Investigation of the effect of the modulation in a boundary layer through DNS simulation and comparison with experimental results

Théodore Meynard, theodore.meynard@usp.br
Andrés Gaviria M, 4ndres.gaviria@gmail.com
Marcello A. F. de Medeiros, marcello@sc.usp.br
Escola de Engenharia de São Carlos, Universidade de São Paulo
Avenida Trabalhador são-carlense, 400, Pq Arnold Schimidt
CEP 13566-590 - São Carlos - SP.

Abstract. *A numerical analysis of the nonlinear evolution of a wavetrain on an incompressible boundary layer was performed by comparison with the experimental results, given in Medeiros (2004)[4]. Experiment was simulated using Direct Numerical Simulations (DNS), code tests are presented and comparisons with experimental results in physical and Fourier space. In experiment, low frequency modes are strongly amplified due to nonlinear interaction also observed in simulations. However sign inversion of the mean flow distortion do not occurs in simulation, and the amplitude of nonlinear amplified modes are lower than in experiment.*

1. INTRODUCTION

Besides more than one century of research, transition to turbulence remains only partially understood. This field can have a huge impact in the transport industry, especially aeronautics. Indeed, the turbulent flow generates a bigger drag than the laminar one and the understanding of the phenomena which induce the transition could significantly improve the efficiency of aircrafts. Moreover, an accurate numerical transition prediction would allow a much better drag estimation. Already exists models for very controlled situations. The transition start is linked to the presence of a perturbation, for example, in the velocity field, when determined frequencies can be amplified by the flow and generate, after a non-linear process, a turbulent flow. The waves in the boundary layer are often an amplification of Tollmien-Schlichting waves.

One of these transition processes is called K-type or fundamental type. This transition comes from the increase of oblique waves of the same amplitude of the fundamental wave, which is parallel to the flow field. If the fundamental wave amplitude is significant compared to the flow, the fundamental wave "catalyses" the oblique waves and their amplitudes increase dramatically. In addition, the non-linearity of the flow creates frequencies other than the fundamental, which increases the corresponding amplitudes, leading to the transition. Another type of transition is called H-type or N-type. Here the oblique waves have a frequency which is half of the fundamental wave. As with the K-type, this fundamental wave "catalyses" the oblique waves amplitudes and trigger the transition. In the two types described before, the oblique waves receive energy from the flow and not from the fundamental wave (see a more detailed explanation in section 2. 2). These two transition types are studied because the transition feature is similar to the natural transition observed experimentally, but are less complicated to understand. The objective now is to make the models more complex to better represent the natural transition and to comprehend the phenomena based on what is understood from the precedent models.

In this paper,the effect of a modulation will be studied. In section 3,.the amplitude modulation will be studied. First a modulation in the flow direction will be introduced. It will show that it can change the transition type. Then comparisons of simulation with experimental result from Medeiros (2004) [4]. The modulation will be in this case in the spanwise direction. In the experiment, the non-linear evolution of a wave train emanating from a point source is studied. The point source introduce a modulation in the of direction of the flat plate leading edge. This wave train is more generic than the K-type or N-type but still far less complicated than the natural transition. This simpler model might provide an important step towards the understanding of natural transition.

There is a qualitative concordance between the experimental and computational results. The difference between the two results seems to be due to the underestimation of the low frequency oblique waves generated by the flow in the simulation. There are already studies showing this numerical underestimation like Spalart (1987)[7].

2. METHODOLOGY

2.1 Code description

For spatial discretisation it was constructed a compact finite differences scheme with spectral like resolution, based on Lele (1992) [3], with fourth-order accuracy. This schemes allows to maintain an established accuracy over a defined range of length scales. The time integration is performed with a standard fourth-order Runge Kutta method. This stencil is stable up to CFL number 1.3. Uniform grid is used in streamwise and spanwise directions, grid in wall normal direction is stretched to increase resolution near the wall and resolve gradients. To control spurious oscillations a tenth-order low pass filter [8] is applied in each iteration at the inner points of the domain.

2.2 Preliminary test

This code have been used successfully in others problems on boundary layer transition and comparisons with experimental results [5]. To illustrate the code and the methodology, a fundamental type with two oblique waves and a fundamental are simulated and the results are compared. This study is in concordance with the transition theory for the fundamental type and the study from Fasel and Konzelmann (1990)[2] and Rist and Fasel (1995) [6].

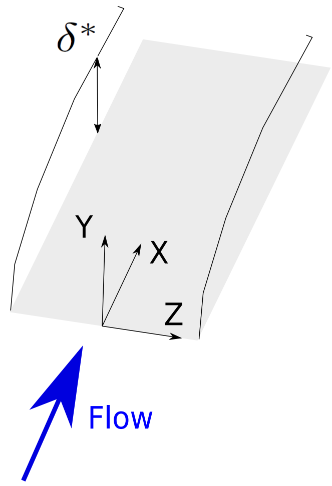


Figure 1: Axis used for the flat plate

For a fixed x and for $y = 0.6\delta^*$ with δ^*, the Blasius boundary thickness, every grid point in the z direction during a determined time can be recorded. The result can be saved in a two dimension matrix. Using the two dimension Fourier transformation, the evolution for each mode of the fundamental and oblique waves can be measured. Doing it for each x in the grid, it is possible to track the amplitude of the modes in the flow direction. Two experiments were made to compare the linear and non-linear reaction. In the figure 2, it is plotted the amplitude for each mode. The coordinates for each plot indicate the coefficient for the frequency ω_0 and the spanwise wave number β_0. For example (1,1) indicate that we are tracking the figure the amplitude for the mode $\omega = 1 \cdot \omega_0 = 9.201 \cdot 10^{-2}$ and $\beta = 1 \cdot \beta_0 = 0.276$. The simulation was done with a perturbation at $Re = \frac{U_\infty \delta^*}{\nu} = 500$ and the amplitude at the perturbation is $1 \cdot 10^{-3}$ for the nonlinear case and $1 \cdot 10^{-6}$ for the linear case. The oblique modes are generated with the same time pulsation but the amplitude a hundred times smaller than the fundamental.

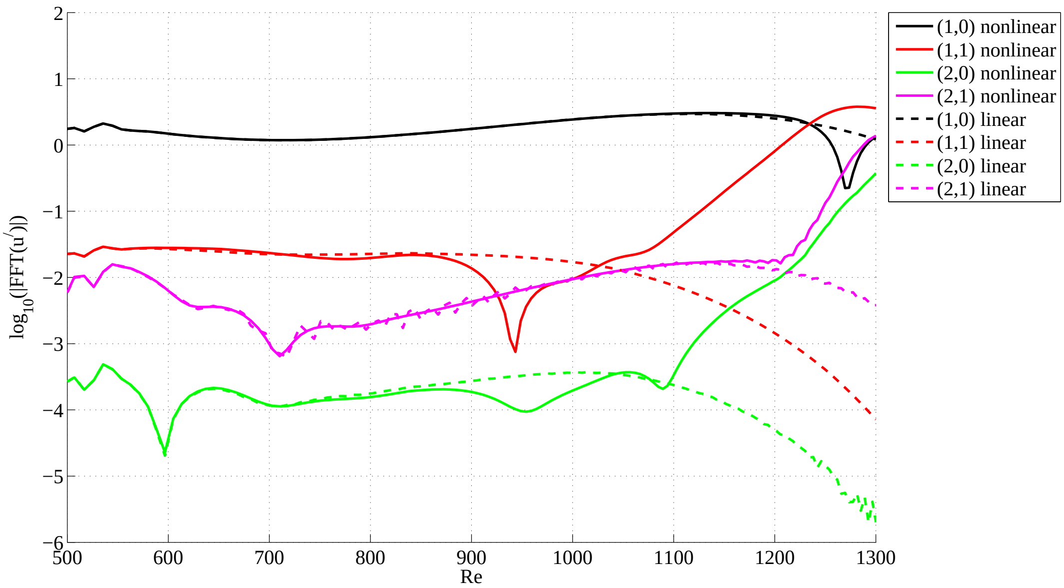


Figure 2: Comparison of normalised amplitude development for each Fourier mode for the linear and non-linear case

The results are in concordance with the theory. In the linear case, the amplitude of the fundamental is not enough to catalyse the oblique modes. On the other hand, for the non-linear case, the fundamental amplitude can catalyse the oblique modes and grow exponentially, starting the non-linear transition process. As it is shown in figure 2 the amplitude of the fundamental for the linear and non-linear cases are identical, which confirms the affirmation that the fundamental just catalyses the oblique wave. The energy for the amplitude comes from the flow and not directly from the fundamental wave but the transfer is possible only when the fundamental wave is significant.

3. RESULTS

3.1 Modulation in the flow direction

First we would like to see the effect of the amplitude modulation in the x direction in the transition process. To implement it, the non-linear case was run this time with a time dependency on the amplitude which create a modulation in the flow direction. This modulation has already been studied by De Paula and al (2013)[1]. The fundamental amplitude for example, which was $A = 1e - 3$, is now$A = 1 \cdot 10^{-3} \cdot \cos^2(\frac{\omega t}{8})$ for fig. 4d, $A = 1 \cdot 10^{-3} \cdot \cos^2(\frac{\omega t}{14})$ for fig. 4c and $A = 1 \cdot 10^{-3} \cdot \cos^2(\frac{\omega t}{28})$ for fig. 4b. The speed disturbance is generated by introducing a wall velocity on the flat plate3. The modulation frequency are respectively each 14, 7 and 4 periods.

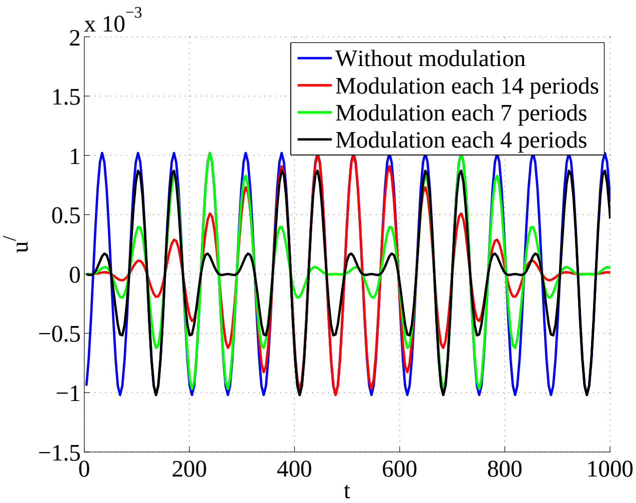


Figure 3: Speed disturbance profile without amplitude modulation and with amplitude modulation each 14,7 and 4 periods

The development of the amplitude for the fundamental and the oblique waves are followed. In addition, others amplitudes which are related to the time dependency on the amplitude can be tracked. Indeed, the fundamental amplitude function f for the 14-period case is now:

$$f_{14}(t) = A(t) \cdot \cos(\omega.t) \tag{1}$$

$$f_{14}(t) = 1.10^{-3} \cdot \sin^2(\frac{\omega \cdot t}{28}) \cdot \cos(\omega \cdot t) \tag{2}$$

$$f_{14}(t) = 1.10^{-3} \cdot \frac{1}{2}(1 - \cos(\frac{\omega \cdot t}{14})) \cdot \cos(\omega \cdot t) \tag{3}$$

$$f_{14}(t) = 1.10^{-3} \cdot \frac{1}{4} \cdot (2 \cdot \cos(\omega \cdot t) - \cos((\omega + \frac{\omega}{14})t) - \cos((\omega - \frac{\omega}{14})t)) \tag{4}$$

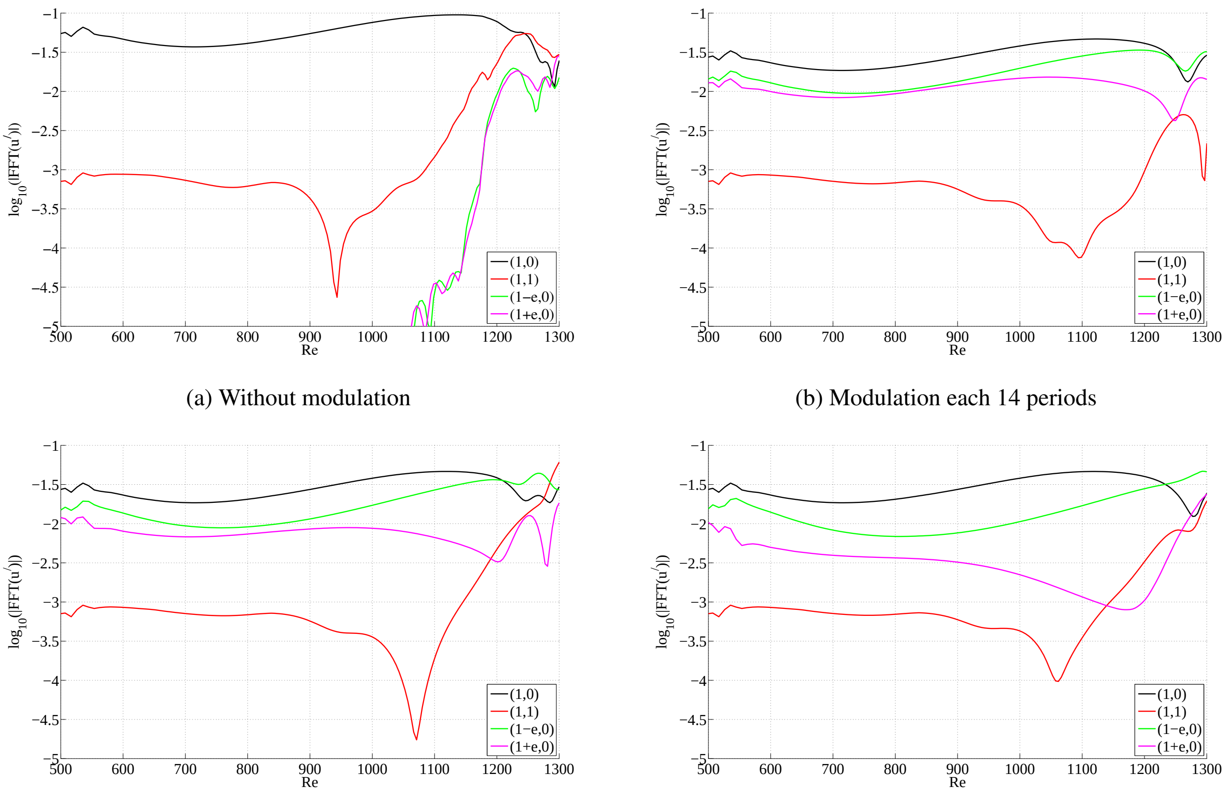
For the modulation each 7 periods it can be transformed to:

$$f_7(t) = 1.10^{-3} \cdot \frac{1}{4} \cdot (2 \cdot \cos(\omega \cdot t) - \cos((\omega + \frac{\omega}{7})t) - \cos((\omega - \frac{\omega}{7})t)) \tag{5}$$

For the modulation each 4 periods it can be transformed to:

$$f_4(t) = 1.10^{-3} \cdot \frac{1}{4} \cdot (2 \cdot \cos(\omega \cdot t) - \cos((\omega + \frac{\omega}{4})t) - \cos((\omega - \frac{\omega}{4})t)) \tag{6}$$

The amplitude modulation introduces two other modes near to the fundamental mode and that is what is represented in the legend with (1+e,0) and (1-e,0).



(a) Without modulation

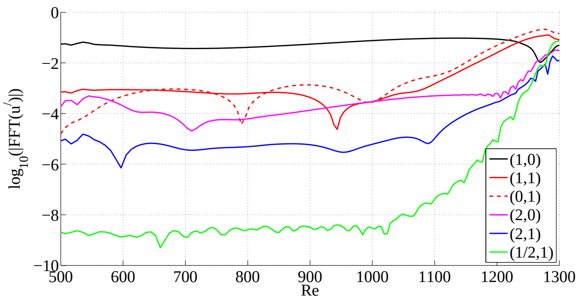
(b) Modulation each 14 periods

(c) Modulation each 7 periods

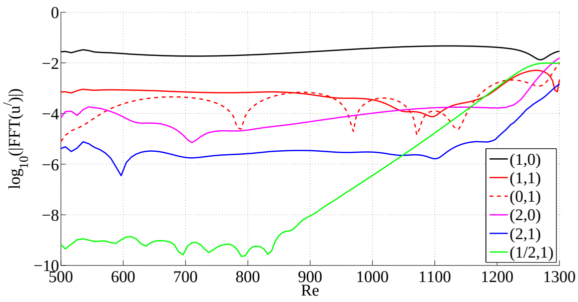
(d) Modulation each 4 periods

Figure 4: Effect of amplitude modulation on the modes' evolution

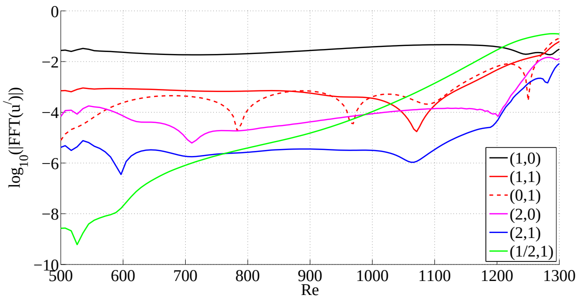
The mode ($\frac{1}{2}$,1) is affected by the modulation. Remember in the N-type it is this oblique mode which triggers the transition. In the fundamental type the mode ($\frac{1}{2}$,1) does not affect the transition as shown in fig. 5a. But in a fundamental type when we introduce an amplitude modulation this mode is catalysed and grow even faster than the mode (1,1) which should be responsible for the transition in a non modulated case as shown in fig. 5b, 5c and 5d. In other word, the modulation seems to influence the transition type.



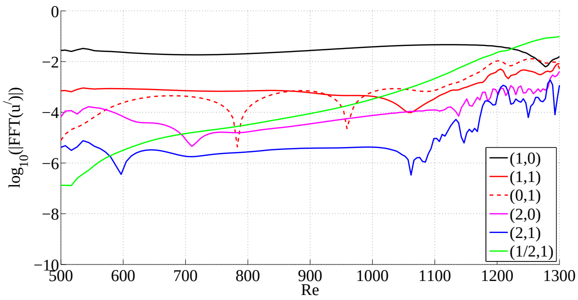
(a) Without modulation



(b) Modulation each 14 periods



(c) Modulation each 7 periods



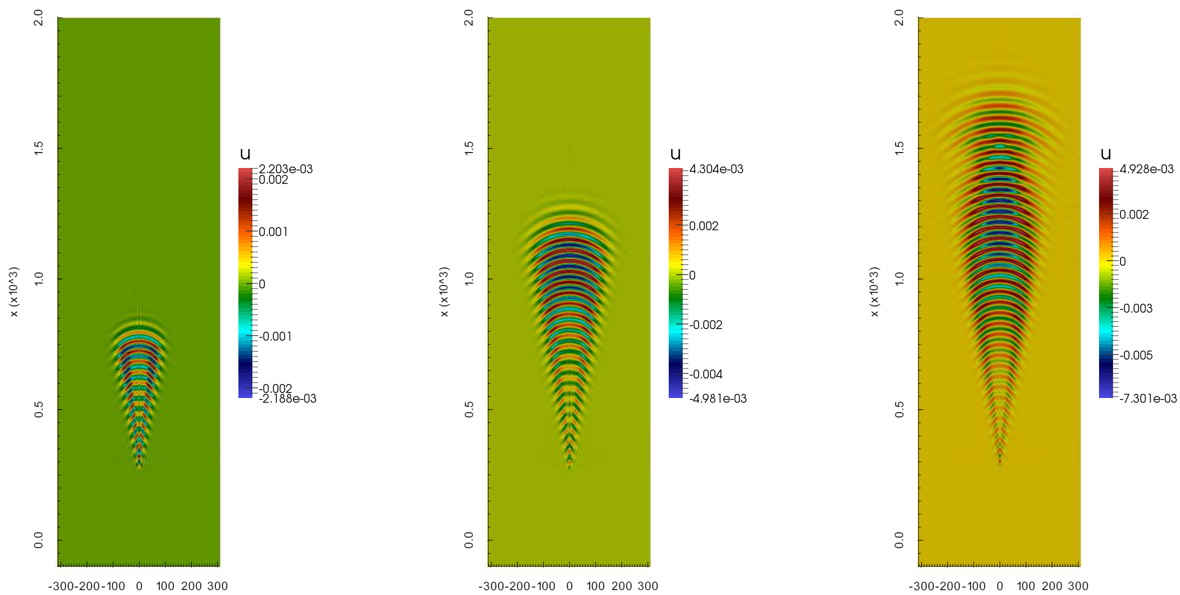
(d) Modulation each 4 periods

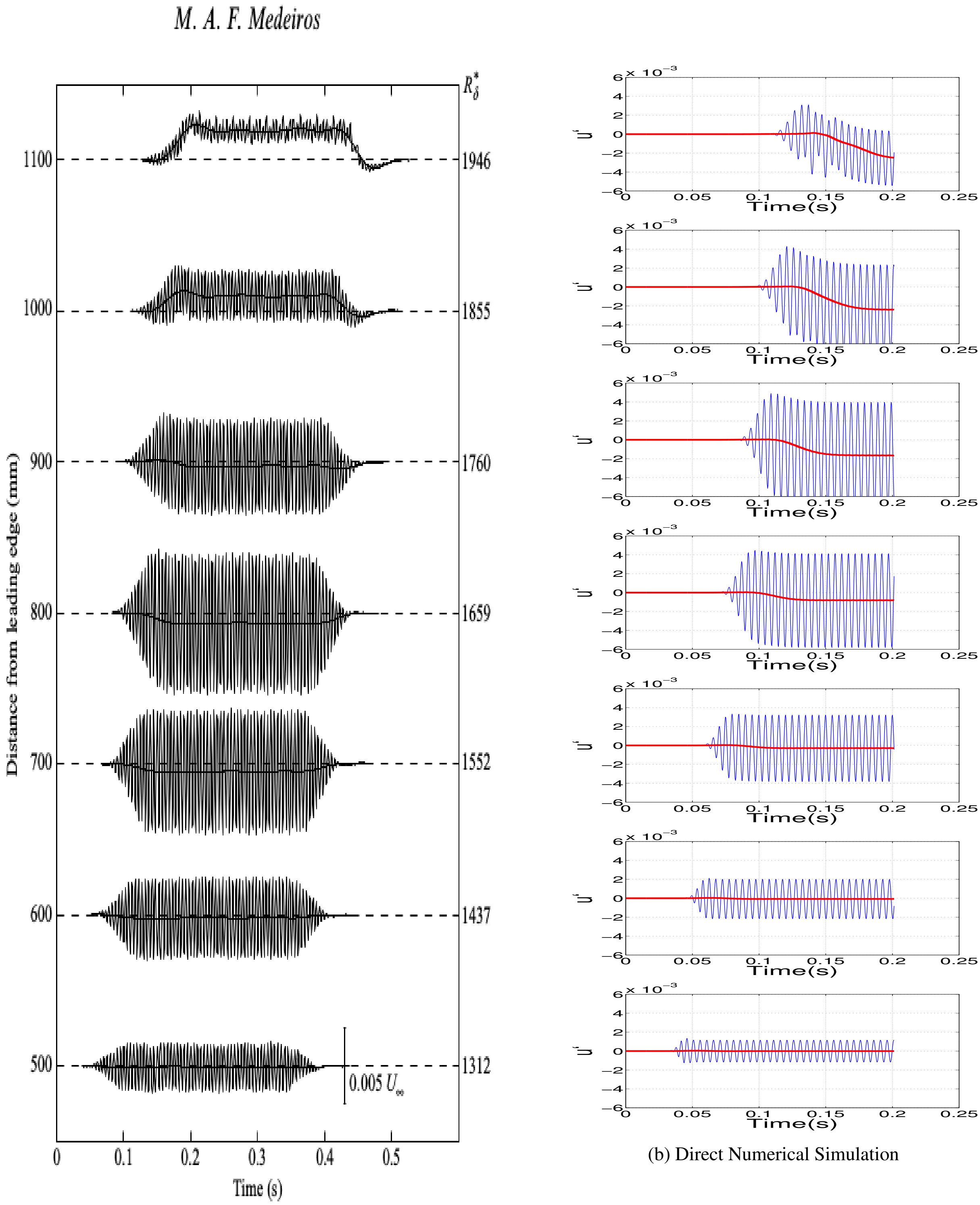
Figure 5: Effect of the amplitude modulation on the transition process : change of transition type

3.2 Wavetrain perturbation

A more realistic scenario with a wave train emanating from a point source is now studied. The objective is to simulate the experimental study from Medeiros (2004) [4]. The main difference from the previous simulation is that we introduce many oblique waves to represent the point source and there is no modulation in the amplitude of the source point. The modulation studied here is in the z direction. It induces a thinner discretisation for the mesh in the flat plate direction. In the previous simulation there are 8 points in the z direction and in this one, 200. This allows us to simulate more precisely the point source by including a band of oblique waves.

Figure 6: Velocity distribuition at $0.6\delta^*$ from the wall



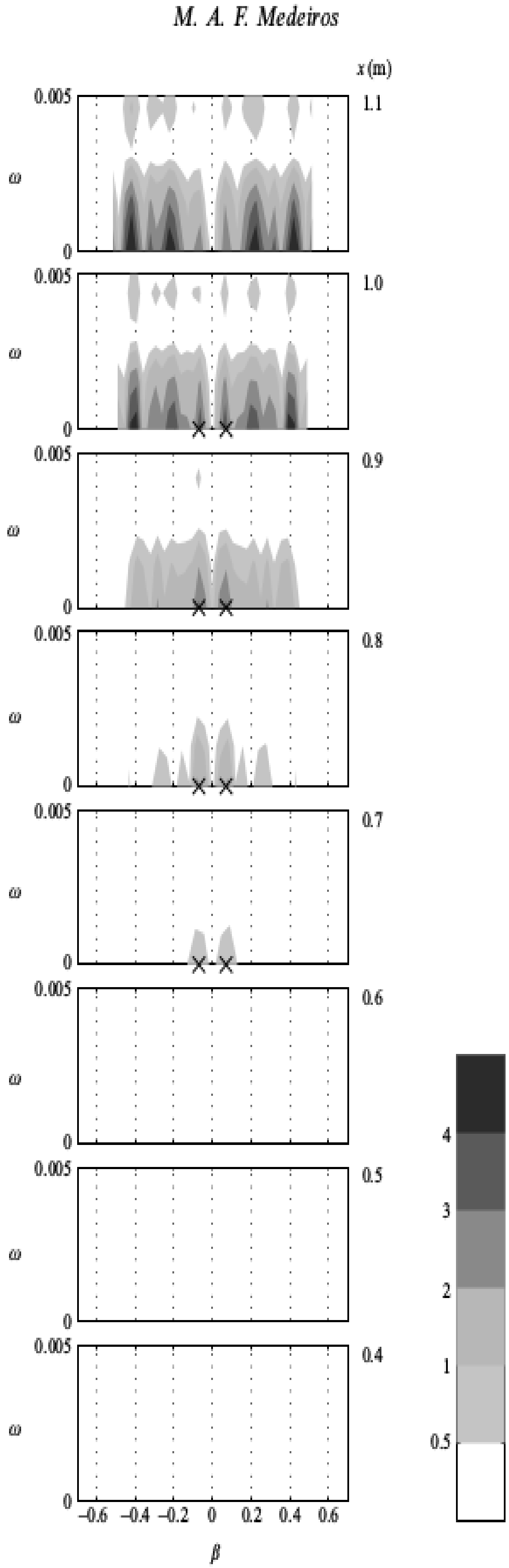


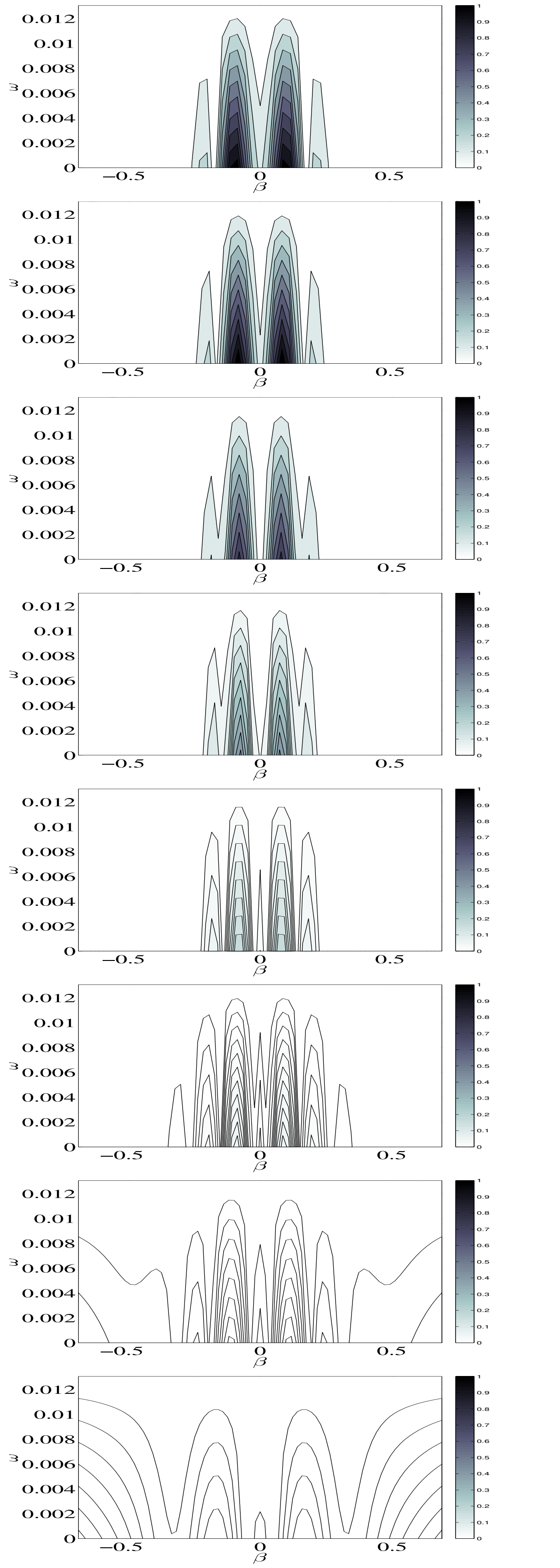
(b) Direct Numerical Simulation

(a) Experimental result from Medeiros (2004)[4]

Figure 7: Streamwise evolution of the wave train along the center line of the plate at $0.6\delta^*$ from the wall

The result are similar with the same negative shift for $R_\delta^* < 1760$ but do not obtain the positive shift as in the experiment for $R_\delta^* < 1760$ (see fig. 7b).





(b) Direct Numerical Simulation

(a) Experimental result from Medeiros (2004)[4]

Figure 8: Streamwise evolution of the three-dimensional mean flow distortion in Fourier space

For the fig. 8 from Medeiros (2004)[4] (reproduced in fig. 8a), the mean flow distortion in Fourier space creates low frequency. The simulation reproduces these induced frequency but they are not similar quantitatively with the experimental measures [4].

The difference could be linked to this low frequency, which is under-represented in the simulation when compared to the experimental results. Using the Fourier transformation we can get the evolution of the modes along the flow. The modes (0,1) and (0,2) are the ones represented in the fig. 8b and the modes (1,1) and (1,2) have the same spanwise wave number but with the fundamental frequency ω_0.

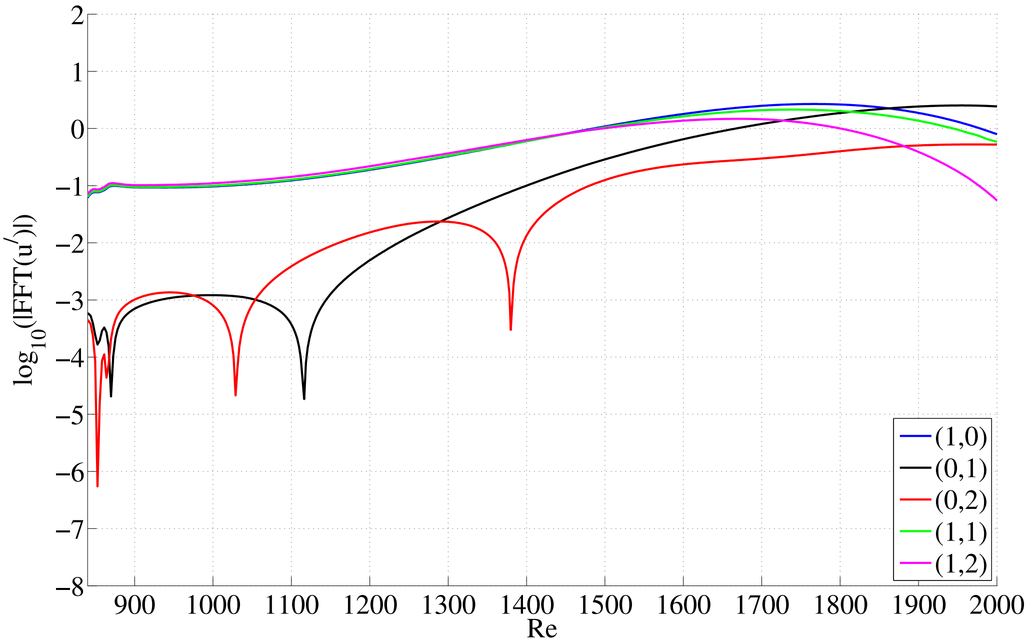
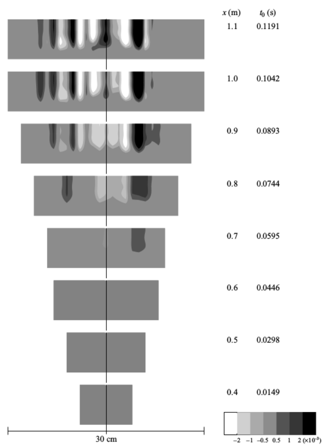
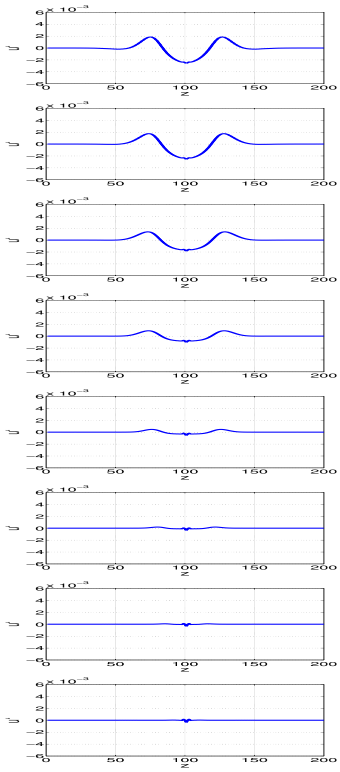


Figure 9: amplitude development for each Fourier modes

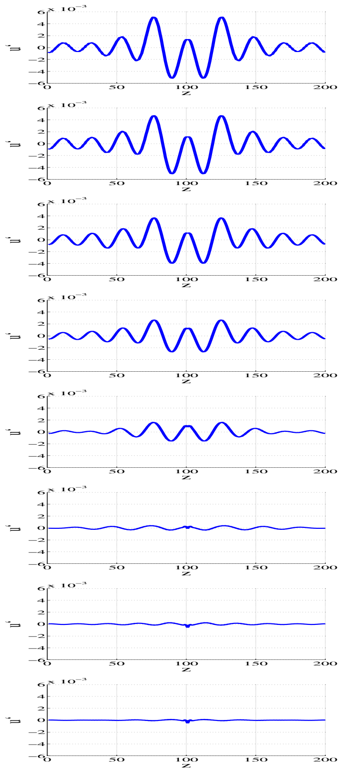
One hypothesis is that the simulation is not reproducing one phenomenon present in the experiment which we are still ignoring. One of this phenomena already noticed when comparing experimental and Direct numerical procedures is the under-representation of the low frequency modes. To try to verify this theory we can compare the mean disturbance velocity at $0.6\delta^*$ from the wall. The experimental result is shown in the fig. 7 from Medeiros (2004)[4] (reproduced in fig. 10a) The experimental flow disturbance is negative in the middle of the plate, which is not present in the simulation10b. The simulation have a positive velocity in the middle of the flat plate. Using the Fourier transformation, the mode (0,2) and its symmetric (0,-2) can be amplified and then, by doing the inverse Fourier transformation, return to the physical space and calculate the mean disturbance velocity. By amplifying these modes we get a profile more in accordance with the experimental results 10c.



(a) Experimental mean flow distortion from Medeiros (2004)[4]



(b) Direct numerical Simulation



(c) Direct Numerical Simulation with a multiplication of the mode (0,2) by 20

4. Conclusion

Simulations show modulation of the fundamental wave can transform the type of laminar turbulent transition. The fundamental type is now acting like an N-type. Comparison between the experimental and simulated wavetrain is quantitatively correct but the amplification of the low frequency oblique modes are underestimated. This phenomena has already been shown by Spalart (1987) [7]. In the current experiment, this underestimation of low frequency oblique mode could be in fact due to additional effects in the experiment which cause irregularity in the amplitude of the oblique waves. To counterbalance this effect, we can use a calibration when the amplification is still linear that will adjust the amplitude of the oblique modes to be closer to the initial experimental disturbance. Then the calibrated perturbation can be used to simulate the experiment and compare the nonlinear effect in the experiment and the simulation. A second hypothesis which will be investigated is the pressure distribution effect along the flat plate. The pressure distribution between the experimental result and the simulation are not exactly identical but it is not clear how much it could have a significant effect in the simulation. In future work others modulations will be simulated and compared with experimental data base, which is available. Also influence of effective experimental conditions will be determined.

5. ACKNOWLEDGEMENTS

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