24th COBEM - 2017

Associação Brasileira de Engenharia e Ciências Mecânicas

24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-1153
MODELING OF ELECTRONIC DIFFERENTIAL SYSTEM FOR VEHICLES WITH REAR WHEEL DRIVE

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Abstract. *The yaw rate analysis is an important variable that affects the vehicle motion stability, therefore many efforts have been done to control it. The electric vehicles have some controllability advantages, due to the fact that they can use independent motors to drive the wheels, so it is not necessary to include a mechanical differential. Actually, they can use only the so-called electronic differential that can be controlled during the trip, thus many different strategies can be programmed in order to control the yaw rate. This paper presents a three degree of freedom model of a four wheel vehicle implemented in MATLAB/Simulink® to analyze the electronic differential behavior. This model uses the nonlinear Magic Formula, the load transfer during curves or drive/break situations, independent steering and wheel torque, considering an in-wheel drive vehicle. The results will present different behavior of the vehicle changing some important parameters and the differential effectiveness.*

Keywords: *Electronic Differential, Tire, Vehicle Model, Steering.*

1. INTRODUCTION

Electric and hybrid electric vehicles present some advantages compared to the combustion vehicles, such as better efficiency, less noise, less oil consumption and lower environment impact. Besides those characteristics the electric drive has more accurate torque and quicker response than the engine. Its torque can be easily measured by the current value and it can be built inside each wheel and controlled independently (Kiencke and Nielsen, 2000; Ando and Fujimoto, 2010; Corrêa *et al.*, 2015).

Considering ordinary vehicles, a single drive system is responsible to move all the traction wheels, thus it is necessary to use mechanical differential to distribute the torque. In electric vehicles, it is possible to use the multi-drive system, vehicles with two or more drivers. Moreover, today the electric drivers can be assembled inside the wheels reducing the mass above the suspension and enabling the independent control, the so called in-wheel drivers (Kiencke and Nielsen, 2000; Ando and Fujimoto, 2010; Corrêa *et al.*, 2015).

This independence is one of many active systems that can be used to improve handling, stability and comfort (Karbalaei *et al.*, 2007). The wheel torque control is used to produce yaw moment in vehicle improving stability and changing the vehicle trajectory. The method explored in this paper is called direct yaw-moment control (DYC) in which the yaw rate and moment are measured to control the vehicle (Karbalaei *et al.*, 2007).

An usual application of DYC showed in many papers (Niasar *et al.*, 2003; Nam *et al.*, 2012; Zhang *et al.*, 2014) is to control the vehicle in order to behave like a linear vehicle model which is more stable and friendlier to the driver (the user). In this paper the desired vehicle behavior is a standard nonlinear vehicle and the actual vehicle has different construction parameters like the CG position. The aim of this control is to afford to the user the same sense driving with different vehicles.

2. VEHICLE MODEL

The electronic differential must be capable of supply different speeds for each wheel depending on the vehicle condition. If the vehicle speed and the forces are very slow and the tires do not slip, as in some robots applications, the speed relation can be described, kinetically, by Ackermann geometry. This geometry reduces the speed of inner wheels and increases the speed of outer wheels during curves. The Ackermann geometry is shown in Fig. 1 and Eq. (1)

to (3) (Genta, 1997; Zhao *et al.*, 2009).

$$R_i = l/tan(\delta_1) \tag{1}$$

$$R_o = l/tan(\delta_2) \tag{2}$$

$$R_i + t_2/2 = R_o - t_2/2 = l/tan(\delta) \tag{3}$$

Where δ_1 and δ_2 are the steering angles of user input in front left and front right wheels, R_i and R_o are the inner and outer curvature radius, t_1 and t_2 are the front and rear axle track length, a and b are the distance between the front and rear axle and the gravity center and l is the wheelbase.

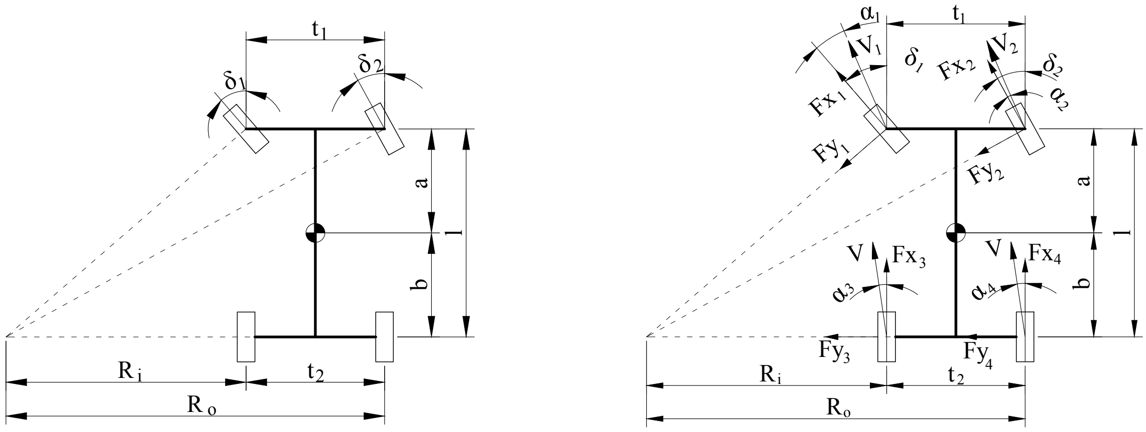

Figure 1. Ackerman geometry (left) and Dynamic four wheel model (rigth). Adapted from (Genta, 1997)

However, if the vehicle has high speed and forces, the dynamic equations must be considered and the Ackermann geometry is not accurate enough. A component that affects the vehicle response in high speed is the tire. According to (Bakker *et al.*, 1987) the tire must slip in order to produce a force. In his paper an equation called Magic Formula was proposed to describe the tire forces and moments as function of three parameters: the lateral slip, longitudinal slip and the normal force applied in the tire.

The lateral slip is measured as the angle between the tire speed and the tire direction (longitudinal tire mid-plane) as shown in Fig. (1). This measure is called slip angle (α). Also, the longitudinal slip is the difference between the actual longitudinal speed and the equivalent rotational tire speed. This value is normalized by the vehicle speed or equivalent rotational tire slip depending on the drive condition, accelerating, Eq. (4), or breaking, Eq. (5), resulting in a slip ratio (σ).

$$\sigma = \frac{r_w \omega_w - V_x}{V_x} \tag{4}$$

$$\sigma = \frac{r_w \omega_w - V_x}{r_w \omega_w} \tag{5}$$

Those two variables (α and σ) are the input (x) for the Magic Formula, Eq.(6), and the constants A, B, C, D, E, S_v and S_h are the tire parameter depending on normal forces. The result y can be either the longitudinal force (x direction), lateral force (y direction) or self-align moment (M_z) acting on the tire, but for each type of y different parameters have to be used.

$$y(x) = D.sin(.tan^{-1}B.(1-E)(x+S_h) + Etan^{-1}(B(x+S_h)))) + S_v \tag{6}$$

As mentioned before, the magic formula depends on the normal forces as shown in Fig. (2), thus these forces must be calculated before solving the tire forces. The normal force (F_z) is calculated by the simulation and consider the load transfer (ΔF_z) during curves or breaks and drivers. As the vehicle has more than three wheels, the normal force is statically undetermined. Thus, in order to solve this problem the model takes into account the flexibility (K) of suspension just in order to determine the normal forces as showed in Eq. (7) and Eq. (8).

$$\Delta F_{z_i} = \frac{K_{t_i}}{\sum_{\forall K} K_{t_k}} \frac{\sum_{\forall K} t_k \Delta F_{z_k}}{t_i} \tag{7}$$

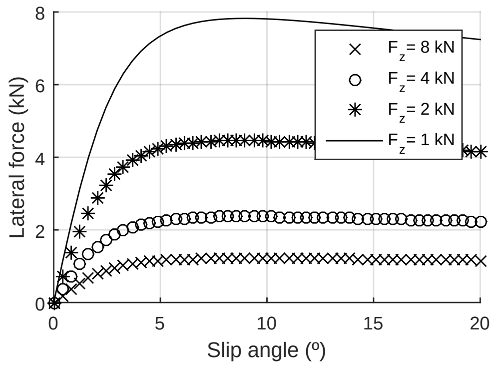

Figure 2. Lateral force in function of slip angle

$$F_{z_i} = F_{z_i}^{st} + \Delta F_{z_i} \tag{8}$$

Where ΔF_{z_i} is the load transfer, K_{t_i} is the stiffness for rolling displacement, k is the number of the wheel, i is the axle number and $F_{z_i}^{st}$ is the normal force in static condition.

The main part of the simulation is to integrate (using MATLAB/Simulink® integrator ODE45) the vehicle acceleration in order to determine the vehicle speed and position. With those speed values it is possible to calculate the wheel speed and considering the user steering angle the slip angle and slip ratio are calculated. After the load transfer is determined and with the user longitudinal force (throttle position) the forces acting in the tire (F_x, F_y and M_z) can be calculated resulting in a CG acceleration that is integrated. This procedure is shown in Fig. (3).

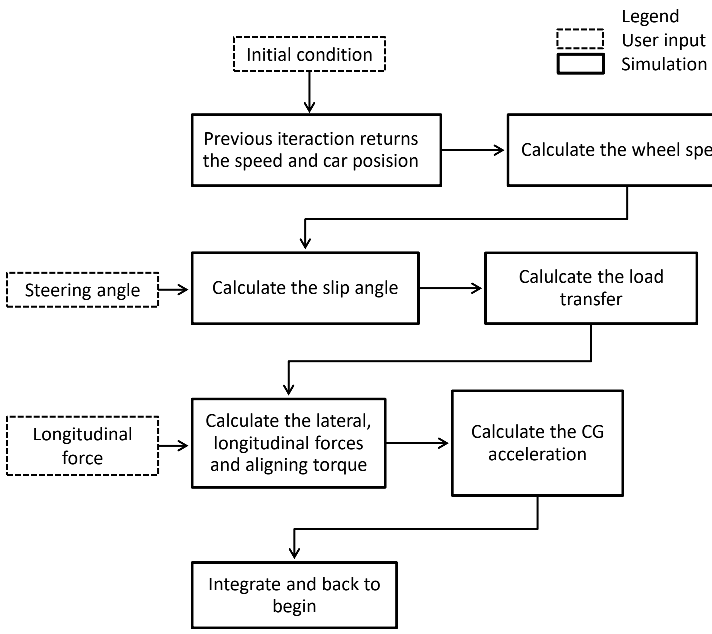

Figure 3. Simulation Procedure

2.1 CONTROL PROPOSAL

The control used in the present work is called Direct Yaw Control. This control is based on the feedback of the yaw dimension (angular in z direction vehicle state), which can be the yaw rate (angular speed) or moment (proportional to angular acceleration), measured by gyroscope and accelerometers, however the control method can be linear as the PID or nonlinear as fuzzy.

This control can be applied in all vehicle wheels as many papers have been doing nowadays (Zhang and Wang, 2016; Shuai *et al.*, 2014b), but ordinary vehicles already have the traction in front wheels, making the process to adapt the transmission design more difficult than in rear wheels. In this work, it is considered an electric motor in each rear wheel as presented by (Eckert *et al.*, 2014). As a result, the controller actuation is limited to the rear wheels.

In some papers like (Shuai *et al.*, 2014a) the design of four wheel and independent torque is considered efficient to control the vehicle. In this present work the limitation in control system (just two independent rear wheel torque) does not block the control propose. A simple simulation presented in the Fig. 4 suggested that the actuation proposed can affect the yaw rate and consequently the yaw moment, the controlled variable. In this simulation the vehicle starts with $10m/s$ and a $+281Nm$ ($3kN$) was applied in outer rear wheel and $-281Nm$ ($-3kN$) was applied in inner rear wheel. The front wheel steer is zero during all the simulation resulting in high slip angles.

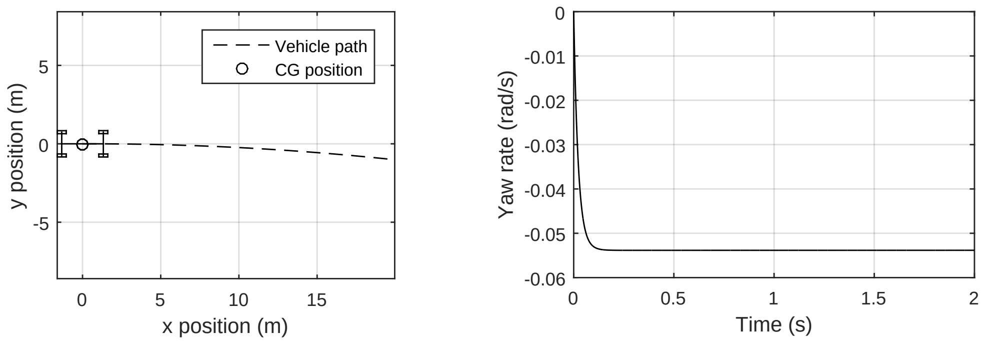

Figure 4. Car position and yaw rate in a simulation with $\delta = 0$, initial speed $= 10m/s$, left rear wheel torque $= 281Nm$ and right rear torque $= -281Nm$.

The structure of proposed controller is based on (Nam *et al.*, 2012; Niasar *et al.*, 2003; Zhang *et al.*, 2014) and showed in Fig.(5). The controller, PID (Zhang and Wang, 2016), has the yaw rate or yaw acceleration as a feedback supplied by the controlled vehicle and the desired value is supplied by the desired vehicle model which is calculated simultaneously. As commented above the controller has two outputs, right and left rear wheel torque, but PID returns just one value. The solution found by (Niasar *et al.*, 2003) is to treat the response as a differential torque (T_{dif}), acoording to Eq. 9 and Eq. 10. Where T_{user} is the desired user torque. In this paper T_{user} will be considered zero, because the T_{user} is applied only in the front wheels, reducing the wheel torque to the T_{dif} as shown in Eq. 11.

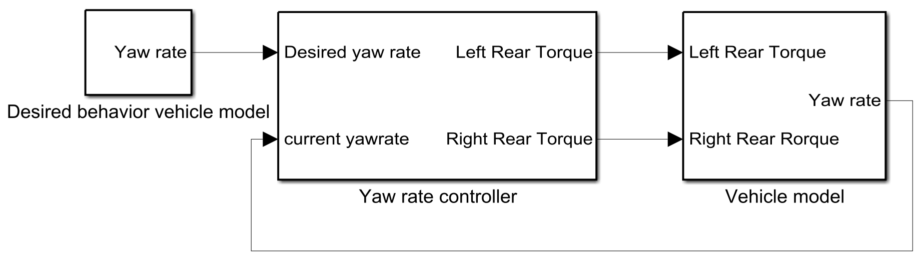

Figure 5. Control Diagram

$$T_l = \frac{T_{user} + T_{dif}}{2} \tag{9}$$

$$T_r = \frac{T_{user} - T_{dif}}{2} \tag{10}$$

$$T_l = -T_r = T_{dif} \tag{11}$$

The desired behavior is calculated at the same time supplying the desired value of the yaw rate. The yaw rate is the integral of the yaw acceleration in time, thus the controller based on yaw acceleration was developed too.

The chosen desired vehicle behavior is standard vehicle (unloaded) presented in Tab. 1 (Genta, 1997). But, in vehicle situation when there are more passengers, there are bags in the trunk, a vehicle has battery pack and motor in the rear axle or even the vehicle is distinct, the center of gravity position will be different than the standard vehicle, thus the behavior will be different too. The loaded vehicle model parameters in this paper will be the same as the standard vehicle, but with $b = 1.6\ m$, changing form 0.8 to 0.6 times wheelbase. The loaded vehicle parameters are shown in Tab. 1.

3. RESULTS

In a conventional vehicle without DYC, the load added into the car can change its dynamic behavior. Three other vehicles conditions were compared by changing b (0.8, 0.5 and 0.2 times l), in other words, a understeer, neutral steer

Table 1. Vehicle Parameters

Parameters	Standard Vehicle	Loaded Vehicle
Wheelbase (l)	$2.660\ m$	$2.660\ m$
Rear axle to gravity center (b)	$2.128\ m$	$1.600\ m$
Gravity center high (h)	$0.570\ m$	$0.570\ m$
Front axle length (t_1)	$1.490\ m$	$1.490\ m$
Rear axle length (t_2)	$1.482\ m$	$1.482\ m$
Wheel radio (r_w)	$0.287\ m$	$0.287\ m$
Caster angle	$5°$	$5°$
Inertia moment (I_g)	$1850\ kg\ m^2$	$185\ kg\ m^2$
Total mass (kg)	$1150\ kg$	$1530\ kg$

and oversteer gradient. In a neutral steer gradient if the driver is increasing the speed and wants to maintain the curvature radio he/she does not have to change the steering angle, but in a oversteer gradient the driver has to reduce the steer angle and in understeer gradient has to the steer angle to maintain the curvature radius. In these simulations the speed starts from 20 m/s (72 km/h) without wheel torque. The results are showed in Fig. 6.

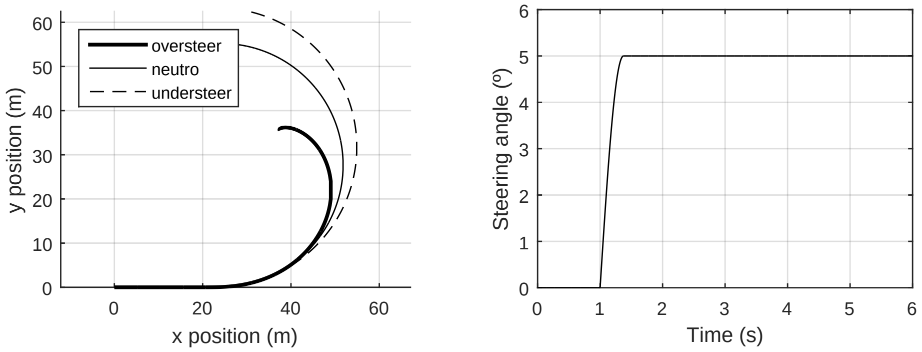

Figure 6. Path (rigth) by changing the center of gravity position in a specific maneuver (left)

As mentioned in section 2.1, the controllers based on the yaw rate and yaw acceleration are similar, since the first is the integral of the second. Thus these two types of controller are simulated. The initial longitudinal speed was 20 m/s (72 km/h), the steer angle was 5 ° and a torque in front axle was applied in order to maintain the constant speed. The PID configuration values were and the values were : $k_p = 0$, $k_i = 3\ 10^6$ and $k_d = 0$ for the yaw acceleration control and $k_p = 0.35\ 10^6$, $k_i = 2\ 10^6$ and $k_d = 0$ for yaw rate control. The results are presented in four variables: x and y position, the yaw rate, and the rear right wheel force.

By analyzing the controlled and uncontrolled vehicle it is possible to notice that these two types of controllers corrected the vehicle path and both results were almost the same. But, in order to quantify the distance between the current and desired path, the method mean squared error (MSE) is computed as shown in Eq. 12. Other measures are the curvature radii of center of gravity path and the max distance error, as shown in Tab. 2.

$$MSE = \frac{\sum_{i=1}^{n}(x_{d_i} - x_i)^2 + (y_{d_i} - y_i)^2}{n} \tag{12}$$

Table 2. Comparing standard vehicle to loaded vehicle - circular path

Vehicle type	Mean Squared Error	Curvature radii	Max. distance error
Standard	—	$68.34\ m$	—
Uncontrolled	$23.25\ m$	$66.16\ m$	$7.23\ m$
Yaw Moment Control	$0.60\ m$	$68.33\ m$	$1.23\ m$
Yaw Rate Control	$0.63\ m$	$68.34\ m$	$1.26\ m$

As expected, the MSE value for the controlled vehicle was less than the uncontrolled one. Also, curvature radii and the maximum distance error were lower than the uncontrolled one. Moreover, for the curvature radii and the max distance error, the yaw moment control performed better than the yaw rate control, but it does not mean that the controller is better.

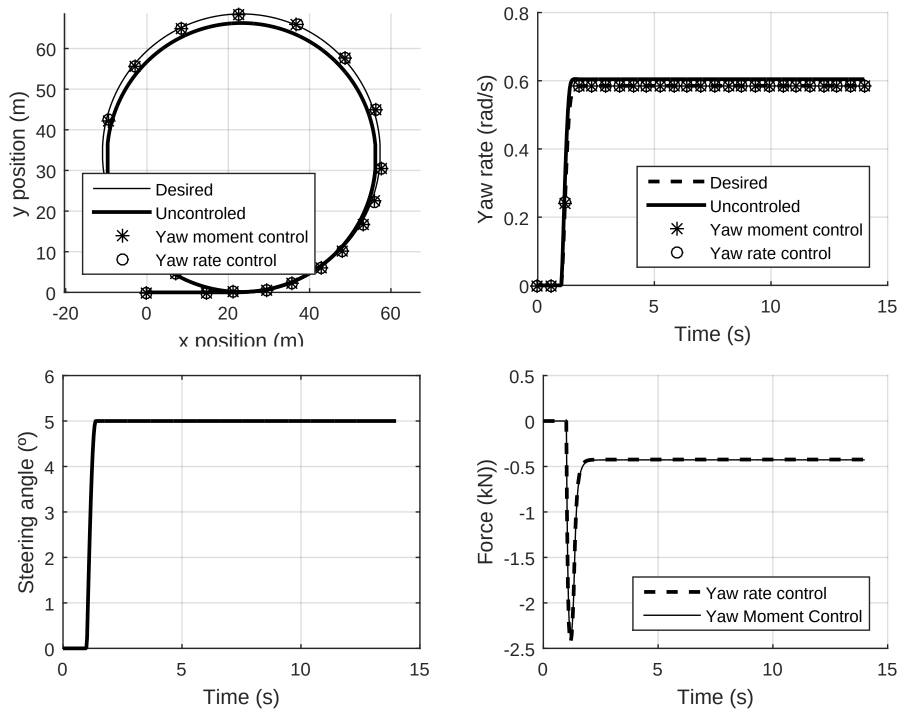

Figure 7. Control results. a) Vehicle path, b) Yaw rate value, c) User steering angle, d) Force in rear right wheel

Also, to visualize the vehicle behavior in nonlinear condition, another simulation was developed using the PID controller making it possible to understand whether the yaw moment control is more accurate than the yaw rate control. In this new simulation the conditions were maintained the same, but the maneuver was different. The max steering angle was higher than the first simulation and the maneuver steers the vehicle for both sides. The maneuver is called 0.7Hz d-well maneuver. The results are shown in Fig. 8 and in the Tab. 3.

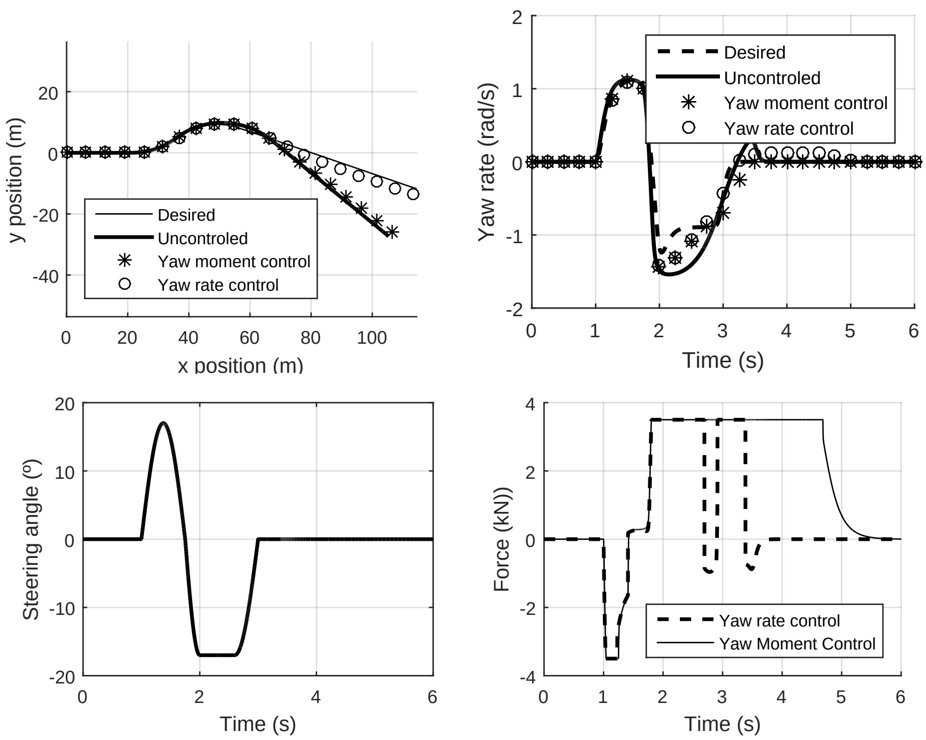

Figure 8. Control results. a) Vehicle path, b) Yaw rate value, c) User steering angle, d) Force in rear right wheel

Although both controllers behavior in a circular path were almost the same, in two side steer maneuver like the d-weel

Table 3. Comparing standard vehicle to loaded vehicle - D Weel Maneuver

Vehicle type	Mean Squared Error	Max. distance error
Standard	—	—
Uncontrolled	46.04 m	18.01 m
Yaw Momento Control	36.71 m	16.17 m
Yaw Rate Control	1.58 m	2.21 m

manuever the yaw rate control deals better than the yaw moment control. However it is not recommended to use this control, because the maximum distance error is higher ($2.21m$) and due to the high rear wheel torque.

4. CONCLUSIONS

The three degrees of freedom vehicle model was developed in MATLAB/Simulink® in order to propose a control stability method. This model considers the nonlinear tire behavior, load transfer during curves, braking and acceleration condition and constructive parameters of the vehicle.

The control aim is to make the loaded car, with different gravity center and mass, behavior as an unloaded vehicle does. Although the control actuation was limited by the rear wheels due to minimize the vehicle construction modification, this limitation does not affect the controller based on the yaw rate and yaw moment performance in a light simulated maneuver (circular path). However in aggressive maneuver both controllers showed problems resulting the maximum path error of 2.21 m for the yaw rate control and 16.17 m for the yaw moment control.

For first choice, the PID showed good results, but during aggressive maneuvers it could be better. As the vehicle behavior is nonlinear, a nonlinear control method should be studied. Other problems can be studied such as the dimension modification.

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6. ACKNOWLEDGMENTS

This work was conducted during scholarships supported by the Brazilian Federal Agency for Support and Evaluation of Graduate Education (CAPES), by the National Council for Scientific and Technological Development (CNPq), São Paulo Research Foundation (FAPESP) and by the University of Campinas (UNICAMP).

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