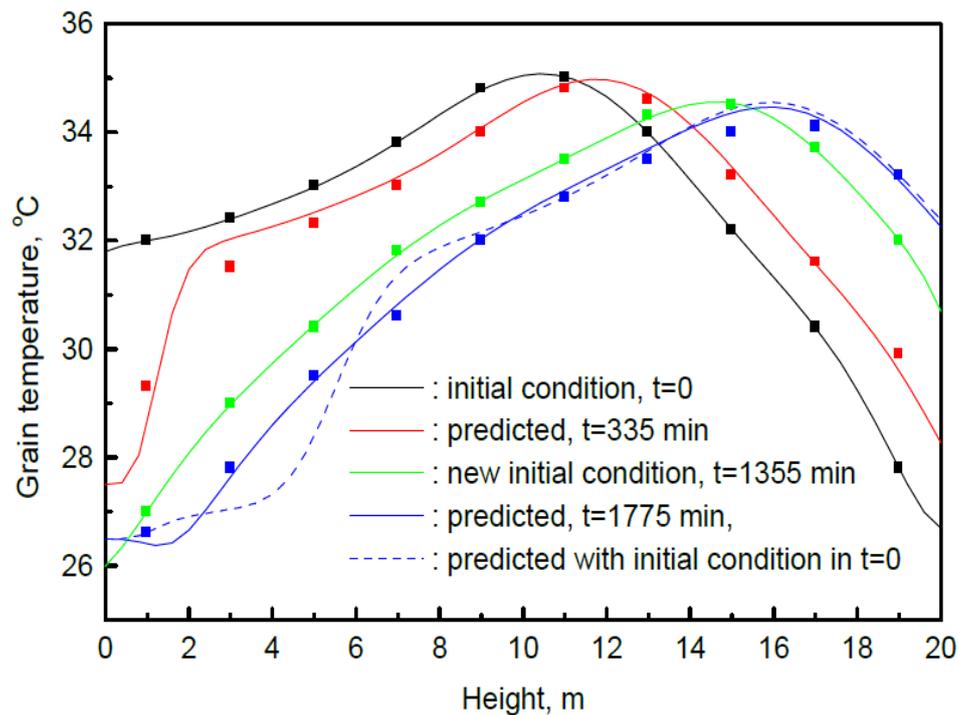


Figure 5. Reduction of the discrepancy between the experimental and predicted data by the use of a last data reading as new initial condition. The points are average experimental temperatures by crosssection; the red and blue solid lines are the solution of the PDE system; the black and green solid lines are fitted curves (a fifth degree polynomial); the dash line is the solution of the PDE system with initial condition in $t = 0$.

of the initial temperature distribution appropriate for the subdomain, centred around the corresponding cable.

5. CONCLUSIONS

The proposed model was able to predict the temperature behavior of the grain mass in rice silo equipped with thermometry and aeration systems. The mathematical model coupled with the automation system can be used for better decision making of the amount of aeration necessary for the system as well as the best time intervals for the procedure, resulting in a more efficient use of resources. In the case of a significant transverse temperature inhomogeneity, a hypothetical division of the silo into independent vertical cylinders centered on the respective cables permits to increase the accuracy.

6. ACKNOWLEDGEMENTS

Thank you UNIJUÍ for the opportunity and a Rice Storage Company in RS, for help and availability in the collection of experimental data.

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