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SELECTING KINEMATIC STRUCTURES USING THE ACTUATED DEGREE-OF-CONTROL

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Abstract. Mechanism design is typically a designer dependent task. Mechanism design methodologies aim to reduce the designer dependency, creating a structured approach to mechanism design. One critical step in enumeration-based mechanism design methodologies is the selection of kinematic structures. Structural characteristics, such as connectivity and degree-of-control can be used to aid the designer in the identification of feasible mechanisms. This paper presents the use of a new structural characteristic to identify feasible kinematic structures, the actuated degree-of-control (ADOC). Initially, a review on structural characteristics is presented, exposing their definitions. Then, the use of the ADOC is shown, exposing its difference in relation to previous structural characteristics. Two case studies are presented to validate the use of the ADOC in the kinematic structure selection process. In the first case study the kinematic structure for a toggle clamp device is selected using the ADOC concept. In the second case study the kinematic structure of a modular variable stiffness actuator is selected using the ADOC concept. Finally, the advantages of using the ADOC in the mechanism design process are discussed.

Keywords: actuated degree-of-control, kinematic chain selection, degree-of-control, connectivity, mechanism design.

1. INTRODUCTION

Mechanism design usually relies on the designer experience, knowledge and creativity. Mechanism design methodologies aim to overcome at some extent the designer dependency, setting a structured approach to develop a feasible mechanism and providing tools and methods for each step.

Enumeration-based mechanism design methodologies (Tsai, 2000; Yan, 1998) present two main steps: enumeration and selection of kinematic structures. The enumeration step generates all possible kinematic structures for a given set of structural requirements, such as the number of links, kinematic pairs and the screw system order. There are several tools to generate kinematic chains (Simoni *et al.*, 2011) and there are also available atlases of kinematic chains in the literature (Tsai, 2000; Ding *et al.*, 2012).

After all kinematic chains have been enumerated, the designer selects the promising kinematic chains. The task of identifying feasible mechanisms in a kinematic chain atlas remains a challenge. Several structural characteristics, such as connectivity, degree-of-control, variety and redundancy can be used to identify promising kinematic chains (Hunt, 1978; Tischler *et al.*, 2001; Belfiore and Di Benedetto, 2000). By using these structural characteristics unfeasible options are eliminated, reducing the quantity of kinematic chains. However, the number of options left can still make it unpractical to explore and analyze all kinematic chains. Thus, new methods for kinematic chain selection are desired.

In this paper, the use of a new kinematic characteristic is explored: the actuated degree-of-control (ADOC). This new structural characteristic is used in two case studies: the design of a toggle clamp device and the design of a modular variable stiffness actuator.

2. REVIEW ON STRUCTURAL CHARACTERISTICS OF KINEMATIC CHAINS

On this section a brief review on mechanism design methodologies is initially presented for a better understanding on where structural characteristics are used in the mechanism design process. Then, it is presented a review on some structural characteristics.

2.1 Mechanism design methodologies based on enumeration techniques

Hartenberg and Denavit (1964) identify three stages that are always present in mechanism design: type synthesis, number synthesis and dimensional synthesis. On the type synthesis step, the type of kinematic pairs that will be used are selected, such as cams, gears, pulleys or linkages. On the number synthesis step, it is established the quantities of links and kinematic pairs and how they are connected. On the dimensional synthesis step, the size of the links and the position of the points of interest are established. Although Hartenberg and Denavit (1964) do not provide a methodology for mechanism design, this division of the mechanism synthesis process in type, number and dimensional syntheses aids in the understanding of mechanism design methodologies.

There are several approaches for mechanism design, such as enumeration-based methodologies (Freudenstein and Maki, 1979, 1983; Olson *et al.*, 1985; Yan and Chen, 1985; Yan and Hwang, 1991; Yan, 1992; Raghavan, 1996; Yan, 1998) and building blocks-based methodologies (Kota and Chiou, 1992; Subramanian and Wang, 1995; Chiou and Kota, 1999; Moon and Kota, 2002; Yan and Ou, 2005; Han and Lee, 2006). Methodologies based on enumeration techniques (Tsai, 2000; Yan, 1998; Ding *et al.*, 2012) usually make use of graph theory to generate all possible kinematic chains that satisfy some structural characteristics, such as mobility, number of independent loops and the screw system order. Once all the kinematic chains are created, the designer selects promising kinematic chains to continue the design process. The two main steps in number synthesis are: enumeration of kinematic structures and selection of kinematic structures.

The enumeration of kinematic structures is a combinatorial problem, thus, the quantity of results to be analyzed grows fast according to the combinatorial parameters. The generation of kinematic structures presents its challenges, such as avoiding isomorphisms while still generating all possible kinematic structures (Simoni *et al.*, 2011). However, there are several methods and tools available in the literature to generate a kinematic chain atlas (Mruthyunjaya, 1979, 1984; Yan and Hwang, 1990; Mruthyunjaya, 2003; Martins *et al.*, 2010; Simoni *et al.*, 2011; Ding *et al.*, 2012; Yan and Chiu, 2015; Ding, 2015; Pozhbelko, 2016). Thus, while developing an efficient algorithm for kinematic chain enumeration is a challenge, the tools available in the literature can be used to automatically generate kinematic chain atlases using a computer. In order to enumerate mechanisms a initial set of structural requirements are needed. Usually the mobility, screw-system order and number of independent loops are chosen since they are structural characteristics that correlate to functional characteristics (Murai *et al.*, 2013; Tischler *et al.*, 2001; Tsai, 2000; Olson *et al.*, 1985; Freudenstein and Maki, 1983).

Regarding the selection of kinematic structures, as the enumeration tends to generate a lot of results, the identification of promising mechanisms is a labourious task. Also, mechanisms are generated by enumerating their abstract representations, such as graphs or using Frank's notation, which makes it harder to correlate the functional requirements to kinematic structure (Kota and Chiou, 1992; Han and Lee, 2006).

Several structural characteristics were created to aid in the mechanism selection, *e.g.*, connectivity, degree-of-control, variety and redundancy (Hunt, 1978; Phillips, 2007; Tischler *et al.*, 1995; Shoham and Roth, 1997; Belfiore and Di Benedetto, 2000; Carboni and Martins, 2007; Martins and Carboni, 2008). As in the enumeration process, it is desirable to have structural requirements that can be correlated to functional requirements. The next section exposes more details on these structural characteristics.

2.2 Connectivity, degree-of-control and redundancy

The connectivity was introduced by Hunt (1978) as the number of relative degrees of freedoms between two members. Phillips (2007) introduced the concept of joint-in-the-bag to illustrate the connectivity concept. The connectivity between every two links is represented by a symmetric matrix $\mathbf{C}_{n \times n}$, being n the number of links in a kinematic chain. There are several works in the literature proposing algorithms to calculate the connectivity of a kinematic chain (Shoham and Roth, 1997; Belfiore and Di Benedetto, 2000; Liberati and Belfiore, 2006; Carboni and Martins, 2007; Martins and Carboni, 2008; Huang *et al.*, 2017).

The degree-of-control concept was introduced by Belfiore and Di Benedetto (2000) as the minimum number of independent actuated kinematic pairs needed to determine the relative position between the two links. The degree-of-control between every two links in the kinematic chain is represented by the symmetric matrix $\mathbf{K}_{n \times n}$. However, the degree-of-control differs from the connectivity because the degree-of-control is not upper bounded by the screw-system order (λ). There are several works in the literature proposing algorithms to calculate the degree-of-control (Belfiore and Di Benedetto, 2000; Carboni and Martins, 2007; Martins and Carboni, 2008).

The redundancy concept was introduced by Belfiore and Di Benedetto (2000) as the number of degree-of-control

that exceeds the connectivity between links i and j. The redundancy between every two links in the kinematic chain is represented by the symmetric matrix $\mathbf{R}_{n \times n}$ and can be obtained by matrix subtraction of $\mathbf{C}$ from $\mathbf{K}$. A kinematic chain is said to present redundancy if there is at least one pair of links i and j where $\mathbf{K}(i,j) > \mathbf{C}(i,j)$.

The definitions of connectivity, degree-of-control and redundancy regard the kinematic chain. Thus, the matrices $\mathbf{C}$, $\mathbf{K}$ and $\mathbf{R}$ do not take into account which is the reference link in a kinematic inversion. Also, the output links and the kinematic pairs that hold the actuators are disregarded as well.

The planar kinematic chain presented in Fig. 1a is used to exemplify the concepts of connectivity ($\mathbf{C}$) and redundancy ($\mathbf{R}$), which are exposed in Eq. 1, respectively. The numbers in bold in the redundancy matrix indicate the pairs of links i and j that present redundancy. The degree-of-control ($\mathbf{K}$) is exposed in Eq. 2. The next section exposes the concept of actuated degree-of-control, presenting the differences between the ADOC and the previously presented structural characteristics.

$$
\mathbf{C} = \begin{bmatrix}
0 & 1 & 2 & 3 & 2 & 1 & 3 & 3 & 3 & 3 & 3 & 3 \\
1 & 0 & 1 & 2 & 3 & 2 & 2 & 3 & 3 & 3 & 3 & 3 \\
2 & 1 & 0 & 1 & 2 & 3 & 1 & 2 & 3 & 3 & 3 & 3 \\
3 & 2 & 1 & 0 & 1 & 2 & 2 & 3 & 3 & 3 & 3 & 2 \\
2 & 3 & 2 & 1 & 0 & 1 & 3 & 3 & 3 & 3 & 2 & 1 \\
1 & 2 & 3 & 2 & 1 & 0 & 3 & 3 & 3 & 3 & 3 & 2 \\
3 & 2 & 1 & 2 & 3 & 3 & 0 & 1 & 2 & 3 & 3 & 3 \\
3 & 3 & 2 & 3 & 3 & 3 & 1 & 0 & 1 & 2 & 3 & 3 \\
3 & 3 & 3 & 3 & 3 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\
3 & 3 & 3 & 3 & 3 & 3 & 3 & 2 & 1 & 0 & 1 & 2 \\
3 & 3 & 3 & 3 & 2 & 3 & 3 & 3 & 2 & 1 & 0 & 1 \\
3 & 3 & 3 & 2 & 1 & 2 & 3 & 3 & 3 & 2 & 1 & 0
\end{bmatrix}
\quad
\mathbf{R} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & \mathbf{2} & \mathbf{2} & \mathbf{1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & \mathbf{2} & \mathbf{2} & \mathbf{1} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & \mathbf{1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & \mathbf{1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & \mathbf{1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & \mathbf{2} & \mathbf{2} & \mathbf{1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & \mathbf{1} & \mathbf{1} \\
\mathbf{1} & 0 & 0 & 0 & \mathbf{1} & \mathbf{2} & 0 & 0 & 0 & 0 & 0 & \mathbf{1} \\
\mathbf{2} & \mathbf{1} & 0 & \mathbf{1} & \mathbf{1} & \mathbf{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{2} & \mathbf{2} & \mathbf{1} & \mathbf{1} & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{1} & \mathbf{2} & \mathbf{1} & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 \\
0 & \mathbf{1} & 0 & 0 & 0 & 0 & \mathbf{1} & \mathbf{1} & 0 & 0 & 0 & 0
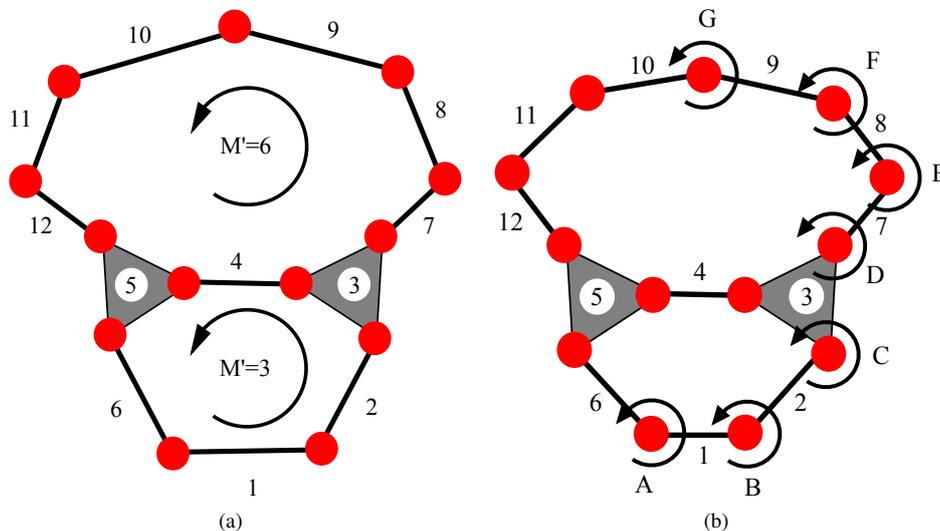\end{bmatrix}
\quad (1)
$$

Figure 1. Planar kinematic chain and an actuated kinematic chain.

3. THE ACTUATED DEGREE-OF-CONTROL

The Actuated Degree-of-Control was proposed by Murai *et al.* (2017) as the quantity of input pairs that affects the relative position between two links in a kinematic chain with the actuated joints defined. This structural requirement can be used to select reference and output links in kinematic chains with and without redundancy. The actuated degree-of-control between every two links in a kinematic chain with actuated kinematic pairs identified is represented by the symmetric matrix $\mathbf{K_A}_{n \times n}$.

To exemplify the concept of actuated degree-of-control, 7 kinematic pairs in the kinematic chain in Fig. 1a are chosen as input pairs, yielding the actuated kinematic chain in Fig. 1b. The degree-of-control of the unactuated chain and the actuated degree-of-control for the actuated chain are respectively

$$
\mathbf{K} = \begin{bmatrix}
0 & 1 & 2 & 3 & 2 & 1 & 3 & 4 & 5 & 5 & 4 & 3 \\
1 & 0 & 1 & 2 & 3 & 2 & 2 & 3 & 4 & 5 & 5 & 4 \\
2 & 1 & 0 & 1 & 2 & 3 & 1 & 2 & 3 & 4 & 4 & 3 \\
3 & 2 & 1 & 0 & 1 & 2 & 2 & 3 & 4 & 4 & 3 & 2 \\
2 & 3 & 2 & 1 & 0 & 1 & 3 & 4 & 4 & 3 & 2 & 1 \\
1 & 2 & 3 & 2 & 1 & 0 & 4 & 5 & 5 & 4 & 3 & 2 \\
3 & 2 & 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 & 4 & 4 \\
4 & 3 & 2 & 3 & 4 & 5 & 1 & 0 & 1 & 2 & 3 & 4 \\
5 & 4 & 3 & 4 & 4 & 5 & 2 & 1 & 0 & 1 & 2 & 3 \\
5 & 5 & 4 & 4 & 3 & 4 & 3 & 2 & 1 & 0 & 1 & 2 \\
4 & 5 & 4 & 3 & 2 & 3 & 4 & 3 & 2 & 1 & 0 & 1 \\
3 & 4 & 3 & 2 & 1 & 2 & 4 & 4 & 3 & 2 & 1 & 0
\end{bmatrix}
\qquad
\mathbf{K_A} = \begin{bmatrix}
0 & 1 & 2 & 3 & \mathbf{3} & 1 & 3 & 4 & 5 & \mathbf{6} & \mathbf{7} & \mathbf{7} \\
1 & 0 & 1 & \mathbf{3} & 3 & 2 & 2 & 3 & 4 & 5 & \mathbf{7} & \mathbf{7} \\
2 & 1 & 0 & \mathbf{3} & \mathbf{3} & 3 & 1 & 2 & 3 & 4 & \mathbf{7} & \mathbf{7} \\
\mathbf{3} & \mathbf{3} & \mathbf{3} & 0 & \mathbf{3} & \mathbf{3} & 4 & 5 & \mathbf{6} & \mathbf{7} & \mathbf{7} & \mathbf{7} \\
\mathbf{3} & 3 & \mathbf{3} & \mathbf{3} & 0 & \mathbf{3} & 4 & 5 & \mathbf{6} & \mathbf{7} & \mathbf{7} & \mathbf{7} \\
1 & 2 & 3 & \mathbf{3} & \mathbf{3} & 0 & 4 & 5 & \mathbf{6} & \mathbf{7} & \mathbf{7} & \mathbf{7} \\
3 & 2 & 1 & \mathbf{4} & \mathbf{4} & 4 & 0 & 1 & 2 & 3 & \mathbf{7} & \mathbf{7} \\
4 & 3 & 2 & \mathbf{5} & \mathbf{5} & 5 & 1 & 0 & 1 & 2 & \mathbf{7} & \mathbf{7} \\
5 & 4 & 3 & \mathbf{6} & \mathbf{6} & \mathbf{6} & 2 & 1 & 0 & 1 & \mathbf{7} & \mathbf{7} \\
\mathbf{6} & 5 & 4 & \mathbf{7} & \mathbf{7} & \mathbf{7} & 3 & 2 & 1 & 0 & \mathbf{7} & \mathbf{7} \\
\mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & 0 & \mathbf{7} \\
\mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & \mathbf{7} & 0
\end{bmatrix}
\qquad (2)
$$

The numbers in bold in the actuated degree-of-control matrix indicate the elements that differ from the degree-of-control matrix. This difference arises because when calculating the degree-of-control matrix, the actuators placement is not taken into account. Thus, the degree-of-control analysis is not constrained to a set of previously selected input pairs.

In the ADOC matrix in Eq. 2, the element $\mathbf{K_A}(i,j)$ indicates the quantity of actuators that affects the relative position between links i and j. For instance, the relative position between links 5 and 7 is controlled by actuators A, B, C and D. The actuators A, B and C define the position of all links in the subchain composed of links 1 to 6, establishing the position between links 5 and 3. The actuator D defines the position between link 3 and 7. Thus, these 4 actuators together control the relative position between links 5 and 7, yielding $\mathbf{K_A}(5,7) = 4$. Similar analysis can be done to every two links in the actuated kinematic chain, considering the actuators placement and the mobility of the kinematic chain and its subchains.

The ADOC can be used to select reference and output links according to the number of desired actuators controlling the relative motion between those links. Therefore, it can be used during the kinematic structure selection phase, aiding the designer in the identification of feasible options. On the other hand, the degree-of-control matrix cannot be used for this purpose because depending on the actuators placement the number of actuators that affects the positioning between two links, $\mathbf{K_A}(i,j)$, can differ from the degree-of-control, $\mathbf{K}(i,j)$.

Regarding the connectivity, while the connectivity is upper bounded by the screw system order, the actuated degree-of-control is not. As the degree-of-control, the ADOC is upper bounded by the kinematic chain mobility. Also, differently from the ADOC, the connectivity does not depend on the actuators placement. Thus, while the connectivity can be used to analyze the degree-of-freedom between two links, it does not indicate how many actuators affect the relative position between two links.

4. CASE STUDIES

This section presents two case studies in which the actuated degree-of-control is used to aid in the identification of feasible mechanisms. As the ADOC matrix is used in to identify feasible selections of output and reference links, design decisions made previously or after this step are not shown in details. More on the design of these case studies are shown in Costa *et al.* (2017a); Rosa *et al.* (2017).

4.1 Kinematic chains for toggle clamping devices

Clamping devices are a fixture element that hold the workpiece during machining operation. A toggle clamp is a type of clamping device characterized for using kinematic chain singularity to deliver a quick fix and release.

A review on toggle clamping devices and a new classification for toggle clamping devices based on mechanism theory is presented by Costa *et al.* (2017b). This section presents details on the selection of kinematic structures for a novel toggle clamping device. More details on the design process are presented by Costa *et al.* (2017a).

The structural requirements chosen for kinematic chain enumeration are mobility three ($M = 3$), two independent loops ($\nu = 2$) and a planar kinematic chain. Each mobility has a function, being:

- One mobility to actuate the toggle clamping device. When this mobility is actuated, it fixes or releases the workpiece.

- One mobility to adjust the fixing pressure to a desired value. When this mobility is actuated, the pressure that the clamping arm applies to the workpiece increases or decreases.

- One mobility to auto-adjust to different workpiece sizes. This mobility allows the toggle clamping device to automatically adjust to different workpiece sizes, without requiring any setup operation.

The number of independent loops was chosen as two because a state or the art survey showed that there is no commercial toggle clamping device or patent of a toggle clamping device that has mobility three and two independent loops. Therefore, any toggle clamping device which the kinematic chain presents these structural characteristics is an innovative design.

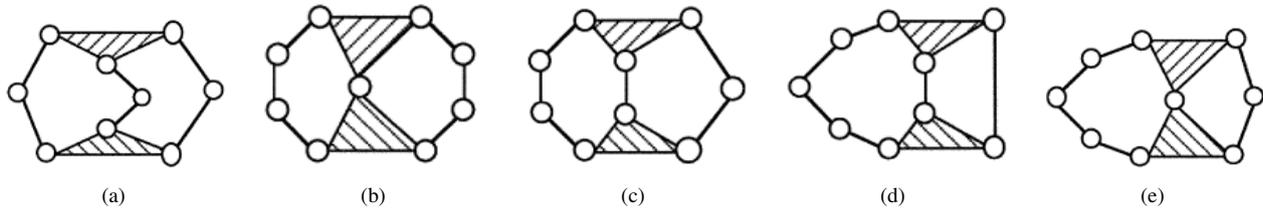All possible non-isomorphic parallel planar kinematic chains with mobility three and two independent loops are exposed in Fig. 2.

Figure 2. Parallel planar kinematic chains with $M = 3$ and $\nu = 2$. Adapted from Tsai (2000).

To apply the ADOC concept, the kinematic chains must have the input pairs identified. The actuators placement must be a proper one, *i.e.*, the actuators placement should result in all links having their position determined while no conflict among actuators arise.

The symmetries in the kinematic chains in Fig. 2 should be considered to avoid isomorphic actuated kinematic chains. In total, there are more than a hundred possible actuated kinematic chains. Each actuated kinematic chain has its own actuated degree-of-control matrix to be generated and analyzed. Here we analyze only one actuated kinematic chain, however, the method can be applied to all actuated kinematic chain.

The kinematic chain in Fig. 2c is shown again in Fig. 3a with the kinematic pairs labeled. The ADOC matrix when the input pairs are b, g and i is

$$\mathbf{K_A} = \begin{bmatrix} 0 & 2 & 2 & 3 & 3 & 3 & 2 & 1 \\ 2 & 0 & 2 & 3 & 3 & 3 & 2 & 2 \\ 2 & 2 & 0 & 3 & 3 & 3 & 2 & 2 \\ 3 & 3 & 3 & 0 & 1 & 3 & 3 & 3 \\ 3 & 3 & 3 & 1 & 0 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 & 0 & 3 & 3 \\ 2 & 2 & 2 & 3 & 3 & 3 & 0 & 1 \\ 1 & 2 & 2 & 3 & 3 & 3 & 1 & 0 \end{bmatrix} \tag{3}$$

The workpiece is secured between the clamping arm and the fixed link. The toggle clamp mobility should affect the relative position between the clamping arm and the fixed link, as well as the other two adjustment mobilities. Therefore, the relative position between output link (clamping arm) and the reference link (fixed link) should be affected by all three actuators. So, feasible choices of reference and output links are those in which the element in the actuated degree-of-control matrix is three.
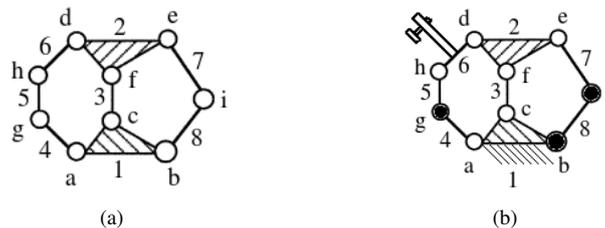

Figure 3. Kinematic chain with kinematic pairs and links labeled and a new toggle clamping device with $M = 3$ and $\nu = 2$.

Combinatorial analysis shows that there are ${}^{8}P_2 = 56$ combinations of output and reference link selection (considering that swapping output and reference links yields different outcomes). This number of combinations can be seen as the number of elements in the ADOC matrix minus the elements in the main diagonal.

However, the feasible pairs of output and reference links are those which $\mathbf{K_A}(i, j) = 3$. Therefore, from the 56 possibilities only 34 are feasible. Figure 3b shows the actuated kinematic chain with link 6 selected as the clamping arm and link 1 selected as fixed link. A functional representation of the toggle clamping device designed is shown in Fig. 4.

The ADOC matrix can also be used when more than one output is desired or when different reference links for different output links are desired.
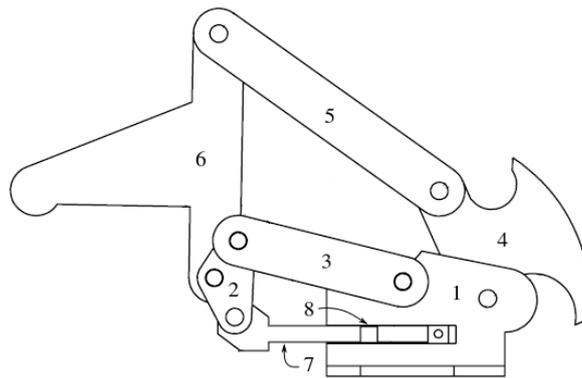
Figure 4. Functional representation of the toggle clamping device designed.

4.2 Kinematic chains for variable stiffness actuator

This section presents details on the selection of kinematic structures for a new modular variable stiffness actuator. More details on the design process is presented by Rosa *et al.* (2017).

A variable stiffness actuator is an actuator that can change the stiffness perceived on the output shaft (Vanderborght *et al.*, 2013; Van Ham *et al.*, 2009). When necessary, the output shaft can be stiff, resulting in a small displacement between the desired position and the position reached when an external torque is applied to the output shaft. The output shaft can also be more compliant, allowing for a large deviation of the output shaft position in relation to the desired position. This compliant configuration is usually used to increase safety in human machine interaction or when collisions with objects in the workspace is a possibility.

A variable stiffness actuator must have at least two degrees-of-freedom, the main mobility (M1), which controls the output shaft desired position; and the secondary mobility (M2), which changes the system stiffness. Regarding the compliant element mobility, there are two approaches: to consider the motion allowed by the compliant element as a mobility and enumerate kinematic chains with mobility 3; to disregard the motion allowed by the compliant element, enumerating kinematic chains with mobility 2, and later add the compliant element at a convenient part of the kinematic chain. The approach chosen was to enumerate kinematic chains with mobility 2.

The number of independent loops was chosen as two, such that one loop transmits the power from M1 to the output shaft and the other loop holds the variable stiffness elements. All possible parallel non-isomorphic planar kinematic chains with mobility two and two independent loops are exposed in Fig. 5.
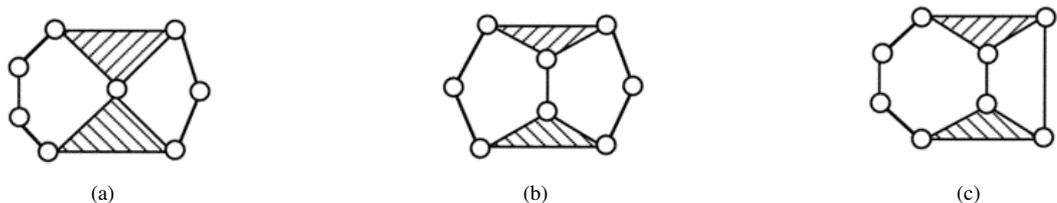

Figure 5. Parallel planar kinematic chains with $M = 2$ and $\nu = 2$. Adapted from Tsai (2000).

There are several combinations of actuators placement in the three kinematic chains presented in Fig. 5. Here only one actuated kinematic chain is analyzed. For the kinematic chain in Fig. 5b, the chosen input pairs are a and b (see Fig. 6a).
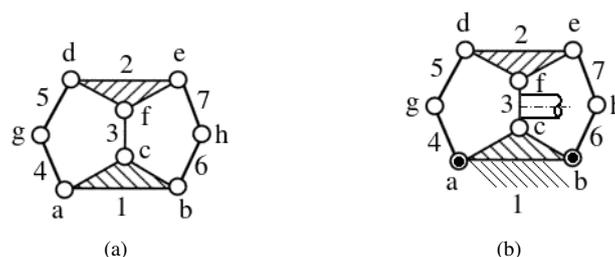

Figure 6. Kinematic chain with kinematic pairs and links labeled and the kinematic structure of a new modular variable stiffness actuator.

The actuated degree-of-control matrix for this actuated kinematic chain is

$$\mathbf{K_A} = \begin{bmatrix} 0 & 2 & 2 & 1 & 2 & 1 & 2 \\ 2 & 0 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 0 & 2 & 2 & 2 & 2 \\ 1 & 2 & 2 & 0 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 0 & 2 & 2 \\ 1 & 2 & 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 0 \end{bmatrix} \tag{4}$$

The position of the output shaft should depend on: the desired position set by M1; the stiffness of the system, controlled by M2. Therefore, both actuators must affect the position of the output shaft. Thus, feasible combinations of reference link and output link on the ADOC matrix are those which $\mathbf{K_A}(i,j) = 2$.

Combinatorial analysis shows that there are $^7P_2 = 42$ combinations of output and reference link selection. However, not all of those combinations satisfy the number of actuators that controls the relative position between output and reference links. Therefore, from the 42 possibilities only 38 are feasible. In this case study the actuated degree-of-control does not filter as many unfeasible combinations as in the the case study in Sec. 4.1 This happens because the kinematic chain in this case study presents variety zero while the kinematic chain in Sec. 4.1presents variety 1. Nevertheless, the actuated degree-of-control is capable of filtering 4 unfeasible combinations.

Link 1 is chosen as reference link and link 3 is chosen as output link since $\mathbf{K_A}(1,3) = 2$ (see Fig.6b). The main mobility, M1, is placed at input pair a, while the stiffness control mobility, M2, is placed at input pair b. The compliant element is added between links 6 and 1 in a manner that as M2 is actuated, it changes the system stiffness. A functional representation of the modular variable stiffness actuator designed is shown in Fig. 7.
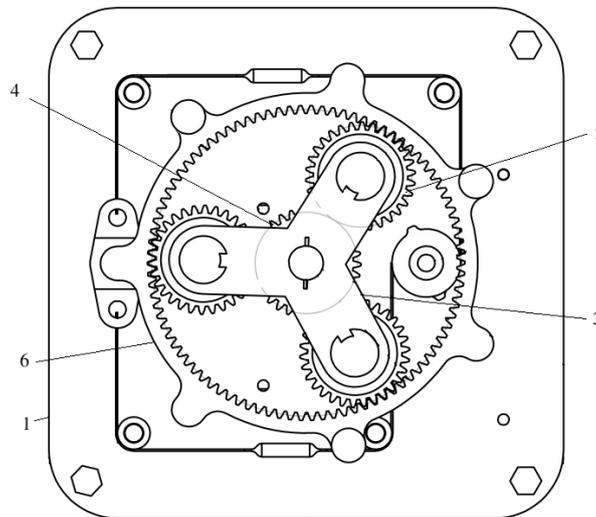

Figure 7. Functional representation of the modular variable stiffness actuator designed.

5. CONCLUSIONS

This paper presented two case studies on how the actuated degree-of-control, a kinematic chain structural characteristic previously proposed, can aid the designer during the kinematic structure selection phase.

In the toggle clamp design case study, for a single actuated kinematic chain, there were 56 possible combinations of reference and output links selection. However, only 34 of them satisfied the actuation requirement for the output link. These 34 feasible options can be easily identified using the actuated degree-of-control matrix. When considering all five kinematic chains, the number of different actuators placements is over a hundred. For each one of these actuated kinematic chain an actuated degree-of-control matrix is generated, yielding 56 combinations of reference and output link selection. Thus, there are thousands of possible kinematic structures with actuators placed and output and reference links selected. Using a computer to automatically generate and analyze the actuated degree-of-control matrix can reduce the designing time.

In the modular variable stiffness actuator case study, for the chosen actuated kinematic chain, there were 42 possibilities of reference and output links selection. However, as the actuated degree-of-control matrix showed, 4 of those

combinations were unfeasible. Thus, the actuated degree-of-control is effective to indicate feasible kinematic structures regarding the output and reference links selection.

Future works include to develop an algorithm to calculate the actuated degree-of-control matrix for a kinematic chain with input pairs identified.

6. ACKNOWLEDGEMENTS

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