COBEM-2017-1722
NUMERICAL SIMULATION OF MIXED CONVECTION IN POROUS MEDIA INDUCED BY VISCOUS DISSIPATION

Pedro Vayssière Brandão
Ricardo Dias dos Santos
Leonardo Santos de Brito Alves
Laboratório de Reologia - Universidade Federal Fluminense
pedrovayssiere@id.uff.br
ricardos@gmail.com
leonardo.alves@gmail.com

Abstract. *Linear stability analysis shows that the two dimensional modes are the most unstable for this physical problem, hence this paper presents a numerical simulation of a two dimensional throughflow in a porous media, where the viscous dissipation is taken into account. The boundaries of the channel are considered as being isothermals, where the bottom is hotter than the top, originating so a vertical temperature gradient. Here are presented some cases where the temperature gradient exceeds the critical value, originating so the secondary flow, that is the mixed convection. In order to do that, the streamfunction formulation of the governing equations are solved numerically, where the finite difference method is used and the time integration is done by using a Runge-Kutta method. The results are compared with some results found in the literature, for the onset of mixed convection to this kind of problem, obtained by linear stability analysis.*

Keywords: *Direct numerical simulation, instability due to viscous dissipation, non-linear stability analysis, porous medium*

1. INTRODUCTION

The onset of the convection in porous media represents an important field on fluid mechanics and heat and mass transfer. This subject has been widely studied over the years, and it started with the classical problem of natural convection, known as Rayleigh-Benard problem. The classical Rayleigh-Benard problem consisted in study the occurrence, or not, of the natural convection in a layer of fluid heated from below.

Horton and Rogers Jr (1945) and Lapwood (1948) have extended the classical problem in order to study the natural convection of a fluid layer in a porous media. Prats (1966) has studied the effect of an horizontal fluid flow on the onset of the convection. More recently some studies have included different effects as contributors to the onset of the secondary flow. Barletta *et al.* (2009) have shown that the viscous dissipation can induce a transition to an unstable configuration, even though neglecting external heat sources. Alves *et al.* (2014) presented the effects of viscous dissipation on the convective instability of viscoelastic fluid flows in porous media.

Linear stability theory has shown to be a good tool to predict the transition to instability in the cases in which the transition does not depend on the amplitude of the perturbation acting on the system. When it does not occur, one could employ either a weakly nonlinear analysis, or a fully nonlinear analysis. One possible way to perform a non linear stability analysis is employing a Direct Numerical Simulation (DNS) of the disturbed governing equations, as was done by Celli *et al.* (2016).

This work presents a direct numerical simulation of the governing equations of the mixed convection in a throughflow, induced by a vertical temperature gradient and by viscous dissipation. Since Alves *et al.* (2014) pointed out that all modes are equally unstable for this problem, it can be treated as a two dimensional one without loss of generality. The steady state of the velocity and temperature profiles are disturbed, and the growth or decaying of the perturbation is observed depending on the control parameters. The focus of this study is to analyse the linear and non linear behaviors of the disturbances on simulating the fully nonlinear problem, for this purpose are reported here supercritical and sub-critical cases, but not so far from the marginal stability configuration. For the linear cases here reported, is possible to calculate some parameters of the perturbation development, taken from the non linear simulation, and compare with those obtained from the linear stability theory. Therefore, some parameters related to the perturbations are compared with those present in Alves *et al.* (2014), for the case such that problem reduces to a newtonian one.

2. MATHEMATICAL FORMULATION

The governing equations are written in a dimensionless form according to the quantities

$$(x,z)=\frac{(x^*,z^*)}{H},\quad t=\frac{t^*}{\sigma H^2/\alpha},\quad \vec{u}=\frac{\vec{u}^*}{\alpha/H},\quad T=\frac{T^*-T_h}{T_0^*-T_h}, \tag{1a}$$

$$Ra=\frac{g\beta(T_0^*-T_h)KH}{\nu\alpha},\quad Ge=\frac{g\beta H}{c},\quad Pe=\frac{U_0 H}{\alpha} \tag{1b}$$

where starred quantities are dimensional, H is the layer thickness, ν is the kinematic viscosity, U_0 is a uniform stream wise velocity, T_0 and T_h are the pescribed lower and upper wall temperature, respectively, α is the average thermal diffusivity, c is the specific heat of the fluid, and σ is the ratio between the average volumetric heat capacity of the porous medium, and the volumetric heat capacity of the fluid, β is the fluid thermal expansion coefficient. Therefore, the governing equations and the boundary conditions are given by

$$\nabla\cdot\mathbf{u}=0, \tag{2a}$$

$$\nabla\times\mathbf{u}=Ra\,\nabla\times(T\vec{e}_z), \tag{2b}$$

$$\frac{\partial T}{\partial t}+\mathbf{u}\cdot\nabla T=\nabla^2T+\frac{Ge}{Ra}\mathbf{u}\cdot\mathbf{u}, \tag{2c}$$

$$z=0:\qquad w=0,\quad T=1, \tag{2d}$$

$$z=1:\qquad w=0,\quad T=0. \tag{2e}$$

The steady states of temperature and velocity are used as initial condition for the numerical simulation, and can be written as following

$$\mathbf{u}_b=(Pe,0,0) \tag{3}$$

$$T_b(z)=1-z+\frac{GePe^2}{2Ra}(1-z)z \tag{4}$$

Here, T is the temperature, $\mathbf{u}$ is the vector velocity , Ra the Rayleigh number, Ge is the Gebhart number, Pe is the Peclet number, and the subscript b stands for basic solution, that is the steady state solution.

3. NUMERICAL METHODOLOGY

As long as the buoyancy force is less important then the gravitational force, the natural convection, in this case mixed convection, will not occur. From the moment that the buoyancy force exceed an certain critical value, should be possible to observe the onset of natural convection. From the theory of linear stability analysis, is possible to obtain the critical value of the control parameters, above which the problem becomes unstable, what means that any infinitesimal perturbation grows exponentially. In the present case, this represents that if the control parameters exceed their critical values, one could observe the grow of the infinitesimal perturbation and hence the beginning of the natural convection. In analogy, if one is below the critical value of the parameters, any perturbation decay exponentially and the transition for the unstable configuration does not occur, in this case the problem is said to be stable.

With the purpose to observe the onset of mixed convection, one can perturb some variable, such as the temperature, and depending on the control parameters is observe a transition from the steady solution to an unstable configuration. In this case the temperature is perturbed on the energy Equation with a source term.

Since the problem is two-dimensional, the Equation 2.can be solved using the streamfunction formulation. So the Equation 2b and 2c can be rewritten as

$$\nabla^2\psi=-Ra\frac{\partial T}{\partial x} \tag{5}$$

$$\frac{\partial T}{\partial t}+\frac{\partial\psi}{\partial z}\cdot\nabla T=\nabla^2T+\frac{Ge}{Ra}\left(\frac{\partial\psi}{\partial x}^2+\frac{\partial\psi}{\partial z}^2\right)+\delta T \tag{6}$$

where ψ is the stream function, and it is defined as follows

$$u=-\frac{\partial\psi}{\partial z} \tag{7}$$

$$w=\frac{\partial\psi}{\partial x} \tag{8}$$

u and w being the velocity components in x and z directions respectively. And δT is the perturbation function. It is oscillatory on time, and has the following form

$$\delta T = \gamma \sin(\omega t) \sin(\pi x) \sin(\pi z) \exp(-\beta((x - x_c)^2 + (z - z_c)^2)) \quad (9)$$

Considering that ω is the frequency of the perturbation and γ and β are the control parameters related to the form and the magnitude of the perturbation. It is known from the stability analysis that if the input parameters are supercritical, the perturbation grows in time or space depending on their values. If the control parameters configure a transition to a convective instability, the perturbation grows in space for instance.

In order to solve these equations, finite difference method is used and the time integration is done by a third order runge-kutta method. On the energy equation, the advective term for the temperature is discretized by using a third order upwind scheme and the diffusive terms are discretized using a fourth order centered finite difference discretization. The Poisson Equation 5 is here solved using the Gauss-Seidel method. Looking for a better convergence, the relaxation technique is applied.

To deal well with inlet and outlet boundary conditions, different approaches were proved. The results presented here, are relative from the choice in which the inlet is fixed as inflow and the condition is equal to the steady state condition, and in order to simulate an infinite horizontal channel, and to avoid numerical reflections to the rest of the domain, an artificial domain is created, where all fluctuations are damped and the solution tend to a target value a priori defined. After doing that the outlet boundary condition is defined as constant properties, that is zero derivative. A more detailed explanation as well a derivation of this kind of treatment of boundary conditions can be found in Freund (1997) and Richards *et al.* (2004). All results presented here are relative to the same domain, namely 15x1 (x coordinate x y coordinate), and the mesh is uniform with 101 points per unit of length, that is 1501x101. The buffer region is considered just in the tree units of length of the end of the domain, that is from 12 to 15.

4. RESULTS

As said before, the steady state is perturbed with a source term and the development of the perturbation is observed as the time marchs. If the control parameters (Peclet, Rayleigh and Gebhart numbers) are criticals, the perturbation grows in time or space, depending on the case. In this case only spatial growth is taken account, when the disturbances grow spatially it is said that the problem is convectively unstable. Thus, in order to observe and measure the growth or not of the perturbation, the problem is disturbed with a constant time frequency, and the spatial behavior is analysed.

After disturbing the steady solution, the solution of the disturbed problem can be analysed as a composed solution, including both the steady state solution and the impulse response of the perturbation.

$$\mathbf{q} = \mathbf{q}_b + \mathbf{q}_p \quad (10)$$

Considering that the perturbation can be written as a superposition of normal modes, and since the most unstable mode prevail over the others, the observed impulse response should be of the form:

$$\mathbf{q}_p = \hat{\mathbf{q}}\, e^{i(\alpha x - \omega t)} \quad (11)$$

Where $\alpha = \alpha_r + i\alpha_i$ is the complex wave number being α_r the wave number and α_i the spatial growth rate, and $\omega = \omega_r + i\omega_i$ the complex frequency, where ω_r represents the oscillatory frequency and ω_i the temporal growth rate.

In order to analyse spatial growth, one should freeze the time and observe the spatial behavior of the disturbance response. Freezing the time, and looking strictly to the disturbance response, it oscillates in space with wave number α_r and grows/decay in space with a rate α_i.

Here are presented the results for a stable case and a convectively unstable case. The results presented are in terms of the perturbation in the z direction (w_p), but since w_b is equal to zero, $w = w_p$. And it is obtained from the streamfunction from Equation 3.

Figure 1 shows the perturbations growing is space. The different frames are respect to different instants of time. This case is referred to $\omega = 32$, $Ra = 45$, $Pe = 10$ and $Ge = 0.1$, what second Alves *et al.* (2014) should represent a state convectively unstable. Is possible to observe in figure 1 that the perturbation, in fact, grows in space.

Figure 2 represents the growth of the disturbances for a supercritical case. For this combination of Peclet and Gebhart numbers, Rayleigh number equal to 39.5333 should represent a marginal stability configuration according to Alves *et al.* (2014). So $Ra = 45$ represents a supercritical state, more precisely a convectively unstable state. The continuous curve represents the function used to fit the discrete data from the numerical simulation. The function chosen is a combination of a cosine and an exponential, where the arguments are associate with the wave number and the growth rate, respectively.

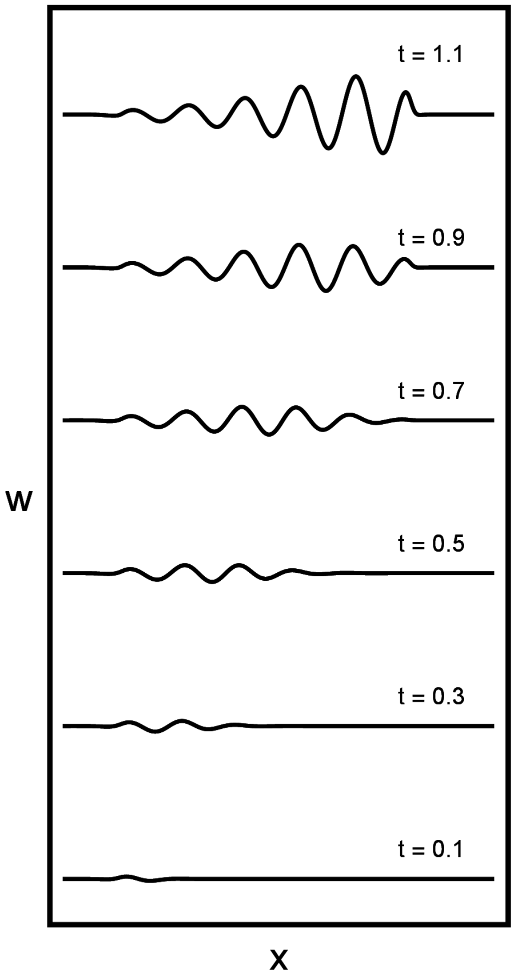

Figure 1: Perturbation relative to $\omega = 32$, $Ra = 45$, $Pe = 10$ and $Ge = 0.1$ growing in space

Table 1: Comparison of literature data (Alves *et al.*, 2014) and the results from the DNS for the growth rate (α_i) and wave number (α_r) of the perturbation considering $\omega_r = 32$, $Pe = 10$ and $Ge = 0.1$

	Ra = 45		Ra = 36	
	α_i	α_r	α_i	α_r
Alves *et al.* (2014)	-0.2997221716	3.178178815	0.1698089501	3.187723785
Present paper	-0.2984240818	3.182821523	0.1716128696	3.190058018
Relative deviation	4.33x10^{-3}	1.46x10^{-3}	1.06x10^{-2}	7.32x10^{-4}

Table 2: Comparison of literature data (Alves *et al.*, 2014) and the results from the DNS for the growth rate (α_i) and wave number (α_r) of the perturbation considering $\omega_r = 32$, $Ra = 45$, $Pe = 10$

	Ge = 1		Ge = 0	
	α_i	α_r	α_i	α_r
Alves *et al.* (2014)	-0.2980867508	3.178544337	-0.4579439071	3.127803768
Present paper	-0.2968359157	3.183185147	-0.4572939004	3.133000200
Relative deviation	4.19x10^{-3}	1.46x10^{-3}	1.42x10^{-3}	1.66x10^{-3}

Figure 3, instead, represents the decay of a perturbation when the parameters combination are sub-critical, that is, for the same values of Peclet and Gebhart number of first, Rayleigh number is below the marginal one($Ra = 39.5333$). In

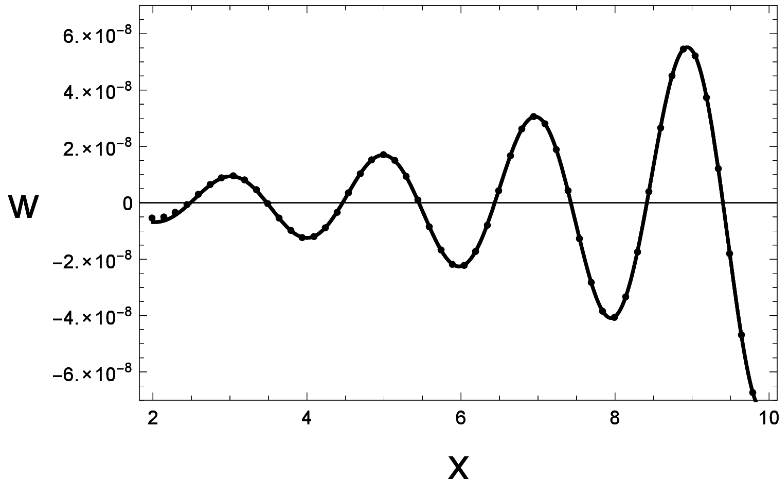

Figure 2: Perturbation relative to $\omega_r = 32$, $Ra = 45$, $Pe = 10$ and $Ge = 0.1$ growing in space

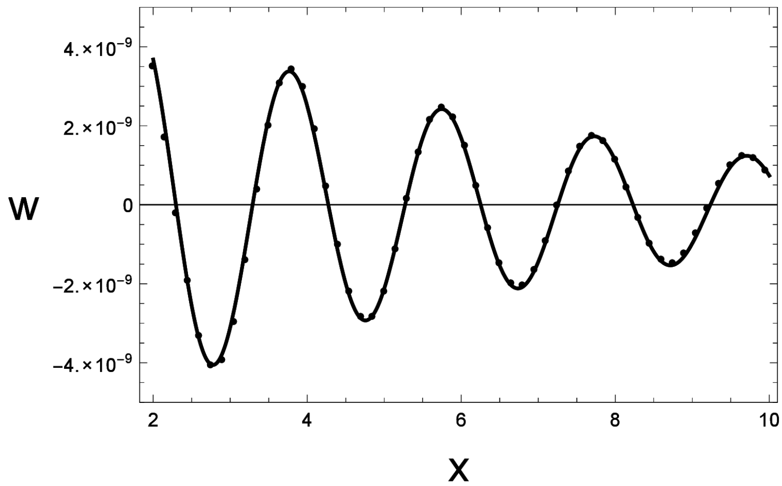

Figure 3: Perturbation relative to $\omega_r = 32$, $Ra = 36$, $Pe = 10$ and $Ge = 0.1$ decaying in space.

this case any infinitesimal perturbation should decay in space, and it is exactly what is observed here.

The calculation of the perturbation parameters, for instance wave number and growth/decay rate, as well the comparison between the literature and the present data, are shown in table 1.

Moreover, table 2 presents the influence of viscous dissipation in the growth rate. Since viscous dissipation is one of the causes of the heat source here, as viscous dissipation increases, the temperature gradient increases as well, and the transition to instability should become easier. In other words, viscous dissipation destabilizes the problem. Therefore, for the same parameters combination, increase Gebhart number should represent a destabilization, or an increase on the growth rate.

Table 2 presents the growth rates for a vanishing Gebhart number and for Gebhart number equal to one. It is possible to see that $Ge = 1$ represents a bigger growth rate of the perturbation in respect of $Ge = 0$, as expected.

5. CONCLUSIONS

This paper proposed to investigate the transition to an unstable configuration, that is the appearance of a secondary flow, of a throughflow in a porous media due to a vertical temperature gradient and by viscous dissipation. In order to do that the fully nonlinear disturbed problem was simulated numerically, and some results were compared with those present in the literature.

The linear behavior of the growth or decay of the perturbations, are in good agreement with those found in the literature. The relative deviation of the cases presented here, are of the order of 10^{-2} to 10^{-4}.

As expected, viscous dissipation assumes a role of destabilize the problem. It was possible to observe clearly this role when comparing a case without viscous dissipation and with a strong viscous dissipation.

As future work, it is desired to go further with this study and analyse the time evolution of the instabilities, the absolutely unstable state, as well to investigate more the non linear effects and the interaction between the different modes that can appear.

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