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SPECTRUM SENSITIVITY OF THE FLOW AROUND A SPRING-MOUNTED CYLINDER

Daiane Iglesia Dolci

Bruno Souza Carmo

Department of Mechanical Engineering, Poli, University of São Paulo

Av. Prof. Melo Morais, 2231, Cidade Universitária, São Paulo 05508-030, SP, Brazil

dolci@usp.br, bruno.carmo@usp.br

Abstract. In this paper, we calculate the sensitivity of the spectrum to structural modifications for a flow around spring-mounted cylinder, in order to identify what regions of the domain are more sensitive to structural changes. To obtain the mathematical expression for the sensitivity calculation an optimization problem is considered. The sensitivity is a function of the perturbation eigen-modes (direct modes) and its respective adjoints (adjoint modes). The numerical verification of the direct modes is achieved by comparing their respective eigenvalues with results of the literature. Next, the adjoint modes are verified by comparing its respective eigenvalues with the eigenvalues associated to the direct modes. The sensitivity analysis is applied in the flow past by an elastically mounted cylinder in the laminar flow regime. In the numerical simulations, the Spectral/hp element is used for spatial discretization and the Arnoldi is employed method to compute the eigenvalues.

Keywords: Sensitivity analysis, Adjoint, Fluid-Structure Interaction

1. INTRODUCTION

Sensitivity analysis is concerned with the effect of the variation of parameters of a system over the state variables, and can characterize the system behaviour by evaluating the gradient of the state variable with respect to the modified parameter. The sensitivity functions are derived based in an optimization problem, which involves: objective functional, state variables, control variables and adjoint variables. In spectrum sensitivity, the objective functional is the eigenvalue associated to the perturbation eingen-modes (direct modes). The control variable is given by structural modifications and the state variable is the direct mode.

Sensitivity of the spectrum to structural modifications has been studied for the flow around stationary cylinder and it was first addressed by Giannetti and Luchini (2007). The problem chosen was the flow around of a circular cylinder with the Reynolds number next to first instability ($Re_c \cong 47$); the regions of the field that are more sensitive to localized structural changes were defined. This analysis basically consists in evaluating the product between the direct and adjoint modes.

The direct modes are obtained from the linearized Navier-Stokes equations and there are many examples of this type of calculation in the literature for flow around stationary structures (Giannetti and Luchini (2006), Giannetti and Luchini (2007), Marquet and Jacquin (2008), Meliga and Chomaz (2011), Meneghello and Huerre (2015)). Although there are important results of stability for fluid flow, in fluid-structure interaction (FSI) problems the studies are recent and challenging. In the last years, relatively few works have presented results over modal linear stability analysis. Cossu and Morino (2000) were the pioneers to study the first stability for a flow around a cylinder mounted on elastic supports. The authors analyzed the leading eigenvalues for low mass ratio m^* and suggested a critical Reynolds number Re_c lower than

that of the stationary cylinder case. Using Direct non-linear simulations, Mittal and Singh (2005) showed that for certain natural frequencies of the spring-mass system, vortex shedding and self-excited vibrations of the cylinder are possible for $Re_c < 47$. They also identified $Re_c < 47$ for low mass ratio from asymptotic stability analysis. Navrose and Mittal. (2016) investigated the lock-in phenomenon by evaluating the influence of the mass ratio (m^*), they defined the critical mass ratio m_c^* for which the modes are coupled.

While there has ben advances in modal analysis, for non-modal analysis, specifically sensitivity analysis, results have not yet been presented. Therefore, the application of such methodologies is very promising to assist in active control mechanisms to intensify or to suppress the dynamic response of flexible structures immersed in a flow.

2. GOVERNING EQUATIONS

2.1 FLUID-STRUCUTRE EQUATIONS

Consider a viscous two-dimensional flow around a spring-mounted cylinder with boundary conditions correctly satisfied. The Navier-Stokes equations governing an incompressible flow are:

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \mathbf{u} \cdot \mathbf{u} - \frac{1}{Re}\nabla^2 \mathbf{u} + \nabla p = \mathbf{0}, \tag{1}$$

$$\nabla \cdot \mathbf{u} = 0. \tag{2}$$

The vector $\mathbf{u}(\mathbf{x}, t)$ represents the velocities, $p(\mathbf{x}, t)$ is the pressure, $Re = UD\rho/\mu$ is the Reynolds Number, U is the free stream velocity, D is the cylinder diameter, μ fluid dynamical viscosity and ρ is fluid density.

The motion of the cylinder mounted on elastic supports free to move only in the transverse direction y is governed by the following nondimensional equation:

$$\frac{\pi m^*}{4}\ddot{\upsilon} + \frac{\pi^2 \gamma m^*}{V_r}\dot{\upsilon} + \frac{\pi^3 m^*}{V_r^2}\upsilon = F_y(\mathbf{u}, p), \tag{3}$$

where the variables $\upsilon = \upsilon(t)$, $\dot{\upsilon} = \dot{\upsilon}(t)$ and $\ddot{\upsilon} = \ddot{\upsilon}(t)$ represent respectively the vectors of displacement, velocity and acceleration of the structure. The coefficient m^* is the nondimensional mass of the body, γ is the structural damping, $V_r = U/(f_n D)$ is the reduced velocity, f_n is the natural frequency of the structure in vacuum, and F_y is an instantaneous nondimensional fluid force acting on the cylinder described by:

$$F_y(\mathbf{u}, p) = \int_{\partial \Omega_w} \mathbf{n} \cdot \left\{ -p\mathbf{I} + Re^{-1}(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \right\} dS_w. \tag{4}$$

2.2 LINEAR STABILITY

We are interested in the sensitivity for the eigenvalue associated to less stable perturbation mode. Thus, to obtain the eigenvalues and their perspectives eigen-modes, consider that the state variables $\mathbf{q} = [\mathbf{u}, p, y, \dot{\upsilon}]^T$ of a FSI problem are given as the sum of a base flow $\mathbf{Q} = [\mathbf{U}, P]^T$ with a perturbation $\mathbf{q}' = [\mathbf{u}', p', y', \dot{\upsilon}']^T$, i. e., it is assumed that the base flow is steady and the displacement of the structure is given by the perturbation. Therefore, taking $\mathbf{q} = \mathbf{Q} + \mathbf{q}'$ and substituting in (1-3) we have:

- Base Flow equations:

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \mathbf{U} \cdot \mathbf{U} - \frac{1}{Re}\nabla^2 \mathbf{U} + \nabla P = \mathbf{0}, \tag{5}$$

$$\nabla \cdot \mathbf{U} = 0, \tag{6}$$

- Perturbations equations:

$$\frac{\partial \mathbf{u}'}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{u}' + \nabla \mathbf{U} \cdot \mathbf{u}' - \frac{1}{Re}\nabla^2 \mathbf{u}' + \nabla p' + \underbrace{\nabla \mathbf{u}' \cdot \mathbf{u}'}_{\text{Non-linear term}} \quad = \quad 0. \tag{7}$$

$$\nabla \cdot \mathbf{u}' \quad = \quad 0, \tag{8}$$

$$\dot{\upsilon}' \quad = \quad \boldsymbol{\upsilon}_1', \tag{9}$$

$$\frac{\pi m^*}{4}\dot{\upsilon}_1' + \frac{\pi^2 \gamma m^*}{V_r}\upsilon_1' + \frac{\pi^3 m^*}{V_r^2}\upsilon' \quad = \quad F_y(\mathbf{u}', p'). \tag{10}$$

Assuming that $\nabla \mathbf{u}' \cdot \mathbf{u}'$ is of a smaller order than that of the perturbation in the short time scale, the nonlinear terms can be neglected. Besides that, assuming that the solution of the direct modes is $\mathbf{q}' = \widehat{\mathbf{q}}exp(\lambda t)$, since the problem is linear, we have the linearized system:

$$(\lambda \mathbb{B} + \mathbb{L})\widehat{\mathbf{q}} = \begin{cases} \lambda \widehat{\mathbf{u}} + \mathbf{U} \cdot \nabla \widehat{\mathbf{u}} + \nabla \mathbf{U} \cdot \widehat{\mathbf{u}} - \frac{1}{Re}\nabla^2 \widehat{\mathbf{u}} + \nabla \widehat{p} & = & \mathbf{0}, \\ \nabla \cdot \widehat{\mathbf{u}} & = & 0 \\ \boldsymbol{\lambda}\widehat{y} - \widehat{y}_1 & = & 0 \\ \boldsymbol{\lambda}\frac{\pi m^*}{4}\dot{\widehat{y}}_1 + \frac{\pi^2 \gamma m^*}{V_r}\widehat{y}_1 + \frac{\pi^3 m^*}{V_r}\widehat{y} - F_n(\widehat{\mathbf{u}}, \widehat{p}) & = & 0, \end{cases} \tag{11}$$

in which λ is the complex eigenvalue and $\widehat{\mathbf{u}}$ is the associated eigen-mode.

3. SPECTRUM SENSITIVITY TO STRUCTURAL MODIFICATIONS

Structural sensitivity was first addressed by Giannetti and Luchini (2007). This sensitivity analysis allows to investigate how a given eigenvalue λ (relative to the eingen-mode $\widehat{\mathbf{u}}$) changes as the spatial linearized operator $\mathbb{L}$ changes.

To obtain an analytical expression for the structural sensitivity, let us consider the generalized eigenvalue problem $(\boldsymbol{\lambda}\mathbb{B} - \mathbb{L}) \cdot \widehat{\mathbf{q}} = \mathbf{0}$, and a variation $\delta\mathbb{L}$ of the operator $\mathbb{L} = \mathbb{L}(\mathbf{U})$. For this problem, we want to find a structural modification $\delta\mathbb{L}$ which produces the greatest eigenvalue drift $\delta\lambda$. Thus, we start defining the Lagrangian functional:

$$\mathcal{L}(\lambda, \widehat{\mathbf{q}}, \mathbb{L}, \widehat{\mathbf{q}}^\dagger) = \lambda - \langle \widehat{\mathbf{q}}^\dagger, (\boldsymbol{\lambda}\mathbb{B} + \mathbb{L}) \cdot \widehat{\mathbf{q}}\rangle, \tag{12}$$

where λ is the objective function, $(\boldsymbol{\lambda}\mathbb{B} - \mathbb{L}) \cdot \widehat{\mathbf{q}} = \mathbf{0}$ is the constraint that must be satisfied, $\mathbb{L}$ is the control variable and $\widehat{\mathbf{q}}$ is the state variable. The inner product of two complex vector fields in a domain Ω is defined as $\langle \mathbf{a}, \mathbf{b}\rangle = \int_\Omega \mathbf{a} \cdot \mathbf{b}d\Omega$.

At an optimum point, the functional $\mathcal{L}(\lambda, \widehat{\mathbf{q}}, \mathbb{L}, \widehat{\mathbf{q}}^\dagger)$ becomes stationary, and its gradient with respect to any variable s is defined by:

$$\frac{\partial \mathcal{L}}{\partial s} = \lim_{\epsilon \to 0} \frac{\mathcal{L}(s + \epsilon \delta s) - \mathcal{L}(s)}{\epsilon}. \tag{13}$$

Taking the first variation with respect to the adjoint/Lagrange multiplier $\widehat{\mathbf{q}}^\dagger$, we have:

$$\frac{\partial \mathcal{L}}{\partial \delta\widehat{\mathbf{q}}^\dagger}\delta\widehat{\mathbf{q}}^\dagger \quad = \quad \frac{\int_\tau \int_\Omega (\widehat{\mathbf{q}}^\dagger + \epsilon\delta\widehat{\mathbf{q}}^\dagger)(\boldsymbol{\lambda}\mathbb{B} + \mathbb{L}) \cdot \widehat{\mathbf{q}}) - \widehat{\mathbf{q}}^\dagger(\boldsymbol{\lambda}\mathbb{B} + \mathbb{L}) \cdot \widehat{\mathbf{q}})d\Omega d\tau}{\epsilon} =$$

$$= \quad \int_\tau \int_\Omega \delta\widehat{\mathbf{q}}^\dagger(\boldsymbol{\lambda}\mathbb{B} + \mathbb{L}) \cdot \widehat{\mathbf{q}})d\Omega d\tau = 0.$$

Requiring that the constraint $(\boldsymbol{\lambda}\mathbb{B} + \mathbb{L}) \cdot \widehat{\mathbf{q}} = \mathbf{0}$ should be satisfied for all domain Ω and time τ.

The first variation with respect to the direct field $\mathbf{q}'$ is:

$$\frac{\partial \mathcal{L}}{\partial \delta \mathbf{q}} \delta \mathbf{q} \;\;=\;\; \underbrace{\frac{\partial \mathcal{J}}{\partial \mathbf{q}} \delta \mathbf{q}}_{\mathbf{I}} - \underbrace{\langle \widehat{\mathbf{q}}^\dagger, (\boldsymbol{\lambda} \mathbb{B} + \mathbb{L}) \cdot \widehat{\mathbf{q}} \rangle}_{\mathbf{II}} . \tag{14}$$

The direct mode $\widehat{\mathbf{q}}$ does not appear explicitly in the inner product, then it is not possible to compute the gradients with respect to $\widehat{\mathbf{q}}$ directly. However, applying integration by parts in $\mathbf{II}$, the direct modes appear explicitly and the gradients can be computed by admitting that the constraints given by the adjoint equations are satisfied. Therefore, applying the inner product definition for continuous operators and using integration by parts, we obtain the adjoint equations for FSI problems:

$$\lambda \mathbf{u}^\dagger + \mathbf{U} \cdot \nabla \mathbf{u}^\dagger - \nabla \mathbf{U} \cdot \mathbf{u}^\dagger - Re^{-1} \nabla^2 \mathbf{u}^\dagger + \nabla p^\dagger \;\; = \;\; 0 \tag{15}$$

$$\nabla \cdot \mathbf{u}^\dagger \;\; = \;\; 0 \tag{16}$$

$$\lambda v^\dagger - v_1^\dagger \;\; = \;\; 0 \tag{17}$$

$$\lambda \frac{\pi m^*}{4} \dot{v}_1^\dagger - \frac{\pi^2 \gamma m^*}{V_r} v_1^\dagger + \frac{\pi^3 m^*}{V_r} v^\dagger - F_y(\mathbf{u}^\dagger, -p^\dagger) \;\; = \;\; 0. \tag{18}$$

Lastly, let us to compute the variation of $\mathcal{L}$ with respect to the structural modification $\delta \mathbb{L}$. For this variation to be done properly, we must take into account that the eigenvalue λ is itself a function of the operator $\mathbb{L}$. Therefore, the greatest eigenvalue drift $\delta \lambda$ associated with the spatial linearized operator $\delta \mathbb{L}$ is obtained by setting:

$$\frac{\partial \mathcal{L}}{\partial \mathbb{L}} \delta \mathbb{L} = \frac{\partial \lambda}{\partial \mathbb{L}} \delta \mathbb{L} - \langle \widehat{\mathbf{q}}^\dagger, \delta \mathbb{L} \widehat{\mathbf{q}} \rangle = 0 \quad \Rightarrow \quad \delta \lambda = \frac{\partial \lambda}{\partial \mathbb{L}} \delta \mathbb{L} = \langle \widehat{\mathbf{q}}^\dagger, \delta \mathbb{L} \widehat{\mathbf{q}} \rangle. \tag{19}$$

Computing the variation with respect to λ, we have:

$$\frac{\partial \mathcal{L}}{\partial \lambda} \delta \lambda = \delta \lambda - \langle \widehat{\mathbf{q}}^\dagger, \mathbb{B} \widehat{\mathbf{q}} \rangle \delta \lambda. \tag{20}$$

Taking the normalization $\langle \widehat{\mathbf{q}}^\dagger, \mathbb{B} \widehat{\mathbf{q}} \rangle = 1$, then $\frac{\partial \mathcal{L}}{\partial \lambda} \delta \lambda = 0$ is satisfied.

WAVEMAKER

Giannetti and Luchini (2007) identified the wavemaker region as the region in the space susceptible to structure modifications that produces the strongest drift of the eigenvalue. Here, we want to adapt this concept applied for FSI problems. Therefore, to determine the wavemaker region in the flow, we assume $\delta \mathbb{L} = C(\mathbf{x})$. Besides that, we suppose that $C(\mathbf{x}) = \mathbf{f}$ is a force localized in the space, performing $C(\mathbf{x}) = \delta(\mathbf{x} - \mathbf{x}_0) \mathbf{C}_0$, where $\mathbf{C}_0$ is a coefficient, $\mathbf{x}_0$ is the cartesian position where the force acts, and $\delta(\mathbf{x} - \mathbf{x}_0)$ is a Kronecker delta function.

Suppose that $C(\mathbf{x})$ is small compared to $\mathbb{L}$, so we can rewrite the shift of the eigenvalue $\delta \lambda$ as:

$$\delta \lambda = \frac{\partial \lambda}{\partial \mathbb{L}} \delta \mathbb{L} = \langle \widehat{\mathbf{q}}^\dagger, \delta \mathbb{L} \widehat{\mathbf{q}} \rangle = \langle \widehat{\mathbf{q}}^\dagger, C(\mathbf{x}) \widehat{\mathbf{q}} \rangle, \tag{21}$$

where

$$|\delta \lambda| = |\langle \widehat{\mathbf{q}}^\dagger, C(\mathbf{x}) \widehat{\mathbf{q}} \rangle| \leqslant ||\widehat{\mathbf{q}}^\dagger|| ||\widehat{\mathbf{q}}|| ||\mathbf{C}_0||. \tag{22}$$

Therefore, the eigenvalue drift due to the localised feedback mechanism can be obtained by the product between the direct and adjoint velocities.

4. NUMERICAL METHODOLOGY

The geometry of interest is a two-dimensional circular cylinder of diameter $D = 1$. This solid body is immersed in an uniform flow of magnitude U_∞ parallel to the x-axis, pointing to the $x+$ direction. The origin of the coordinate system is at the center of the cylinder (See Figure 1) with dimensions: $x+ = 45$ to downstream, $x- = -25$ to upstream and vertical $y\pm = 25$. In the numerical simulations, a rectangular computational domain is used.

The numerical results were obtained using in the Nektar++ software, which is a open-source code based on the spectral/hp element method (Karniadakis and Sherwin (2005)) and implemented in C++. Seventh-degree polynomials are employed as basis functions in the two-dimensional simulations. A second order stiffly stable time-stepping scheme (Karniadakis *et al.* (1991)) is employed to advance the solution in time. The eigenvalues are obtained by solving a generalized eigenvalue problem by the Arnoldi method (Saad (1992)).

Following Li et al. (2002), for numerical simulations of a flow around in a two-dimensional spring-mounted cylinder the non-inertial frame of reference is used. In this method, the coordinates system is fixed to the structure.
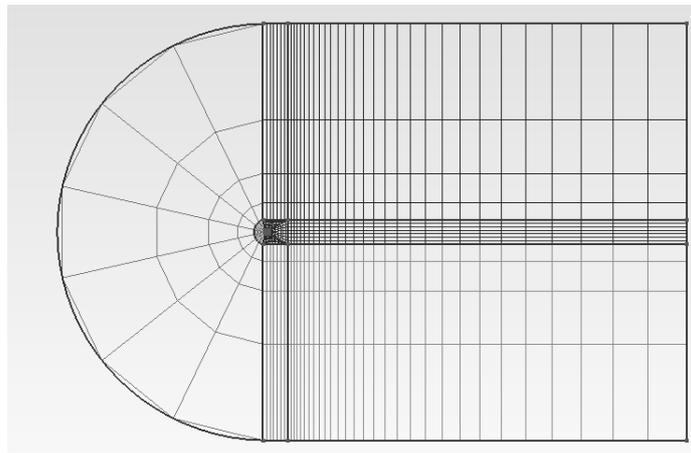

Figure 1. Domain used in the simulations.

5. RESULTS

5.1 STABILITY ANALYSIS

Flow past a stationary cylinder becomes unstable at $Re_c \cong 47$ (Jackson (1987) Giannetti and Luchini (2007), Marquet and Jacquin (2008)); at this point the flow undergoes a supercritical Hopf bifurcation (the first instability) that leads to a two-dimensional time-periodic flow. However for flow around spring-mounted cylinder, Mittal and Singh (2005) showed that for certain natural frequencies of the spring-mass system, vortex shedding and self-excited vibrations of the cylinder are possible for $Re_c < 47$. This result is confirmed from linear stability analysis (Cossu and Morino (2000), Meliga and Chomaz (2011), Zhang and Jiang (2015), Navrose and Mittal. (2016)). The papers about stability analysis for a flow around a spring-mounted cylinder show that the mass of the oscillator plays a major role in the lock-in.

To verify our stability analysis algorithm, a comparison with Zhang and Jiang (2015) at a subcritical $Re = 33$ is performed. The cylinder is free to vibrate in the transverse direction. Figure 2 shows the instability boundaries of the coupled system for different mass ratios (m^*). Notice that the results are in good agreement with Zhang and Jiang (2015), showing that the proposed linear model is capable of capturing the instabilities.
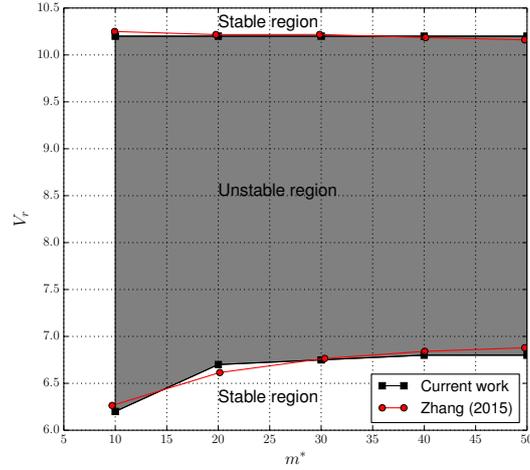
Figure 2. Instability boundaries for $Re = 33$ compared with Zhang et al. (2015).

5.2 SENSITIVITY ANALYSIS

The spectrum sensitivity to structural changes is computed for $Re = 33$, reduced velocity $V_r = 8$ and mass ratio $m* = 500$. To identify the wavemaker region, the direct and adjoint modules were computed (See Figure 3). The eigenvalues associated to this direct and adjoint modes are exposed in the Table below.

Table 1. Least stable eigenvalue associated to Direct and Adjoint modes.

Direct	Adjoint
$8.0842e - 04 \pm 7.8412e - 01i$	$7.8889e - 04 \pm 7.8731e - 01i$

(a)

(b)
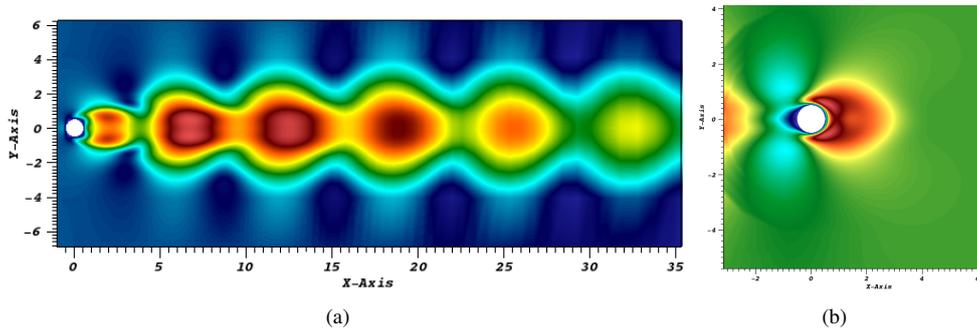
Figure 3. Spatial distribution of the perturbation velocity field modulus $||\hat{\mathbf{u}}||$ (a) and its adjoint $||\mathbf{u}^*||$ (b).

The sensitivity calculation is carried out, Figures 4 and 5 shows the spatial distribution of the spectrum sensitivity to structural changes for a flow around spring-mounted cylinder and around a stationary cylinder respectively. Note that near the cylinder the sensitivity fields are similar. However, to downstream the fields show differences: Figure 4 shows that when the cylinder is free to move, the wavemaker region is strong near to the cylinder but is also significant to downstream of the body, and the sensitivity calculation of the flow around the fixed structure shows that the wavemaker region occurs only near to the cylinder.
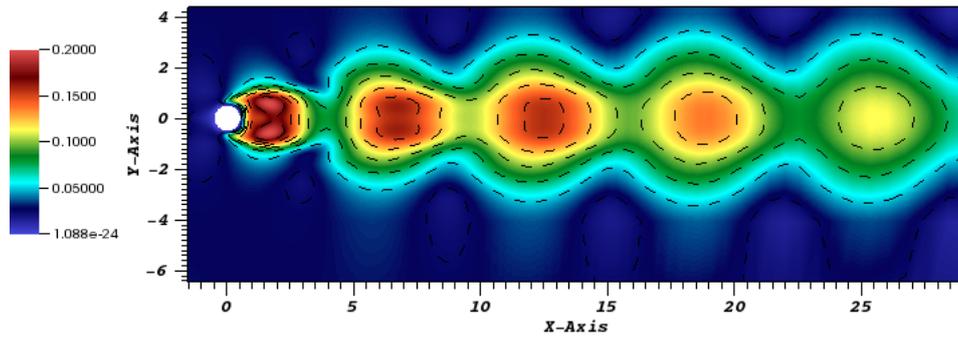
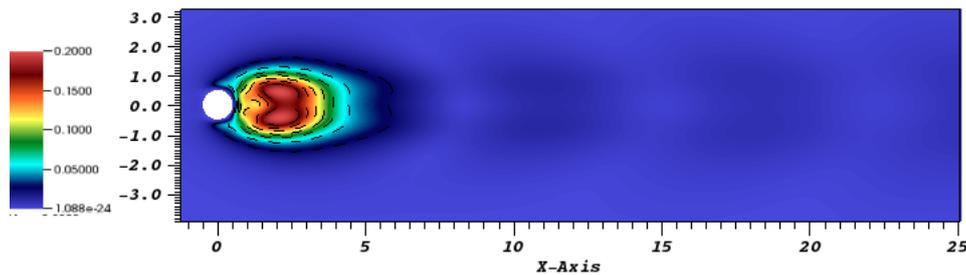Figure 4. Sensitivity to spatially localized feedbacks at $Re = 33$, $m^* = 500$ and $V_r = 8$

Figure 5. Sensitivity to spatially localized feedbacks at $Re = 33$. (Stationary cylinder)

6. CONCLUSION

This work introduced the linearized and adjoint systems for FSI problems. Besides that, a mathematical expression to compute the spectrum sensitivity to structural modifications was presented. Next, the direct modes were verified numerically by comparing their respective eigenvalues with literature results. The adjoint modes were numerically verified by comparing their eigenvalues with the eigenvalues associated to the direct modes, showing a good concordance. Lastly, wavemaker regions of the flow around a spring-mounted cylinder were obtained from the sensitivity calculations. These regions were compared with the case in which the flow is around of a fixed cylinder. We observe that in both cases (free to move and fixed cylinder) the region more sensitive to structural changes is near to the cylinder, but on the case in which the structure is free to move, the far wake region also presents a significant sensitivity. So, in the next steps, we can apply the spectrum sensitivity for other Reynolds numbers, evaluating the influence of the mass ratio and reduced velocity parameters in the wavemaker regions.

7. ACKNOWLEDGEMENTS

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