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# ATTITUDE CONTROL OF A PENDULUM USING TWO REACTION WHEELS

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***Abstract.*** *The attitude control using reaction wheels as actuators has been one of the most popular ways to stabilize and to reject external disturbances in aerospace devices. From the controlled change of the angular momentum rate using the reaction wheels, it is possible to control the oscillation and direction rate of change on rigid bodies. This paper is going to show the modeling of a two reaction wheels pendulum and its linear and nonlinear control. The present paper presents a two reaction wheel pendulum been controlled in two different situations: in the inverted position and with tracking control. A comparison between PID controller with some adaptations and a sliding mode control is also performed.*

***Keywords:*** *Reaction wheels, attitude control, nonlinear control, pendulum*

## 1. INTRODUCTION

The control and stabilization of mechanisms using reaction wheels have been common due to its simple configuration and reliability for applications in aerospace industry (Rui *et al.*, 2000; Jepsen *et al.*, 2009). This kind of actuator is commonly used in systems that require an attitude change, such as, robots (Walker and Wee, 1991; Dubowsky and Papadopoulos, 1993) and satellites (Nudehi *et al.*, 2008; Ismail and Varatharajoo, 2010). Since reaction wheels can store angular momentum which may aid in the stabilization and rejection of external disturbance torques.

The most common way of using a reaction wheel to produce torque is from angular velocity variation that causes the amplitude variation of angular momentum. In this scenario, it is possible from specific angular velocities to control oscillation and direction rate of rigid bodies in space through the application of controlled torques in the reaction wheel. The basic physical principle consists in the validity of the conservation of angular momentum and in the equilibrium of its rate of variation equal to the involved torques (Sperry, 1913; Deimel, 1950; Scarborough, 1958; Cannon, 1967; Ardema, 2005; Tenenbaum, 2006; Goldstein *et al.*, 2011). In order to show this concept, the two reaction wheels pendulum (2-RWP) consists of a rigid pendulum with a disk free to spin attached to each end. The disk free to spin is the reaction wheel that is the system actuator. The two reaction wheels pendulum is also an underactuated nonlinear system and in this kind of system the number of actuators is less than the degrees of freedom of the system what makes the system attitude control harder.

In this paper, it is applied the sliding mode control methodology that is a simple approach of robust control able to treat models uncertainties (Slotine and Li, 1991). Therefore, it was chosen to develop a sliding mode controller for the reaction wheel pendulum mainly to evaluate how this controller would deal with the actuator saturation problem found when it is desired to control 2-RWP in positions far from equilibrium points, tracking control, for example. The main operation principle of SMC techniques is to use gain switchings in the control laws in order to modify controlled systems dynamics so that the states are carried and maintained on a surface of the specified state space. The main advantage of this controller is its insensibility to plant parameters variations and to external disturbances (Utkin, 1978; Slotine and Li, 1991; Jinkun and Xinhua, 2011). This control technique changes the nonlinear dynamic of the system through the application of a discontinuous control signal that forces the system to slide along a cross-section of the normal behavior of the system. Thus, the sliding mode control is a specific type of variable structure control system in which the control law is a nonlinear function. Thus, this paper presents a mathematical model for the 2-RWP, the design of a PID controller and of a sliding mode controller in order to control 2-RWP in the inverted position and tracking a desired trajectory. Finally it is presented the final remarks.

## 2. MODEL OF THE TWO REACTION WHEELS PENDULUM

This section presents the motion equations of 2-RWP. Figure 1 shows a 2-RWP with mass $m_p$, length $\ell$ and fixed in $O$. For modeling was adopted an inertial frame described by the axis $(x, y, z)$ and the orthonormal basis $\{\hat{i}, \hat{j}, \hat{k}\}$ and three moving frames. The first moving frame ($\mathscr{B}_1$) is described by axis $(x_1, y_1, z_1)$ and orthonormal basis $\{\hat{i}_1, \hat{j}_1, \hat{k}_1\}$ rotating solidarity to the pendulum with the angle $\theta$. The second moving frame ($\mathscr{B}_2$) is described by $(x_2, y_2, z_2)$ and by the orthonormal basis $\{\hat{i}_2, \hat{j}_2, \hat{k}_2\}$ rotating with angle $\alpha$ solidarity to one of the reaction wheels. This reaction wheel has a mass $m_1$, radius $r_1$ and it is fixed in $A$. The third moving frame ($\mathscr{B}_3$) is described by $(x_3, y_3, z_3)$ and by the orthonormal basis $\{\hat{i}_3, \hat{j}_3, \hat{k}_3\}$ rotating solidarity with second reaction wheel with angle $\beta$, with a mass $m_2$ and radius $r_2$.

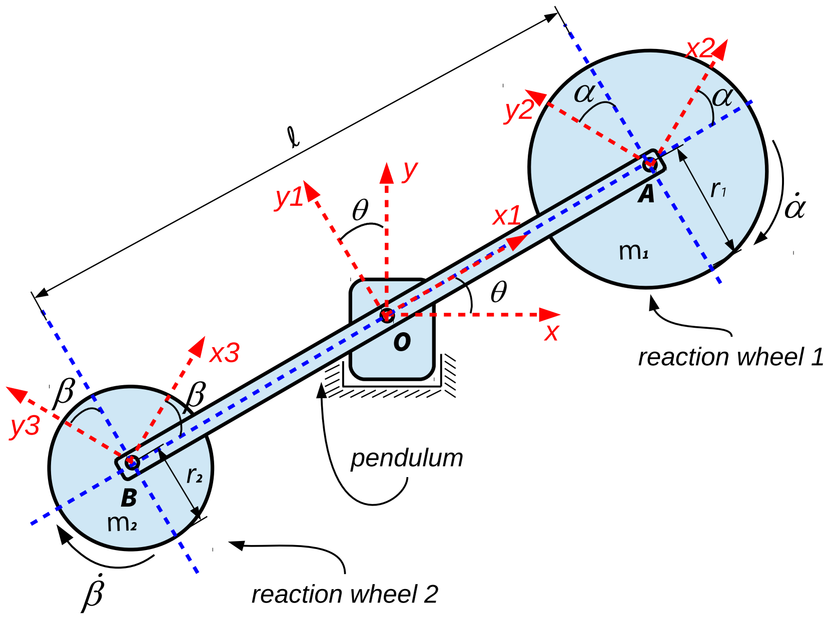



Figure 1. Two reaction wheels pendulum.

The angular momentum of the rigid pendulum described in moving frame $\mathscr{B}_1$ is:

$$_{\mathscr{B}_1}\mathscr{H} = I^O_{z_{eq}}\dot{\theta}\hat{k}_1 \tag{1}$$

where $I^O_{z_{eq}} = \frac{1}{12}m_p\ell^2 + \frac{1}{2}\left(m_1r_1^2 + m_2r_2^2\right) + \frac{1}{4}\left(m_1\ell^2 + m_2\ell^2\right)$ is the second moment of inertia of the pendulum and both reaction wheels computed relatively to the fixed point $O$ using Steiner's Theorem and $\dot{\theta}$ is the angular velocity of the pendulum. The rate of the angular momentum, $\dot{\mathscr{H}}$ is equal to the torque sum relative ($_{\mathscr{B}_1}\mathbf{T}_O$) to the fixed point $O$:

$$_{\mathscr{B}_1}\dot{\mathscr{H}} \equiv {}_{\mathscr{B}_1}\mathbf{T}_O = \frac{d}{dt}\left(_{\mathscr{B}_1}\mathscr{H}\right) + \underbrace{_{\mathscr{B}_1}\mathbf{\Omega} \times {}_{\mathscr{B}_1}\mathscr{H}}_{=0} \tag{2}$$

The mathematical model of 2-RWP is:

$$I^O_{zeq}\ddot{\theta} = m_2g\frac{\ell}{2}\cos\theta - m_1g\frac{\ell}{2}\cos\theta + T_1 + T_2 \tag{3}$$

It is also performed the same analysis for both reaction wheels, where the rate of change of the angular momentum must be equal to the torques sum applied on it, and thus, for both reaction wheels:

$$T_1 = {}_{\mathscr{B}_2}\dot{\mathscr{H}}_{RW1} = I^A_{zr1}\ddot{\alpha}; \qquad T_2 = {}_{\mathscr{B}_3}\dot{\mathscr{H}}_{RW3} = I^B_{zr2}\ddot{\beta} \tag{4}$$

where $I^A_{zr1} = \frac{1}{2}m_1r_1^2$ is the moment of inertia of reaction wheel 1 and $I^B_{zr2} = \frac{1}{2}m_2r_2^2$ is the moment of inertia of reaction wheel 2. For simulations, the parameters used were $m_1 = 0.95$ kg, $r_1 = 0.15$ m, $m_2 = 0.42$ kg, $r_2 = 0.1$ m, $m_p = 0.25$ kg and $\ell = 1$ m. The torques $T_1$ and $T_2$ are provided by DC motors coupled to the reaction wheels are given by:

$$\tau\ddot{\alpha}(t) + \dot{\alpha}(t) = KV_\alpha(t) \tag{5}$$

$$\tau\ddot{\beta}(t) + \dot{\beta}(t) = KV_\beta(t) \tag{6}$$

Equations (5) and (6) can be coupled to the motion equation of 2-RWP shown in Eq. (3) that can be rewritten as:

$$\ddot{\theta} = \frac{m_2 g \frac{l}{2}}{I^O_{zeq}} \cos\theta - \frac{m_1 g \frac{l}{2}}{I^O_{zeq}} \cos\theta + \frac{I^A_{zr1}}{I^O_{zeq}} \left( \frac{K}{\tau} V_\alpha - \frac{1}{\tau} \dot{\alpha} \right) + \frac{I^B_{zr2}}{I^O_{zeq}} \left( \frac{K}{\tau} V_\beta - \frac{1}{\tau} \dot{\beta} \right) \tag{7}$$

The time constant $\tau$ and stationary gain $K$ were obtained from a characterization of a DC motor Akiyama model AK555/390ML 12S18200C. In order to do this characterization, equations (5) and Eq. (6) can be generically rewritten as:

$$\tau\dot{\omega}(t) + \omega(t) = KV(t) \tag{8}$$

where $\dot{\omega}(t)$ is the angular acceleration of the DC motor and $\omega(t)$ is the angular velocity of the DC motor. The Laplace transform is applied in Eq. (8) in order to obtain the transfer function that relates the angular velocity of the DC motor with the voltage applied on it. The transfer function is shown bellow:

$$\frac{\omega(s)}{V(s)} = \frac{K}{\tau s + 1} \tag{9}$$

So, for the characterization of the DC motor it was applied a voltage by a step input in the DC motor and measured its response obtaining $K = 521.6$ and $\tau = 5.88$ s that can be seen in Fig. 2 where to obtain those values it has been done three experimental measurements of time and angular velocity of the DC motor coupled to the reaction wheel. After obtaining this measures it was estimated the transfer function shown in Eq. (9) to compare with the experimental ones. The values of $K$ and $\tau$ were used in all simulations done in this paper.

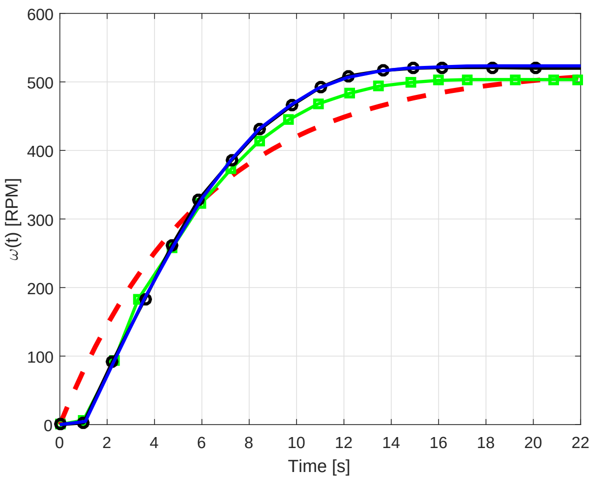



Figure 2. Characterization of the DC motor where —○— indicates experiment 1, — indicates experiment 2, —□— indicates experiment 3 and – – indicates the calculated transfer function

## 3. ATTITUDE CONTROL

The attitude control of the two reaction wheels pendulum is performed based on PID and sliding mode approaches that are explained in next section.

### 3.1 PID controller

It is well known that a PID controller is made up of three actions: proportional (P), integral (I) and derivative (D). These three actions correspond to the proportional, integral and derivative effect on the signal of the active error (Ogata, 2010). The parameters of a PID controller are easy to adjust and its construction is robust for the industrial and aerospace environment (De Lauro Castrucci *et al.*, 2011). In this case of the 2-RWP it is desired to control pendulum angular position $\theta(t)$ by applying two actuation signals assuming the same voltage applied in each DC motor that spins each reaction wheel, then it is necessary to control the voltage applied in the DC motors.

To design PID controller it was chosen as project parameters for the inverted case a maximum overshoot of 15% and a rise time of 5 seconds. PID controller equation used in *Simulink*® from *Matlab*® toolbox *PID Tuner* is shown in Eq.

(10).

$$C(s) = k_p + k_i \frac{1}{s} + k_d \frac{N}{1 + N\frac{1}{s}} \tag{10}$$

where $k_p$ is the potential gain, $k_i$ is the integral gain, $k_d$ is the derivative gain and $N$ is the filter coefficient. For the inverted case, the gains obtained were $k_p = 0.0032$, $k_i = 0.0018$, $k_d = 0.0011$ and $N = 120.4$. For the tracking a trajectory case it was decided that the PID controller should have a maximum overshoot 5% and a rise time of 2 seconds. In this way, the gains used for simulating with a tracking trajectory were $k_p = 2$, $k_i = 0.5$, $k_d = 1$ and $N = 100.4$. A scheme of the controller implemented in *Simulink*® can be seen in Fig. 3. Figure 3 shows the PID control block described by Eq. (10) and the model described by Eq.(7) coupled with the DC motor model. Figure 4 presents the complete model.

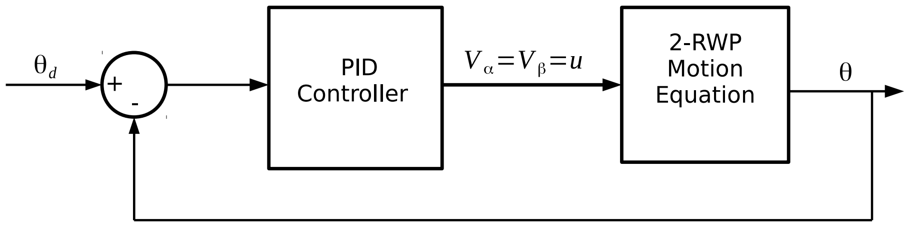



Figure 3. Representative scheme of PID controller implemented in *Simulink*® from *Matlab*®

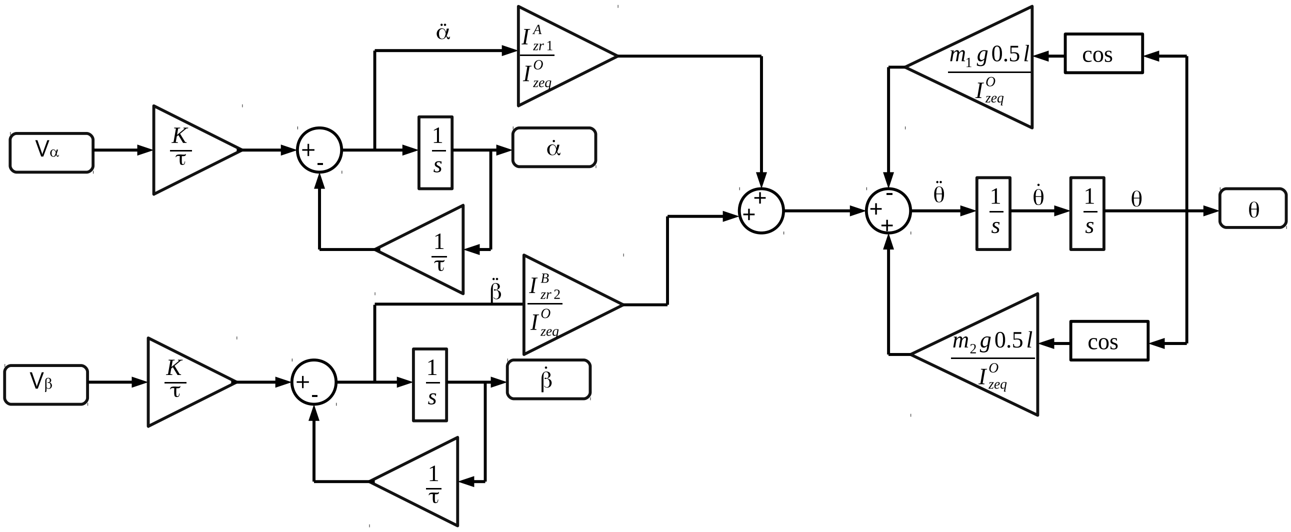



Figure 4. Representative scheme of the complete model of the 2-RWP and DC motors coupled.

### 3.2 Sliding Mode Controller

In order to design a sliding mode controller is necessary to choose a sliding surface in which all trajectories of the systems converges to this surface defined. This surface dynamic is chosen that all trajectories inside this surface converge to its desired value. The tracking error $e = \theta_d - \theta$ is associated with the desired trajectory and the sliding surface is defined by equation $\sigma(e, t) = 0$.

$$\sigma = \dot{e} + \gamma e \tag{11}$$

where $\gamma$ is a positive constant associated to the closed loop bandwidth (Slotine and Li, 1991). For a second order system, equation (11) represents a straight line of sliding. The sliding surface defined should have its values trends to zero, to converge with a finite time interval. From the sliding surface presented in Eq. (11) it can be calculated its derivative:

$$\dot{\sigma} = \ddot{e} + \gamma \dot{e} \equiv \ddot{\theta}_d - \ddot{\theta} + \gamma \dot{e} \tag{12}$$

Equation (12) can be substituted 2-RWP motion equation presented in Eq. (7) and here it is made a project assumption as it has been done for PID controller that it is going to be considerate that the same voltage is applied in both DC motors, so $V_\alpha = V_\beta = u$ that is the control variable where it is obtained that:

$$\dot{\sigma} = \ddot{\theta}_d + \gamma\dot{e} - \frac{K\left(I^A_{zr1} + I^B_{zr2}\right)}{I^O_{zeq}\tau}u - \frac{(m_2 - m_1)g\frac{l}{2}}{I^O_{zeq}}\cos\theta + \frac{I^A_{zr1}}{I^O_{zeq}\tau}\dot{\alpha} + \frac{I^B_{zr2}}{I^O_{zeq}\tau}\dot{\beta} \tag{13}$$

And the controller can be adopted as:

$$u = \frac{I^O_{zeq}\tau}{K\left(I^A_{zr1} + I^B_{zr2}\right)}\left(\ddot{\theta}_d + \eta\mathrm{sgn}(\sigma) + \gamma\dot{e} - \frac{(m_2 - m_1)g\frac{l}{2}}{I^O_{zeq}}\cos\theta + \frac{I^A_{zr1}}{I^O_{zeq}\tau}\dot{\alpha} + \frac{I^B_{zr2}}{I^O_{zeq}\tau}\dot{\beta}\right) \tag{14}$$

At this point, it is necessary to select a Lyapunov function $\mathcal{V}(\sigma)$ that is positive defined and has a negative derivative in relation to time in the attraction region where a candidate is:

$$\mathcal{V}(\sigma) = \frac{1}{2}\sigma^2 \tag{15}$$

With this Lyapunov function presented, the sliding mode will exist if $\dot{\mathcal{V}}(\sigma)$ is negative definite and:

$$\dot{\mathcal{V}}(\sigma) = \sigma\dot{\sigma} < 0 \tag{16}$$

In order to prove that Eq. (14) that is the controller adopted can be substituted in Eq. (13) and the resulting can be substituted in Eq. (16) where it is obtained that:

$$\dot{\mathcal{V}}(\sigma) = \sigma\left(-\eta\mathrm{sgn}(\sigma)\right) = -\eta|\sigma| < 0 \tag{17}$$

where $\eta$ is a positive arbitrary constant and sgn($\sigma$) is a signal function defined as:

$$\mathrm{sgn}(\sigma) = \begin{cases} 1 \text{ if } \sigma > 0 \\ -1 \text{ if } \sigma < 0 \end{cases} \tag{18}$$

In a practical point of view, a sliding mode cannot exist because this would require that the controller switches at an infinite frequency. In the presence of switching imperfections the discontinuity in controller feedback produces a particular behavior known as chattering (Slotine and Li, 1991). To avoid chattering, Slotine and Li (1991) have proposed to smooth signal function sgn($\sigma$) used in the control law where it is established a boundary layer around sliding surface. Thus, in order to restrain chattering phenomenon it is adopted a saturated function sat($\sigma$) instead of signal function sgn($\sigma$).

$$\mathrm{sat}(\sigma) = \begin{cases} 1 \text{ if } \sigma > \Delta \\ k\sigma \text{ if } |\sigma| \leq \Delta, \quad k = \frac{1}{\Delta} \\ -1 \text{ if } \sigma < -\Delta \end{cases} \tag{19}$$

Thus, applying saturation function in the controller equation presented in Eq. (14) and rewriting it, it is obtained that:

$$u = a\left(\ddot{\theta}_d + \eta\mathrm{sat}(\sigma) + \gamma\dot{e} - b\cos\theta + c\dot{\alpha} + d\dot{\beta}\right) \tag{20}$$

where $a = \frac{I^O_{zeq}\tau}{K\left(I^A_{zr1}+I^B_{zr2}\right)}$, $b = \frac{(m_2-m_1)g\frac{l}{2}}{I^O_{zeq}}$, $c = \frac{I^A_{zr1}}{I^O_{zeq}\tau}$ and $d = \frac{I^B_{zr2}}{I^O_{zeq}\tau}$. This controller was implemented in *Simulink*® from *Matlab*® and a scheme of how it was implemented can be seen in Fig. 5. Figure 5 presents the Sliding Mode Controller block that is described by Eq. (20) and the model described by Eq. (7) coupled with the DC motors model. The complete model was illustrated in Fig. 4 and the details of implementation of sliding mode controller can be seen in Fig. 6

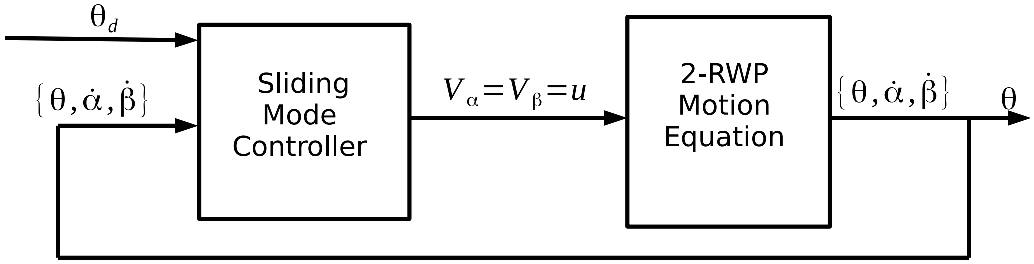



Figure 5. Representative scheme of sliding mode controller implemented in *Simulink*® from *Matlab*®

The sliding mode control parameters used in the simulations were: $\eta = 35$, $\gamma = 5$ and $\Delta = 0,5$.

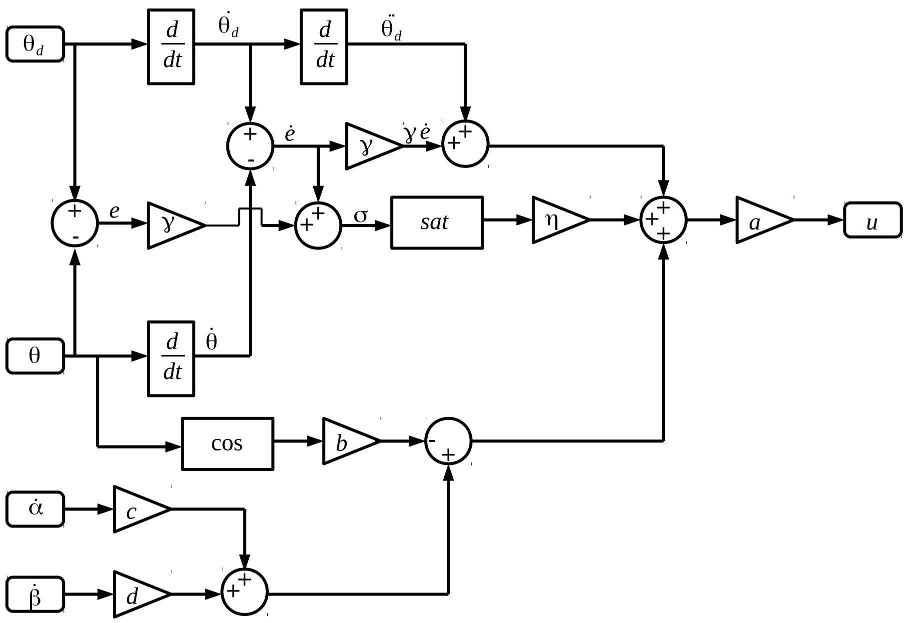



Figure 6. Representative scheme of the sliding mode controller equation implemented in *Simulink*® from *Matlab*®

## 4. Numerical Results

In this section it is presented the simulations for the cases of interest of this paper, to control 2-RWP in its inverted position and to perform a tracking a desired trajectory control. It also has been added a disturbance in the tracking control case in order to see the robustness of the sliding mode controller.

As it is one of the interests of this paper to evaluate if the actuators are operating close to saturation conditions it is calculated the energy applied by the controllers in the control signal using Eq. (21). It can be observed by the voltage signals applied in the tracking case that the 2-RWP operates closer to saturation conditions than in the inverted case.

$$E = \int_{-\infty}^{+\infty} |V(t)|^2 dt \tag{21}$$

where $E$ is the energy in the signal and $x(t)$ is time signal.

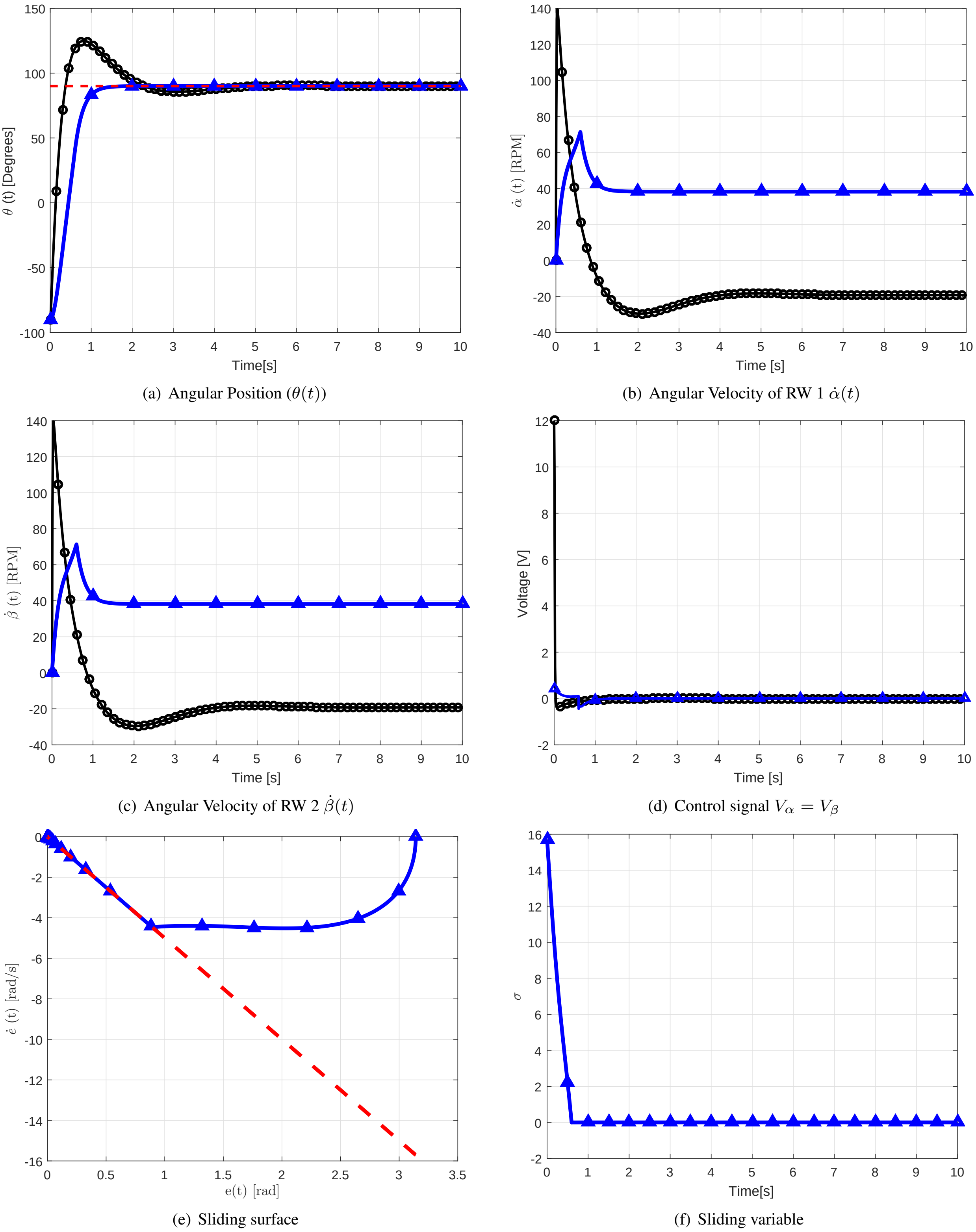
Table 1. Energy spent by each controller in each case

| Controller and case | Energy $[Vs]$ |
|---|---|
| PID inverted case | 0,1507 |
| SMC inverted case | 0,0039 |
| PID tracking case | 13,8238 |
| SMC tracking case | 0,5439 |
| PID tracking case with disturbance | 21,8147 |
| SMC tracking case with disturbance | 4,5368 |

In the condition that is applied a disturbance in the system it can be noticed that the energy spent is increased by both controllers. It also can be observed that PID controller has a higher energy spent than sliding mode control, this also can be seen when it is observed the control signal shown in Fig. 7 (d) , Fig. 8 (d) and 9 (b) where the PID controller demands in the beginning of the simulations a higher control signal what can be seen by the values obtained in Tab. 1.

### 4.1 Control in the inverted position

Firstly, Figure 7 shows the results for controlling 2-RWP in the inverted position for both controllers designed. Figure 7 (a) shows the angular position control and it can be seen that sliding mode control has a better performance and reaches the reference with no overshoot. Figure 7 (b) and Fig.7 (c) show each reaction wheel angular velocity that must be equal as for instance it is being used the same voltage in both DC motors. Figure 7 (d) presents the control signal necessary to control 2-RWP for each controller. Another interesting thing to notice is that Fig.7 (e) illustrates the sliding surface where it can be seen that the system starts in a position far from the sliding surface but the controller designed is able to take the system to the sliding surface and control it. And Fig.7 (f) shows as expected that the sliding variable after a period of time goes to zero.




(a) Angular Position ($\theta(t)$)

(b) Angular Velocity of RW 1 $\dot{\alpha}(t)$

(c) Angular Velocity of RW 2 $\dot{\beta}(t)$

(d) Control signal $V_\alpha = V_\beta$

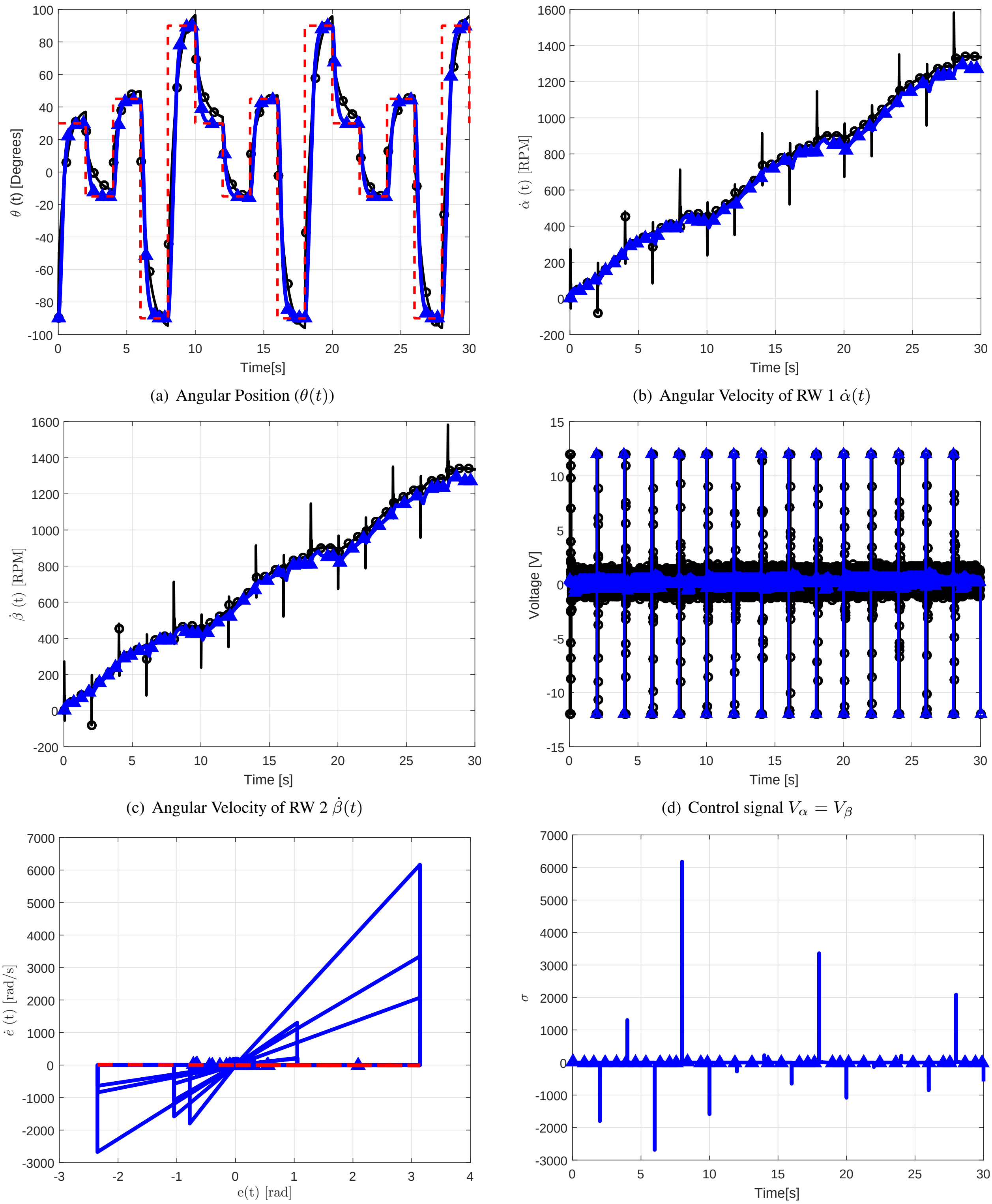
(e) Sliding surface

(f) Sliding variable

Figure 7. Simulation results for controlling 2-RWP in the inverted position using PID control $-\circ-$ and SMC $-\triangle-$. $--$ indicates reference used, in this case $\theta = 90^o$ and $--$ indicates the desired sliding surface.

### 4.2 Tracking a desired trajectory control

Figure 8 presents the performance of both controllers for tracking a desired trajectory. Figure 8 (a) presents results for position control of 2-RWP and Fig. 8 (b) and Fig. 8 (c) illustrate both reaction wheels angular velocities. Figure 8 (d) presents the control signal used by both controllers.




(a) Angular Position ($\theta(t)$)

(b) Angular Velocity of RW 1 $\dot{\alpha}(t)$

(c) Angular Velocity of RW 2 $\dot{\beta}(t)$

(d) Control signal $V_\alpha = V_\beta$

(e) Sliding surface

(f) Sliding variable

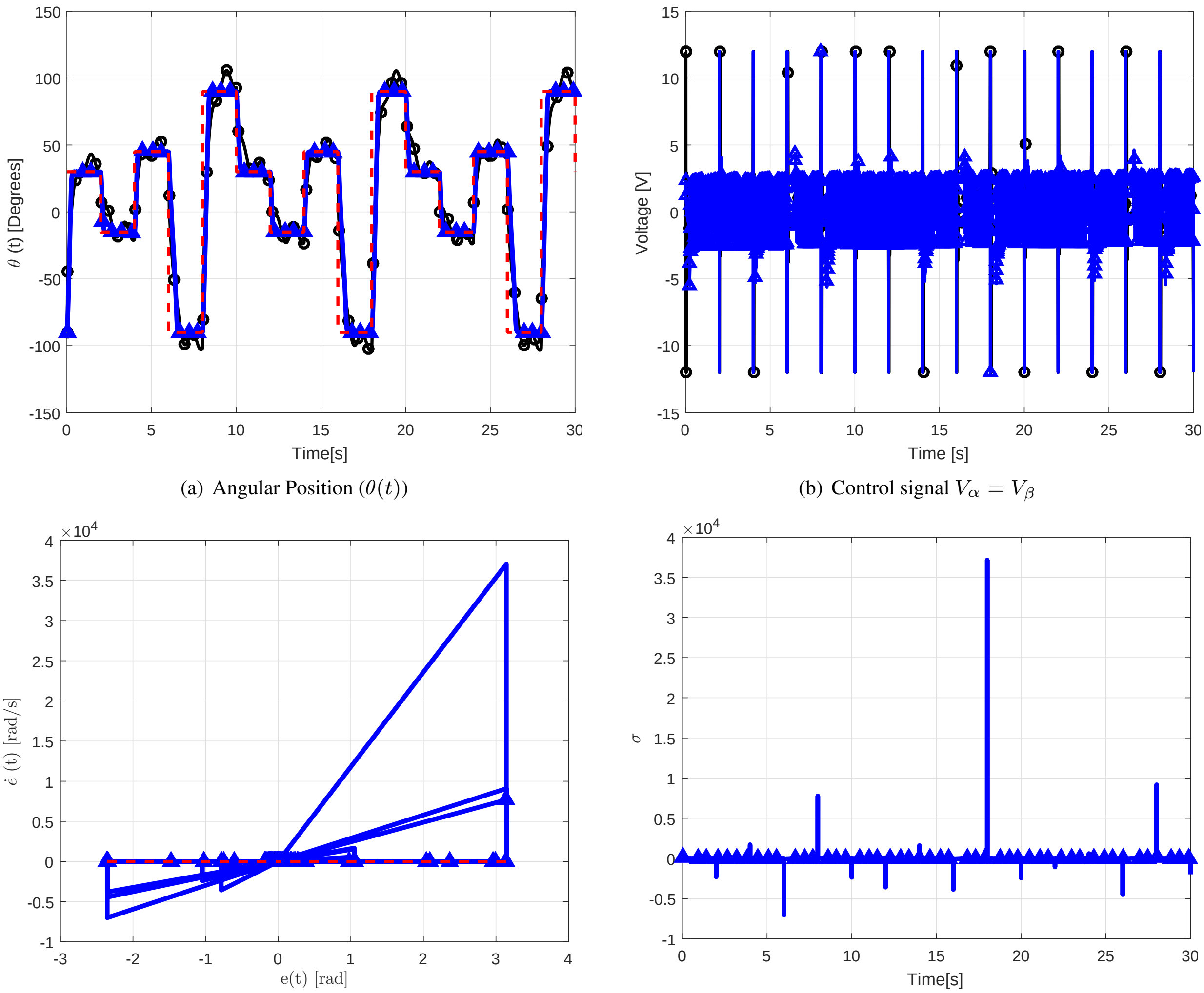
Figure 8. Simulation results for tracking control of 2-RWP using PID control $-\circ-$ and SMC $-\triangle-$. $--$ indicates reference used, in this case $\theta = 90^o$ and $--$ indicates the desired sliding surface.

Figures 8 (e) and 8 (f) present for the sliding mode control the sliding surface and the sliding variable. It can be noticed that when the reference is changed the system is takes for a point far from the sliding surface and the control law proposed is able to bring the system back to the sliding surface.

Another interesting simulation to do is the addition of a disturbance term in the 2-RWP motion equation in order to evaluate how both controllers designed would deal with this. So in Eq. (7) would become:

$$\ddot{\theta} = \frac{m_2 g \frac{l}{2}}{I^O_{zeq}} \cos\theta - \frac{m_1 g \frac{l}{2}}{I^O_{zeq}} \cos\theta + \frac{I^A_{zr1}}{I^O_{zeq}} \left( \frac{K}{\tau} V_\alpha - \frac{1}{\tau} \dot{\alpha} \right) + \frac{I^B_{zr2}}{I^O_{zeq}} \left( \frac{K}{\tau} V_\beta - \frac{1}{\tau} \dot{\beta} \right) + A \sin(ft) \tag{22}$$

where $A\sin(ft)$ is the disturbance applied in the 2-RWP and in the simulations it was used $A = 100$ and $f = 2\pi$ rad/s. Figure 9 presents the results for PID controller and for sliding mode control where it can be noticed that the PID has more difficulties to do the tracking control of 2-RWP. The sliding mode control, as expected, has a very similar response than the one shown in Fig. 8 where tracking control has been done without a disturbance showing one of the advantages of using a variable structure controller. Thus, Fig. 9 (a) shows the angular position control and Fig. 9 (b) presents the control signal used by each controller. Already, Figure 9 (c) illustrates the system reaching the desired sliding surface and it can be noticed that when the reference is changed the system goes to a point far from the sliding surface but the control law used, presented in Eq. (14), is able to bring the system back to the sliding surface even with the disturbance. Figure 9 (d) shows the behavior of the sliding variable.




(a) Angular Position ($\theta(t)$)

(b) Control signal $V_\alpha = V_\beta$

(c) Sliding surface

(d) Sliding variable

Figure 9. Simulation results for tracking control of 2-RWP with addition of disturbance using PID control $-\circ-$ and SMC $-\triangle-$. $--$ indicates reference used in (a) and the desired sliding surface in (c).

## 5. FINAL REMARKS

In this paper it was presented a different configuration of the well known reaction wheel pendulum or inertia wheel pendulum where it was decided to use two reaction wheels to control it. It was presented the complete modeling of 2-RWP and a linear and a nonlinear control for the inverted position and for tracking a desired trajectory. It also has been evaluated the energy spent by the controllers and it has been seen that the PID controller spends more energy that the SMC to control 2-RWP. It also could be noticed that for tracking a desired trajectory the system works more close to conditions that would take the system actuators to operate closer to saturation limits.

## 6. ACKNOWLEDGMENTS

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