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SOMMERFELD EFFECT IN A CANTILEVER BEAM WITH DOUBLE NON-IDEAL SOURCES

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Abstract. *A non-ideal or finite power source system is characterized by the fact that the energy source who excites the system suffer at the same time an influence of the excitation itself, changing its normal behavior, especially when occurs a passage through the resonance. This occurs due to the coupling between the mechanical system and the system of the energy source. When near the resonance region, the power supply of this type of system, which can be an electric motor, an engine or a turbine, loses energy feeding only the vibration of the system and not its shaft rotation. The behavior of the ideal type system is already well known in the literature, but various non-ideal systems still presents as challenges for their complete understanding. This work presents the numerical analysis of a non-ideal type system made by a beam and two unbalanced electric motors. The dynamic model was obtained with the use of the Lagrangean formalism and was simulated numerically in MATLAB. For the results, several jump phenomena diagrams were obtained for different configurations of the system, showing the influence of the motors in the mechanical structure.*

Keywords: *Non-Ideal System, Sommerfeld Effect, Jump Phenomena, Limited Power Supply*

1. INTRODUCTION

In 1902, Sommerfeld made the discovery of the first phenomenon later classified as non-ideal. In his experiment, which consisted of a cantilever beam with an unbalanced DC motor at its tip, Sommerfeld realized that the system acted unexpectedly when the angular frequency of the motor approached the region of resonance of the electromechanical structure, where the system abruptly increased its vibration amplitudes. As the system passed this region, the angular frequency of the electric motor returned to their normal behavior and the vibrations amplitudes abruptly dropped. Later, this effect was called Sommerfeld Effect or jump phenomena, being a common effect in non-ideal type systems (Balthazar *et al.*, 2003) (Cveticanin, 2010).

Several systems can be considered as non-ideal, motor-portal frames, crank mechanism with electro-motor systems, rotors, centrifugal vibrators, among others (Felix *et al.*, 2005). In the non-ideal systems, a series of characteristic effects arise when your dynamic analysis is done, such as the jump phenomena, discontinuities of the curves of frequency versus the rotation of the motor and a strict dependency of the parameters of the motor in the response of the system then, according to Balthazar *et al.* (2003), in mathematical modeling one must consider the equations of the motors when considering a non-ideal system, which leads to an increase in degrees of freedom making the system more complex to analyze (Balthazar *et al.*, 2003) (Piccirillo *et al.*, 2014).

After the discovery published by Sommerfeld, there were other studies considering the system as non-ideal. Kononenko (1969) wrote a book dedicated to the subject and other works includes Balthazar *et al.* (2003) and Cveticanin (2010), who wrote reviews considering more current works. For Balthazar *et al.* (2003), the non-ideal systems still are little explored when compared to the ideal systems.

Others works includes Goncalves *et al.* (2014) which presented experimental and numerical results with emphasis on the effect of capture by resonance for stationary condition and non-stationary condition. Balthazar *et al.* (2001) presented remarks on non-ideal systems with two degrees of freedom and one source of excitation. Karthikeyan *et al.* (2015) showed results for the analysis of a rotor with non-ideal drive system, presenting a finite element model where the integrated system was made in bond graph.

The objective of this paper is to present a numerical analysis of the dynamic behavior of a non-ideal type system made of a cantilever beam and two non-ideal sources. Unbalanced DC motors were chosen as the non-ideal source and were considered one at the tip of the beam and the other in the middle. Results for the jump phenomena were obtained for different configurations in order to demonstrate the influence of each motor on the behavior of the system.

2. SYSTEM MODELING

The system under study consists of a cantilever beam where two unbalanced DC motors are installed, one in the middle of the beam (motor 1) and the other at its tip (motor 2), Fig. 1 shows the system described.

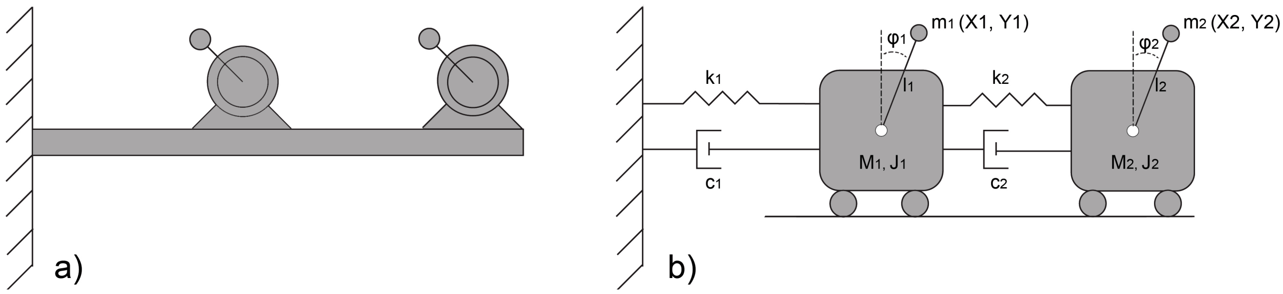

Figure 1. Non-ideal system. a) Real system; b) Equivalent system

Where $M_{1,2}$ is the mass of the electromechanical system, $m_{1,2}$ is the unbalanced mass in the shaft of the motor, $J_{1,2}$ are the motor shaft moment of inertia of the respectively motor. $k_{1,2}$ are the stiffness, $c_{1,2}$ are the damping of the system, $l_{1,2}$ are the length of the unbalanced shaft and $\phi_{1,2}$ are the angle of the respective unbalanced mass.

The Lagrange's energy method was considered to obtain the constitutive equations. In this mathematical method, potential energies and kinetic energies are related by Eq. (1).

$$L = T - V \tag{1}$$

Where L is the Lagrangean function, T is the kinetic energy and V is the potential energy of the system. In sequence, the equations are derived with respect to generalized coordinates q_i as shown by the Eq. (2).

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \left(\frac{\partial L}{\partial q_i}\right) + \left(\frac{\partial P}{\partial \dot{q}_i}\right) = 0 \tag{2}$$

P in the Eq. (2) are the dissipative energies in the system. The kinetic energy of the system is defined by the Eq. (3).

$$T = \frac{1}{2}M_1\dot{x}_1^2 + \frac{1}{2}J_1\phi_1^2 + \frac{m_1}{2}\left(\dot{X}_1^2 + \dot{Y}_1^2\right) + \frac{1}{2}M_2\dot{x}_2^2 + \frac{1}{2}J_2\phi_2^2 + \frac{m_2}{2}\left(\dot{X}_2^2 + \dot{Y}_2^2\right) \tag{3}$$

Where the displacement of the mass is given by x. To define the position of the unbalanced mass of each motor were used $X_{1,2}$ and $Y_{1,2}$, being the same ones defined in the Eq. (4) and Eq. (5).

$$X_{1,2} = x_{1,2} + l_{1,2}sin(\phi_{1,2}) \tag{4}$$

$$Y_{1,2} = l_{1,2}(1 - cos(\phi_{1,2})) \tag{5}$$

The potential energy is given by Eq. (6).

$$V = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2\left(x_1 - x_2\right)^2 \tag{6}$$

And the dissipative energies in the system are given by Eq. (7).

$$P = \frac{1}{2}c_1\dot{x}_1^2 + \frac{1}{2}c_2\left(\dot{x}_1 - \dot{x}_2\right)^2 \tag{7}$$

Using the Euler-Lagrange's equations and coupling the system with the DC motors equations, one obtains the group of equations representing the dynamical system, as shown in the Eq. (8) to Eq. (13).

$$\left(M_1 + m_1\right)\ddot{x}_1 + m_1l_1\ddot{\phi}_1cos(\phi_1) - m_1l_1\dot{\phi}_1^2sin(\phi_1) + k_1x_1 + k_2(x_1 - x_2) + c_1\dot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) = 0 \tag{8}$$

$$\left(J_1+m_1l_1\right)\ddot{\phi}_1+m_1l_1\ddot{x}_1cos(\phi_1)+b_1\dot{\phi}_1-Kt_1i_1=0 \tag{9}$$

$$\frac{di_1}{dt}=\frac{V_1-Kb_1\dot{\phi}_1-R_1i_1}{L_1} \tag{10}$$

$$\left(M_2+m_2\right)\ddot{x}_2+m_2l_2\ddot{\phi}_2cos(\phi_2)-m_2l_2\dot{\phi}_2^2sin(\phi_2)+k_2x_2-k_2x_1+c_2\dot{x}_2-c_2\dot{x}_1=0 \tag{11}$$

$$\left(J_2+m_2l_2\right)\ddot{\phi}_2+m_2l_2\ddot{x}_2cos(\phi_2)+b_2\dot{\phi}_2-Kt_2i_2=0 \tag{12}$$

$$\frac{di_2}{dt}=\frac{V_2-Kb_2\dot{\phi}_2-R_2i_2}{L_2} \tag{13}$$

Equations (10) and (13) are the complete equations of the DC motors, where the electric voltage applied V_1 and V_2 are the parameters that change and control the behavior of the system.

3. NUMERICAL SIMULATIONS

All numerical simulations were performed in MATLAB using the ODE45 integrator, being a fourth order Runge-Kutta method. Initially the equations of the system previously obtained need to be represented in the state space form. Performing the following substitutions: $x_1=x_1$, $x_2=\dot{x}_1$, $x_3=\phi_1$, $x_4=\dot{\phi}_1$, $x_5=i_1$, $x_6=x_2$, $x_7=\dot{x}_2$, $x_8=\phi_2$, $x_9=\dot{\phi}_2$, $x_{10}=i_2$, $u_1=V_1$ and $u_2=V_2$ the equations can be rewritten as follows.

$$\dot{x}_1=x_2 \tag{14}$$

$$\dot{x}_2=\frac{-(m_1l_1\dot{x}_4cos(x_3)-m_1l_1x_4^2sin(x_3)+k_1x_1+k_2(x_1-x_6)+c_1x_2+c_2(x_2-x_7))}{M_1+m_1} \tag{15}$$

$$\dot{x}_3=x_4 \tag{16}$$

$$\dot{x}_4=\frac{-(m_1l_1\dot{x}_2cos(x_3)+b_1x_4-Kt_1x_5)}{J_1+m_1l_1} \tag{17}$$

$$\dot{x}_5=\frac{u_1-Kb_1x_4-R_1x_5}{L_1} \tag{18}$$

$$\dot{x}_6=x_7 \tag{19}$$

$$\dot{x}_7=\frac{-(m_2l_2\dot{x}_9cos(x_8)-m_2l_2x_9^2sin(x_8)+k_2x_6-k_2x_1+c_2x_7-c_2x_2)}{M_2+m_2} \tag{20}$$

$$\dot{x}_8=x_9 \tag{21}$$

$$\dot{x}_9=\frac{-(m_2l_2\dot{x}_7cos(x_8)+b_2x_9-Kt_2x_{10})}{J_2+m_2l_2} \tag{22}$$

$$\dot{x}_{10}=\frac{u_2-Kb_2x_9-R_2x_{10}}{L_2} \tag{23}$$

The following initial conditions were considered: $x_1(0)=0$, $x_2(0)=0$, $x_3(0)=0$, $x_4(0)=0$, $x_5(0)=0$, $x_6(0)=0$, $x_7(0)=0$, $x_8(0)=0$, $x_9(0)=0$ and $x_{10}(0)=0$. Before the simulations were performed, the system needed to have its equations reduced and decoupled so that only one second derivative with respect to time is present in each equation.

The parameters used in the simulations are presented in Tab. 1.

4. RESULTS AND DISCUSSIONS

For the analysis of the system, were considered two different cases regarding the electric motors in order to observe the influence of each motor on the behavior of the vibratory system. As defined by the parameters presented in Tab. 1, both electric motors (limited power supply) were considered identical and their torque-speed-current curve is shown in the Fig. 2.

Table 1. Parameters of the system.

Parameter	**Value**
Mass - M_1 and M_2	0.09 - $[kg]$
Stiffness - k_1 and k_2	400 - $[N/m]$
Viscous damping - c_1 and c_2	0.077 - $[Ns/m]$
Unbalanced mass - m_1 and m_2	0.005 - $[kg]$
Eccentricity of the shaft - l_1 and l_2	0.015 - $[m]$
Armature inductance - L_1 and L_2	0.004 - $[H]$
Torque constant - Kt_1 and Kt_2	0.0066 - $[Nm/A]$
Armature resistance - R_1 and R_2	51 - $[\Omega]$
Motor inertia - J_1 and J_2	$0.9e^{-6}$ - $[kgm^2]$
Back EMF constant - Kb_1 and Kb_2	0.0066 - $[Vs/rad]$
Viscous friction - b_1 and b_2	$2.81e^{-6}$ - $[Nms/rad]$

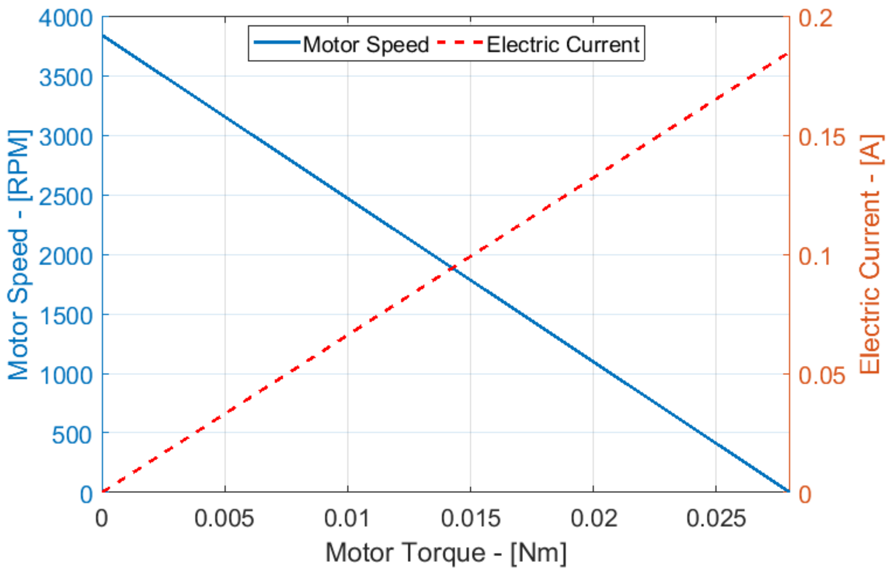

Figure 2. Electric motor torque-speed-current curve

4.1 Case 1: voltage at Motor 1 being changed and voltage at Motor 2 set at 24V

In this test, the voltage of the motor 2 was fixed to observe the behavior of the jump phenomena when acting on the motor 1. Figure 3 shows the jump phenomenon for the increase and decrease of motor 1 voltage. Figure 3a shows the influence of the motor 1 on mass 1 (beam middle) and Fig. 3b shows the influence of the motor 1 on mass 2 (beam tip). Even considering the case where the motor is acting in the middle of the beam, it is observed that the greatest amplitudes of vibration are in the tip of the beam.

Figures 4a and 4b show the jump as a function of the voltage applied in the motor 1 for the mass 1 and mass 2 respectively. Figure 5 presents the relation of the angular frequency with the applied electric voltage in the DC motor, where Fig. 5a considers the increase of the electric voltage and Fig. 5b considers the decrease of the electric voltage. The horizontal lines in Fig. 5 show that even increasing the electric voltage, the angular frequency (motor rotation) does not change their values at all and this lost energy only increases the vibration of the system.

A characteristic observed in this non-ideal system is that the vibration amplitudes of the jump are smaller when done with the decrease of the electric voltage. This behavior is also observed in real experiments as shown in Goncalves *et al.* (2014).

4.2 Case 2: voltage at Motor 2 being changed and voltage at Motor 1 set at 24V

In this case, the voltage of the motor 1 was fixed in 24V. Figures 6a and 6b show the influence of the motor 2 on mass 1 (beam middle) and on mass 2 (beam tip) respectively. Again, the largest amplitudes of vibration of the system were those of the tip of the beam, this is due to the greater influence of the vibrating modes of the cantilever beam, where the greatest influence of each mode of vibration is in the free end of the beam.

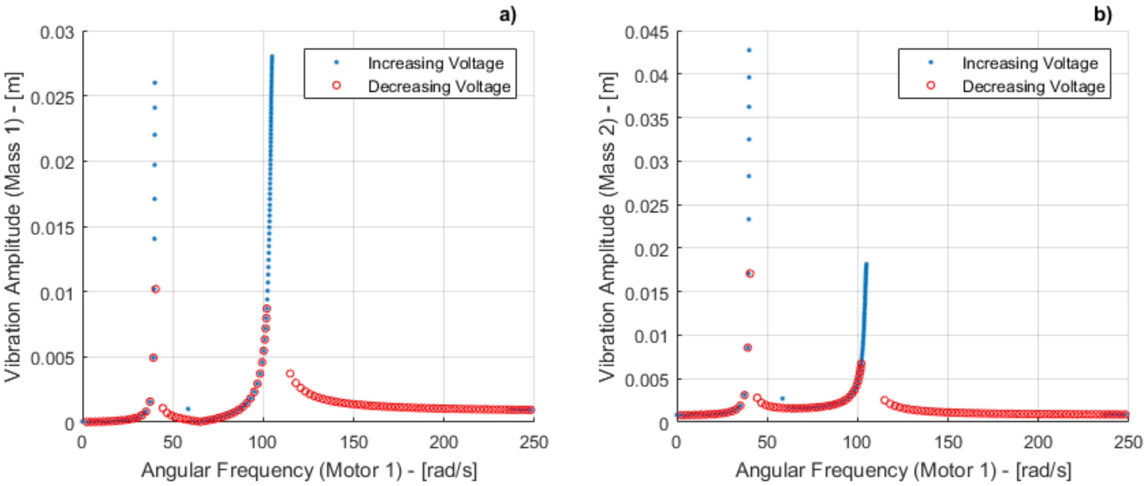

Figure 3. Jump phenomenon. a) Mass 1; b) Mass 2

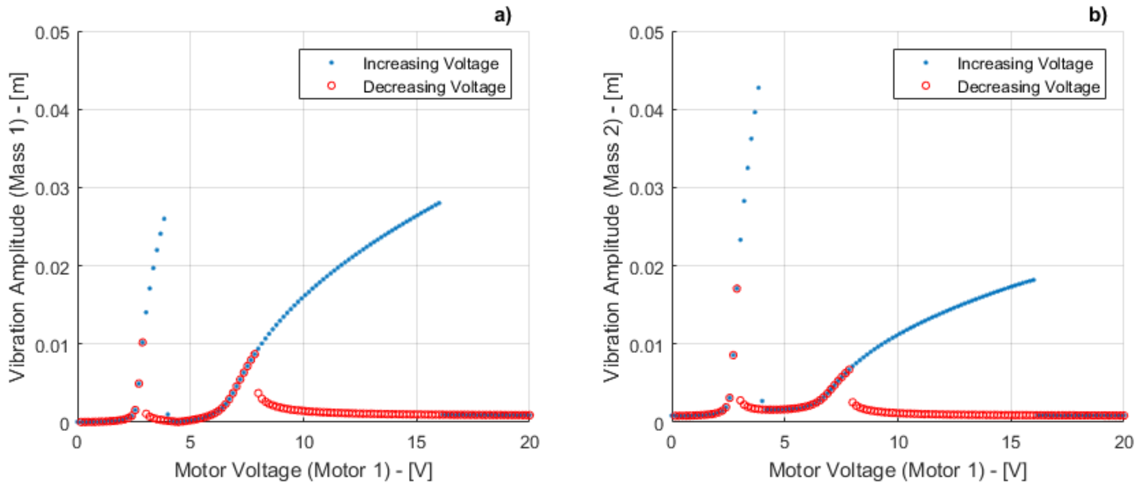

Figure 4. Jump phenomena in function of the voltage applied in the motor. 2 a) Mass 1; b) Mass 2

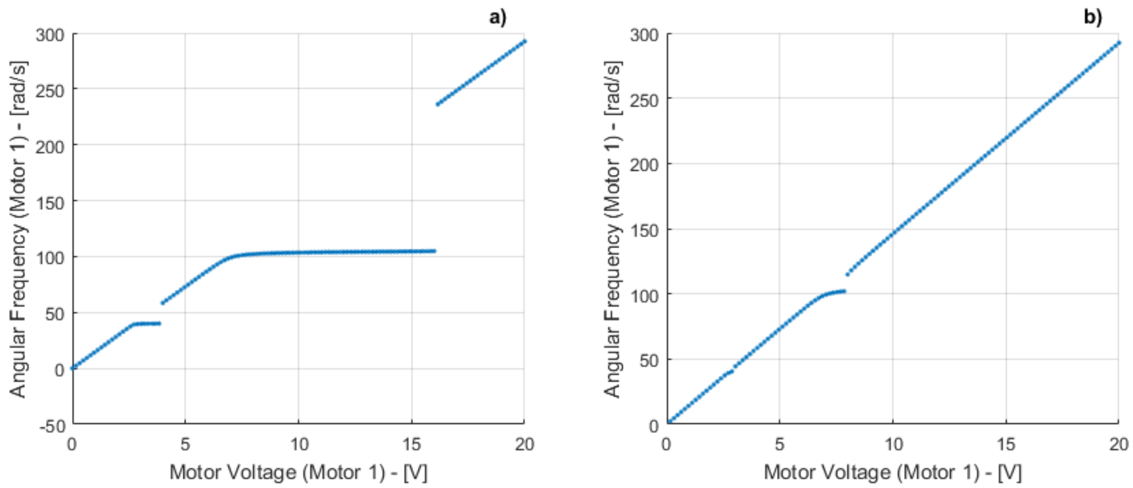

Figure 5. Variation of the angular frequency of the motor with the increase of the electric voltage. a) Increasing voltage; b) Decreasing voltage

Figure 7 relates the applied voltage to the vibration amplitudes, where, Fig. 7a presents the amplitudes of mass 1 and Fig. 7b the amplitudes of mass 2.

As the active excitation source is located at the tip of the beam, the amplitudes of the main jumps were considerably larger than the first case, however, it is noticed that the second jump, which occurs in each test, presented a much smaller

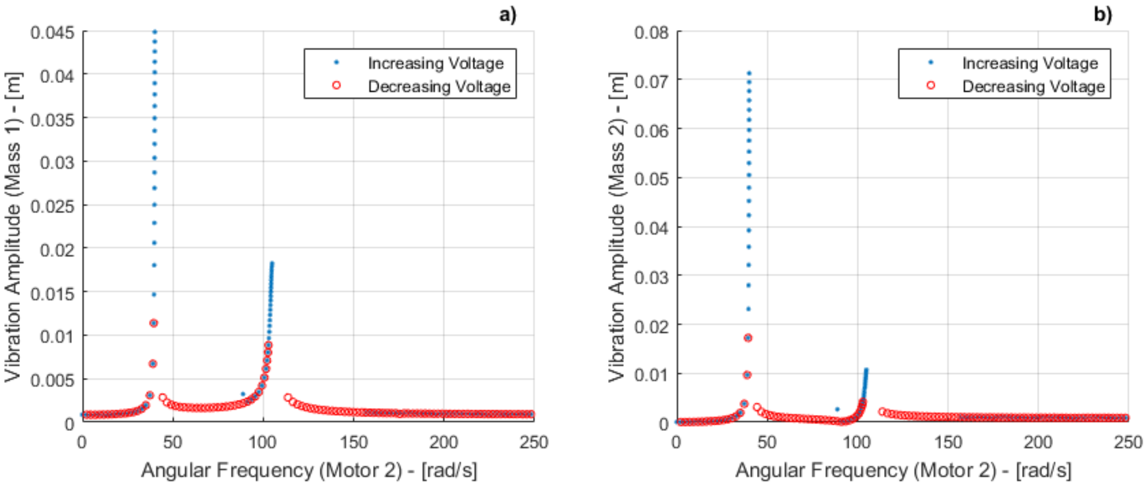

Figure 6. Jump phenomenon. a) Mass 1; b) Mass 2

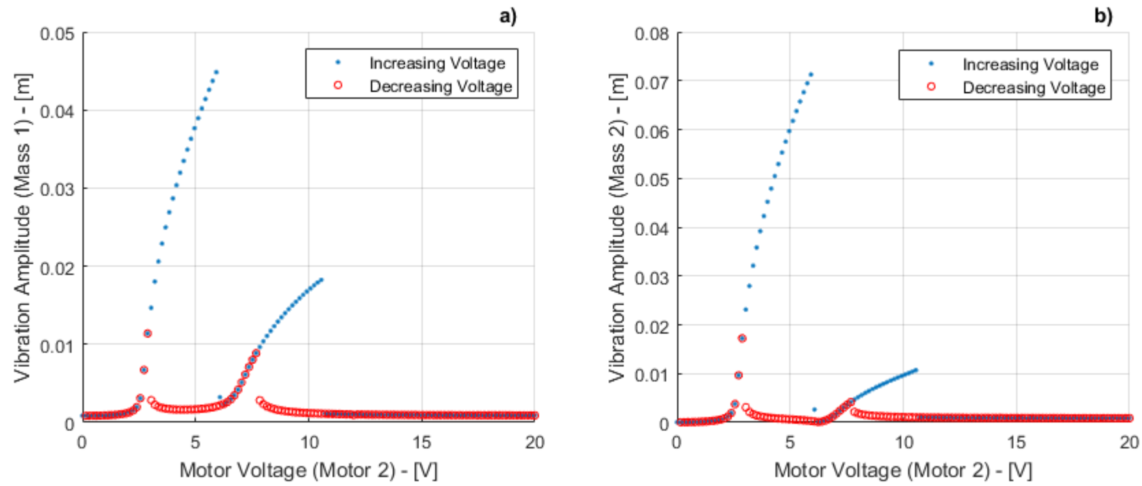

Figure 7. Jump phenomena in function of the voltage applied in the motor. 2 a) Mass 1; b) Mass 2

amplitude when compared to the first case. This fact can be observed in Figs. 8a and 8b, in which show that, in this case, the jumps also occurred at a lower voltage.

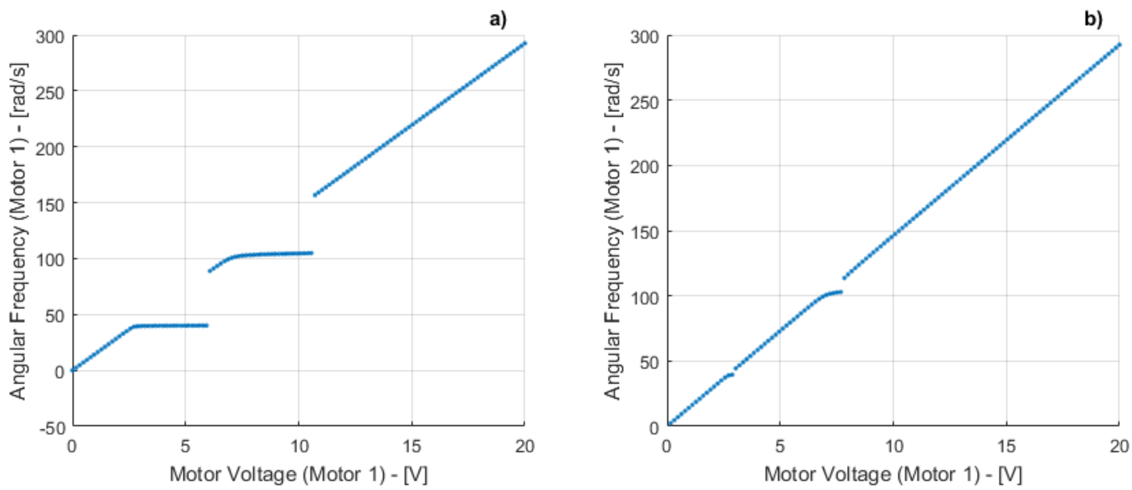

Figure 8. Variation of the angular frequency of the motor with the increase of the electric voltage. a) Increasing voltage; b) Decreasing voltage

5. CONCLUSIONS

This article deals with the mathematical modeling of a non-ideal type system consisting of a cantilever beam and two unbalanced electric motors. Graphs of the jump phenomena were obtained for two different cases: with the voltage of the motor 2 fixed and with the voltage of the motor 1 fixed. With these tests, it was possible to analyze the influence of each motor on the jump effect.

The jump graphs considering the frequency of the motor and considering the applied voltage show the loss of energy in the system, especially when it arrives near the resonance region. Due to the consideration of two masses and two sources of excitation, the system presented two consecutive jumps in all the tests. The intensity of each jump varied according to the non-ideal source. In addition, regardless of the source of excitation, the beam tip is the region with the highest amplitudes of vibration, much of this due to the greater influence of the vibration modes in this region.

There was a minimal influence (less than 1%) on the rotation of the fixed motor during the vibration of the system, showing that the non-ideal behavior only affects the motor which has a change in its supply voltage.

6. ACKNOWLEDGEMENTS

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