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COBEM-2017-0253 NUMERICAL COMPUTATION OF AIRFOIL-GUST COMPRESSIBLE LIFT RESPONSE

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Abstract. *We propose a numerical framework to compute the airfoil-gust lift response and its subsequent leading-edge noise generation due to an incident compressible turbulent flow. This approach is valid for blades with large aspect ratios, general airfoil geometries, three-dimensional supercritical perturbations and compressible subsonic flows. The linearized equation for unsteady potential flow is rewritten as a Helmholtz equation in the transformed Prandtl-Glauert plane, leading to a boundary value problem prescribed by the linearized airfoil theory. The boundary element method is then employed to solve the Helmholtz equation subjected to a set of boundary conditions representative of the problem. Numerical implementation details are discussed including the implementation of the Kutta condition and the accurate capturing of singularities in the leading- and trailing-edge lift responses.*

Keywords: *Leading-edge noise, Amiet's theory, boundary element method*

1. INTRODUCTION

Viscous wake-airfoil interaction represents a noise source mechanism which has been the focus of several analytical, numerical and experimental investigations reported in the literature (Amiet (1975); Paterson and Amiet (1977); Atassi (1984); Roger and Moreau (2005); Mish and Devenport (2006a,b); Moreau and Roger (2009); Devenport *et al.* (2010)) . The development of low-cost, easy-to-run tools which allow quick noise assessments and optimization analyses underlying this phenomenon is desired during conceptual and preliminary design of rotating machines such as fans, wind turbines, contra-rotating open rotors and helicopter blades.

The determination of the unsteady response of the airfoil is required for noise prediction using Curle's analogy (see Curle (1955)). In order to account for complex incident flows and airfoil geometrical features, numerical methodologies were proposed for the flow solution (Gennaretti *et al.* (1997); Zhou and Joseph (2007); Gennaretti *et al.* (2013)). Glegg and Devenport (2010) successfully adopted a panel method to predict turbulence-airfoil interaction noise, obtaining good agreement with Amiet's theory for the low-frequency regime.Besides the good agreement, their technique is not suited to predict the noise at high frequencies, as compressibility begins to exert an important effect on the airfoil unsteady response.

For those cases in which either the flow is compressible and subsonic and the gust wave front is oblique to the airfoil leading-edge Amiet (1976) found a simple approach to compute the airfoil load distribution. Roger and Moreau (2005) revised Amiet's work and extended the pressure field prediction for the three-dimensional subcritical gust, also highlighting the importance of the back-scattering for small reduced frequencies, subcritical regimes and low Mach numbers. Recently, Santana *et al.* (2016) presented analytical iterative corrections for leading- and trailing-edge back-scattering effects at low frequency regimes.

In the current work, a numerical framework is presented for computing the aeroacoustic transfer function due to a gust-airfoil lift response. The present methodology allows the study of leading-edge noise due to an incident turbulent flow. The present approach is valid for blades with large aspect ratios, general airfoil geometries, three-dimensional supercritical perturbations and compressible subsonic flows and it is able to account for the reduction in magnitude of the aeroacoustic transfer function due to a thickness increment. Here, the boundary value problem prescribed by the linearized airfoil theory is rewritten as the canonical Helmholtz equation which is solved by the boundary element method (BEM).

2. THEORETICAL BACKGROUND

2.1 Acoustic radiation

Considering a coordinate system whose origin is placed at the mid-chord and mid-span of an airfoil with span $2d$ and chord $2b$ at $z_s = 0$, the power spectral density (PSD) of the acoustic pressure, valid for large aspect ratio airfoil subject to a gust of the form $w = w_0 \, \exp\{\, \mathrm{i}\,[k_1(U_0 t - x) - k_2 y]\}$, is represented as (see Amiet (1976))

$$S_{pp}(\mathbf{x},\omega) = \left(\frac{\rho_0\, k\, z\, b}{r_o^2}\right)^2 \pi\, U_0\, d\, \Phi_{ww}(k_1,k_2)\, |\mathcal{L}(\mathbf{x},k_1,k_2)|^2 \,, \tag{1}$$

where $\mathbf{x} = (x, y, z)$ is the geometrical observer position, k_1 and k_2 are the hydrodynamic wavenumbers in x and y directions respectively, $r_o^2 = x^2 + \beta^2(y^2 + z^2)$ is the square of the far-field observer position considering compressibility effects. The term $\mathcal{L}$ is aeroacoustic transfer function. The two-dimensional turbulence spectrum Φ_{ww} can be modeled, for example, by the von Kármán or Liepmann spectrum for isotropic turbulence when the airfoil thickness is considered negligible.

The acoustic pressure PSD is combined with the aeroacoustic transfer function $\mathcal{L}$ to predict the noise produced by the unsteady pressure jump along the airfoil surface. The pressure jump is represented by $\Delta p = p^{lower} - p^{upper}$ and the aeroacoustic transfer function is calculated as (see Amiet (1976))

$$\mathcal{L}(\mathbf{x},k_1,k_2) \;=\; \int_0^2 g(x_s',k_1,k_2)\, \mathrm{e}^{-\mathrm{i}kb(x_s'-1)(M_0 - x/r_o)/\beta^2} \mathrm{d}x_s' \,, \tag{2}$$

where $\mathbf{x}_s = (x_s, y_s, z_s)$ is the dipole source position on the surface and $g(x_s, k_1, k_2)$ is the transfer function between the incoming gust of amplitude w_0 and the consequent airfoil pressure jump, written as

$$g(x_s,k_1,k_2) = \frac{\Delta p(x_s,y_s,t)\, \mathrm{e}^{-\mathrm{i}\omega t}\, \mathrm{e}^{\mathrm{i}k_2 y}}{2\,\pi\,\rho_0\,U_0\,w_0} \,. \tag{3}$$

This transfer function can be analytically determined for thin airfoils, however, considerable differences are noticed in the value of the transfer function for finite thickness airfoil geometries. This paper proposes to apply the boundary element method (BEM) to numerically determine the compressible aeroacoustic transfer function $\mathcal{L}$ of an airfoil with thickness large enough to invalidate the thin airfoil assumption but within the framework of linearized theory.

2.2 Airfoil response computation

Assuming small perturbations, the potential flow model can be employed to compute the unsteady airfoil aerodynamic response according to

$$(1 - M_0^2)\phi_{xx} + \phi_{yy} + \phi_{zz} - (2M_0/c_0)\phi_{xt} - (1/c_0^2)\phi_{tt} = 0 \,. \tag{4}$$

In the equation above, M_0 and c_0 are the Mach number and speed of sound, respectively. Note that, from now on, the index notation will be used to denotate partial derivatives with respect to the spatial coordinates and time. The linearity of the problem allows the unsteady load induced by turbulence to be represented by oblique harmonic gusts. Therefore, supposing a potential ϕ written as

$$\phi = \phi' \,\exp\left[\mathrm{i}\left(\omega t + \frac{kM_0}{\beta^2}x - k_2 y\right)\right] \,, \tag{5}$$

where $\phi' = \phi'(x, z)$ is a complex amplitude (see Christophe (2011); Graham (1970)). Hence, the velocity of an oblique sinusoidal vertical-gust can be expressed as

$$\phi_z = w_0 \exp[\mathrm{i}(\omega t - k_1 x - k_2 y)] \,, \tag{6}$$

and the flow velocity becomes $\mathbf{U} = \{U_0,\, 0,\, \phi_z\}$. In the equation above, k_1 and k_2 are the hydrodynamic wavenumbers in chordwise and spanwise, respectively. The term k_1 is also defined as $k_1 \equiv \omega/U_0 = \omega/(M_0\, c_0) = k/M_0$.

By replacing Eq. 5 into Eq. 4 and using the non-dimensionalization

$$\bar{x} = \frac{x}{b}, \qquad \bar{y} = \frac{\beta y}{b}, \qquad \bar{z} = \frac{\beta z}{b} \,, \tag{7}$$

the equation can be represented by the following canonical Helmholtz equation in the transformed Prandtl-Glauert plane

$$\phi'_{\bar{x}\bar{x}} + \phi'_{\bar{z}\bar{z}} + A^2(1 - 1/\theta^2)\phi' = 0 \quad \text{, where} \quad A = \frac{bk_1M_0}{\beta^2} \quad \text{and} \quad \theta = \frac{k_1M_0}{k_2\beta} \,. \tag{8}$$

The parameter θ defines the nature of the problem represented by Eq. 8. If $\theta > 1$, the solution represents the propagation of acoustic waves and the gust is named supercritical. In case of $\theta < 1$, the gust is said to be subcritical and the waves are evanescent. A complete discussion of subcritical gusts is presented by Roger and Moreau (2005).

The aerodynamic response computation is performed considering a rectangular infinite span airfoil placed in a Cartesian reference system with $0 < x < 2b$ chord. It is important to highlight that now the origin of the coordinate system is shifted from that of the acoustic radiation computation. In this sense, the problem is described by the solution of the Helmholtz equation subjected to the following set of boundary conditions

$$\phi'_{\bar{x}\bar{x}} + \phi'_{\bar{z}\bar{z}} + K^2\phi' = 0\,, \quad \text{where} \quad K^2 = \frac{b^2}{\beta^2}\left(\frac{k^2}{\beta^2} - k_2^2\right) \tag{9}$$

$$\phi'(\bar{x}, 0) = 0 \quad , \quad \bar{x} \leq 0 \tag{10}$$

$$\phi'_{\bar{z}}(\bar{x}, 0) = -\frac{w_0 b}{\beta}\exp\left(-\frac{\mathrm{i}k_1 b\bar{x}}{\beta^2}\right) \quad , \quad 0 < \bar{x} < 2 \tag{11}$$

$$\phi'_{\bar{x}} + \frac{\mathrm{i}k_1 b}{\beta^2}\phi' = 0 \quad , \quad \bar{x} \geq 2\,. \tag{12}$$

The boundary conditions represent the Sommerfeld radiation condition, the non-penetration boundary condition and the Kutta condition, respectively (see Roger and Moreau (2005)).

In the present work, we employ the BEM to solve the Helmholtz equation (Eq. 9). The current method is able to account for arbitrary realistic airfoil geometries. The present numerical approach was validated by Miotto *et al.* (2017) by comparing the solutions obtained from the BEM to Amiet's solution for a flat plate.

3. NUMERICAL METHODOLOGY

In the present work, the boundary element method (BEM) is employed to iteratively solve the Helmholtz equation, Eq. 9, subjected to boundary conditions given by Eqs. 10, 11 and 12. For that, two out of the three boundary conditions of the problem are considered in an equivalent Schwarzschild approach (see Schwarzschild (1901); Miotto *et al.* (2017)). Its formulation allows the calculation of the unsteady velocity potential or acoustic pressure distribution along the surface of an airfoil with arbitrary geometry by

$$\varphi(\mathbf{x}) = \oint_S G\frac{\partial\varphi}{\partial\mathbf{n}} - \varphi\frac{\partial G}{\partial\mathbf{n}}\,\mathrm{d}S\,, \tag{13}$$

where the normal derivative points inward to the surface. In the equation above, the frequency domain Green's function is defined as

$$G(\mathbf{x}|\mathbf{x}_s) \equiv -\frac{\mathrm{i}}{4}H_0^{(2)}(K||\mathbf{x}-\mathbf{x}_s||) = -\frac{\mathrm{i}}{4}H_0^{(2)}(Kr)\,. \tag{14}$$

Here, $H_0^{(2)}$ is the Hankel function of zeroth-order and second type written as $H_0^{(2)} = J_0 - \mathrm{i}Y_0$, where J_0 and Y_0 are the Bessel and Neumann functions, respectively. The vector $\mathbf{x}$ stands for the observer position and $\mathbf{x}_s$ for the source position.

To solve the Helmholtz equation, Eq. 9, subjected to boundary conditions given by Eqs. 10, 11 and 12, the computational domain must extend from beyond the realistic airfoil geometry. In this sense, a finite flat plate is placed upstream and downstream of the airfoil leading- and trailing-edges, respectively, as depicted in Fig. 1.

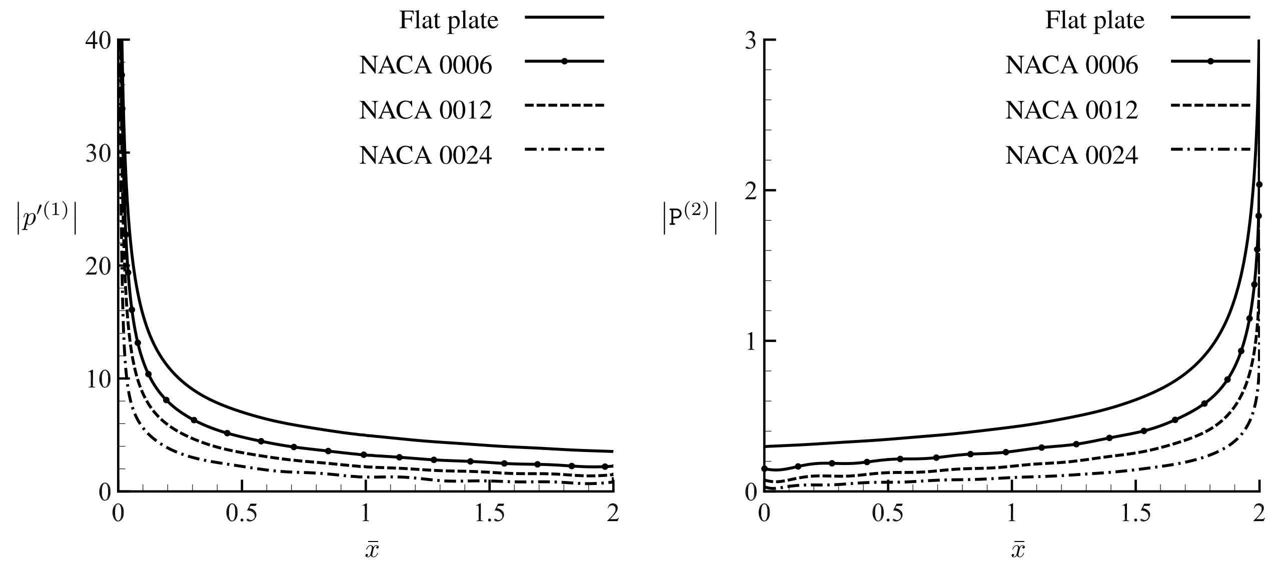
Figure 1. Computational domain over which the BEM is applied.

Both leading- and trailing-edge loading distributions present discontinuities in the lift response at $x = 0$ and $x = 2b$, respectively. Accurately capturing such discontinuities is shown to improve the numerical evaluation of the aeroacoustic transfer function (see Miotto *et al.* (2017)) and, therefore, mesh refinement is employed in the extremities of the airfoil surface.

4. PRELIMINARY RESULTS FOR REALISTIC AIRFOILS

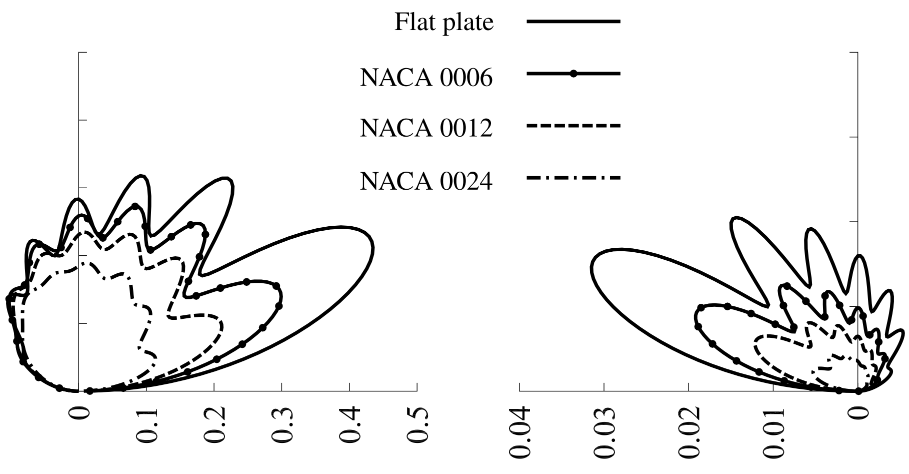
Figure 2 shows the reduction in the loading distribution due to the increment of the airfoil thickness, where both magnitudes of the first-order solution and trailing-edge correction are plotted. Additionally, Fig. 3 presents the respective directivities $\mathcal{L}_1$ and $\mathcal{L}_2$ in order to address a more complete assessment of the problem. We observe from Fig. 3 a significant reduction on the noise radiated towards the main lobe direction for both leading- and trailing-edge scattering.

Admitting a counterclockwise direction for the directivity angle, the amplitude is less affected by the thickness increment for directivity angles $\frac{\pi}{2}$ or higher for $\mathcal{L}_1$. One can even see that the typical cardioid directivity pattern expected for a non-compact source approaches that of a dipolar source for $\mathcal{L}_1$. A perfect cardioid shape will not be obtained due to the secondary diffraction from the finite airfoil chord.



(a) Magnitude of $p'^{(1)}$.
(b) Magnitude of $\mathrm{P}^{(2)}$.

Figure 2. Magnitude of the loading distribution for a flat plate and for several NACA airfoils; $k_1 b = 50$, $k_2 b = 0$ and $M_0 = 0.2$; $-5 \le \bar{x} \le 7$; $J = 2500$.



(a) $|2kbz\mathcal{L}_1/r_o|$
(b) $|2kbz\mathcal{L}_2/r_o|$

Figure 3. Directivity plots for several airfoils under $k_1 b = 50$, $k_2 b = 0$ and $M_0 = 0.2$ (He = 20).

In the work of Devenport *et al.* (2010), results from a numerical method, experimental data and Amiet's solution under low reduced frequencies and subsonic flow were compared against each other. The flat plate model assumed by Amiet was found to overpredict the sound pressure level in comparison to finite thickness airfoils as the reduced frequency is increased. Although their work focused on lower frequencies than those studied here, current results are consistent with their observations, but with a lower noise reduction. Furthermore, cambered airfoils are found to play a minimal effect on the airfoil acoustic scattering in the present work, a trend also observed by Devenport *et al.* (2010) . For these reasons, loading distributions and directivity plots are not presented. In Fig. 3, we observe a reduction of 2 dB between the plate and the NACA 0012 solutions for an observer positioned perpendicularly to the airfoil, and 6 dB at the primary lobe direction, for a normal gust at $M_0 = 0.2$. For high-frequencies, the present results are consistent with the studies from Gill *et al.* (2013).

5. CONCLUSIONS

The numerical approach used in the current paper was proposed and validated by Miotto *et al.* (2017) to compute the aeroacoustic transfer function of realistic airfoil geometries. The methodology is valid for large aspect-ratio non-compact airfoils, subsonic flows and three-dimensional supercritical perturbations. The numerical framework adopts the well-known BEM technique, which can be modified or adapted from already existing commercial or scientific codes. This methodology can be applied as a fast calculation tool for noise assessment and optimization in engineering applications where the leading-edge noise is an issue to be mitigated.

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