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COBEM-2017-1370 MODEL PREDICTIVE CONTROL FOR PATH TRACKING OF A UAV SUBJECT TO AERODYNAMIC DISTURBANCES

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Abstract. *This article presents a strategy to solve the path tracking problem of an unmanned aerial vehicle quadrotor type. The quadrotor is an underactuated mechanical system, nonlinear, has a time-varying behavior and it is constantly affected by aerodynamic disturbances on outdoor flights. Thus, advanced control strategies are required to achieve good performance in autonomous flight. The dynamic model of the vehicle was obtained by the Lagrange-Euler formalism. The proposed control structure is a cascade scheme consisting of a model-based predictive controller to track the reference trajectory and other to stabilize the rotational movements of the aerial vehicle. In order to verify the effectiveness of the proposed control strategy, simulations were performed under the interference of aerodynamic disturbances (e.g. wind gusts), parametric uncertainties and physical constraints of the quadrotor.*

Keywords: *Quadrotor, Model Predictive Control, Path Tracking, Aerodynamic Disturbances*

1. INTRODUCTION

The progress performed in areas of sensors, energy storage and data processing allowed the development of unmanned aerial vehicles (UAVs). These vehicles can be used in civil and military applications, in tasks such as search and rescue, inspections over large areas, security, intervention in hostile environment, as well as in the film and entertainment industry (Raffo *et al*., 2010).

The quadrotor belongs to a class of unmanned aerial vehicles with rotating wings. The system is similar to a helicopter, however, with four sets of rotors/propellers as shown in Fig. 1.

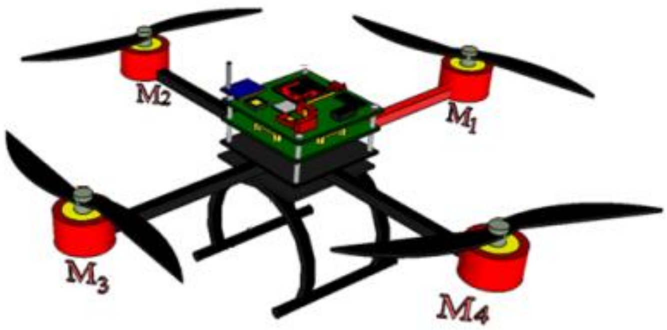

Figure 1. Schematic diagram of a quadrotor (Carrillo *et al*., 2013).

This configuration holds several advantages when compared to the conventional helicopter. As the quadrotor is lifted and propelled by four rotors, it is possible to reduce the individual rotor size and maintain or increase the total vehicle load capacity. In addition, this system does not require angular control on the propellers. Factors such as these allow for a significant reduction in project costs and vehicle maintenance (Hoffmann *et al*., 2007).

Unfortunately, the development of control systems for such vehicles is not a trivial task. The quadrotors have a nonlinear and time-varying behavior and they are constantly affected by aerodynamic disturbances in outdoor flights (Raffo *et al*., 2010).

Many efforts have been made to control the quadrotor and several strategies have been tested around the world. For example, in Bouabdallah *et al*. (2004) two model based control techniques were compared for stabilization of this kind of vehicle: a classical PID approach, which assumed simplified dynamics, and a LQR technique based on a more complete model. In Bouabdallah and Siegwart (2005) the model was split up into two subsystems: the angular rotations and the linear translations. Backstepping and sliding-mode techniques were used to control the vehicle. Lara *et al.* (2006) proposed two control strategies to stabilize the quadrotor, based on PD control and nonlinear nested saturation techniques. In Raffo *et al*. (2010) a model-based predictive control (MPC) was implemented to track the reference trajectory and a nonlinear H∞ controller was designed to control the attitude and altitude of the quadrotor. Alexis *et al*. (2014) proposed an integral MPC for the translational motions followed by an MPC scheme for the quadrotor's attitude motions. The implementation of the MPC scheme was based on the quadrotor dynamics modeling and on utilizing the theory of piecewise affine systems (PWA).

However, few studies have dealt with the problem of robustness in presence of the aerodynamics disturbances and your effects on the path tracking. In addition, experimental systems must be developed in order to evaluate the behavior of the aerial vehicle in a real environment.

In this study, a cascade control strategy is presented using a model-based predictive control. The system use an inner control loop for the rotational subsystem, combined with an outer loop to control the translational movements of the quadrotor. The main reasons for using the predictive controller are: possibility of dealing with multivariable control problems naturally and the ability to take under consideration the system constraints (Maciejowski, 2000).

2. DYNAMIC MODELLING

The dynamic modelling of the quadrotor is obtained under the hypothesis that the vehicle is a rigid body in space, subject to a principal force (thrust) and three torques.

Without loss of generality, the following assumptions can be established: (a) the structure of the quadrotor is rigid and symmetric, (b) the propellers are rigid, (c) the mass center and the body-fixed frame origin are assumed coincident, (d) the thrust and drag forces are proportional to the square of the propeller's speed.

The dynamic equations of the quadrotor were obtained through Euler-Lagrange formalism, as presented in Lima *et al*. (2015). The system can be described by a set of ordinary differential equation (ODE), in accordance with Eq. (1):

$$\dot{\mathbf{X}} = f(\mathbf{X}, \mathbf{U}) \tag{1}$$

where **X** is the state vector and **U** is the input vector. The states are defined by Cartesian positions (x, y, z) in meters, by the attitude angles (ϕ, θ, ψ) in radians and their respective rates $(\dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi})$. The inputs are based on the rotational speed of the propellers $(\Omega_1, \Omega_2, \Omega_3, \Omega_4)$.

The expanded set of equations of the translational subsystem can be defined in accordance with Eq. (2).

$$\dot{\mathbf{X}} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \\ \dot{z} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \left(\sin\psi \sin\phi + \cos\psi \sin\theta \cos\phi\right)\dfrac{U_1}{m} \\ \dot{y} \\ \left(-\cos\psi \sin\phi + \sin\psi \sin\theta \cos\phi\right)\dfrac{U_1}{m} \\ \dot{z} \\ -g + \left(\cos\theta \cos\phi\right)\dfrac{U_1}{m} \end{bmatrix} \tag{2}$$

The dynamic of the rotational subsystem can be represented by Eq. (3). The control inputs are described in Eq. (4).

$$\dot{\mathbf{X}} = \begin{bmatrix} \dot{\phi} \\ \ddot{\phi} \\ \dot{\theta} \\ \ddot{\theta} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ \dfrac{(I_{yy}-I_{zz})\dot{\theta}\dot{\psi}}{I_{xx}} + \dfrac{J\dot{\theta}(\Omega_1\text{-}\Omega_2+\Omega_3\text{-}\Omega_4)}{I_{xx}} + \dfrac{U_2}{I_{xx}} \\ \dot{\theta} \\ \dfrac{(I_{zz}-I_{xx})\dot{\phi}\dot{\psi}}{I_{yy}} - \dfrac{J\dot{\phi}(\Omega_1\text{-}\Omega_2+\Omega_3\text{-}\Omega_4)}{I_{yy}} + \dfrac{U_3}{I_{yy}} \\ \dot{\psi} \\ \dfrac{(I_{xx}-I_{yy})\dot{\theta}\dot{\phi}}{I_{zz}} + \dfrac{U_4}{I_{zz}} \end{bmatrix} \quad (3)$$

$$\mathbf{U} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b\left(\Omega_1^2+\Omega_2^2+\Omega_3^2+\Omega_4^2\right) \\ bl\left(\Omega_2^2-\Omega_4^2\right) \\ bl\left(\Omega_3^2-\Omega_1^2\right) \\ d\left(\Omega_1^2-\Omega_2^2+\Omega_3^2-\Omega_4^2\right) \end{bmatrix} \quad (4)$$

The model parameters present in Equations (2)-(4) are shown in Table 1.

Table 1. Quadrotor model parameters.

Parameter Description	**Symbol**	**Value**
Mass	m	0.65 kg
Half Wingspan	l	0.23 m
Thrust Coefficient	b	$3.1 \cdot 10^{-5}$ N.s²
Drag Coefficient	d	$7.5 \cdot 10^{-7}$ N.m.s²
X-axis Inertia	I_{xx}	$7.5 \cdot 10^{-3}$ kg.m²
Y-axis Inertia	I_{yy}	$7.5 \cdot 10^{-3}$ kg.m²
Z-axis Inertia	I_{zz}	$1.3 \cdot 10^{-2}$ kg.m²
Rotor Inertia	J	$6.0 \cdot 10^{-5}$ kg.m²
Gravitational Acceleration	g	9.81 m/s²

3. CONTROL STRATEGY

The proposed control strategy is based on the decentralized structure of the quadrotor system, which is composed by Eq. (2)-(4). The scheme of the control strategy is depicted in Fig. 2.

Initially, the trajectory to be followed in space is generated by the trajectory generator block. Then, a predictive controller is proposed to control the quadrotor translational movements in an outer loop, using the references provided by the trajectory generator. Finally, another predictive controller for the rotational subsystem is used in an inner loop to perform the quadrotor stabilization.

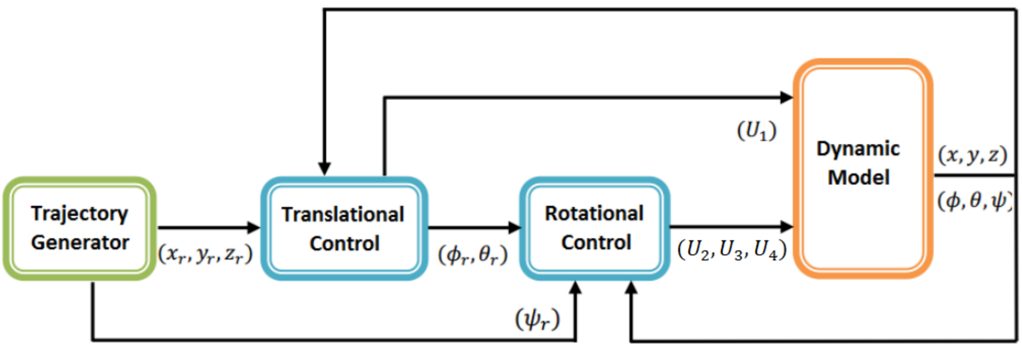

Figure 2. Quadrotor control structure.

3.1 Model Predictive Control

The quadrotor's dynamics was approximated by successive linearization using a first-order Taylor expansion. After the linearization and discretization, was obtained a model in accordance with Eq. (5):

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) \end{aligned} \tag{5}$$

where **x** is the state vector, **u** is the manipulated variables vector and **y** is the controlled variables vector. The matrices **A**, **B** and **C** represent the system matrix, the input matrix and the output matrix, respectively.

The various MPC algorithms propose different cost functions for obtaining the control law. The objective function used in this work can be represented by Eq. (6), where H_p is termed prediction horizon and H_u is defined as the control horizon. This cost function is weighted by the matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ (Maciejowski, 2000).

$$V(k) = \sum_{i=1}^{H_p} \| \hat{y}(k+i \mid k) - r(k+i) \|^2_{\boldsymbol{Q}(i)} + \sum_{i=0}^{H_u-1} \| \Delta\hat{u}(k+i \mid k) \|^2_{\boldsymbol{R}(i)} \tag{6}$$

The term $\hat{y}(k+i \mid k)$ corresponds to the value of the predicted output at the time $k+i$, with the information available at the current instant k. The trajectory reference at each sampling instant is represented by $r(k+i)$ and $\Delta\hat{u} = \hat{u}(k) - u(k-1)$ defines the increment of the manipulated variable.

Equation (6) can be rewritten in matrix form:

$$V(k) = \| \boldsymbol{Y}(k) - \boldsymbol{\Gamma}(k) \|^2_{\boldsymbol{Q}} + \| \boldsymbol{\Delta U}(k) \|^2_{\boldsymbol{R}} \tag{7}$$

where

$$\boldsymbol{Y}(k) = \begin{bmatrix} \hat{y}(k+1 \mid k) \\ \vdots \\ \hat{y}(k+H_p \mid k) \end{bmatrix} \quad \boldsymbol{\Gamma}(k) = \begin{bmatrix} r(k+1 \mid k) \\ \vdots \\ r(k+H_p \mid k) \end{bmatrix} \quad \boldsymbol{\Delta U}(k) = \begin{bmatrix} \Delta\hat{u}(k \mid k) \\ \vdots \\ \Delta\hat{u}(k+H_u-1 \mid k) \end{bmatrix} \tag{8}$$

The weighting matrix $\boldsymbol{Q}$ is positive-definite and $\boldsymbol{R}$ is a positive semi-definite matrix, both indicated in Eq. (9). The $\boldsymbol{Q}$ matrix allows the output variables to be weighted according to their relative importance. The matrix $\boldsymbol{R}$ is applied in order to penalize the increments in control actions. Thus, for high values of $\boldsymbol{R}$, the closed-loop response becomes slower (Camacho and Bordons, 2004)

$$\boldsymbol{Q} = \begin{bmatrix} Q(1) & 0 & \cdots & 0 \\ 0 & Q(2) & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & Q(H_p) \end{bmatrix} \qquad \boldsymbol{R} = \begin{bmatrix} R(1) & 0 & \cdots & 0 \\ 0 & R(2) & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & R(H_u-1) \end{bmatrix} \tag{9}$$

The model prediction of the outputs controlled can be obtained simply as:

$$\hat{y}(k+1 \mid k) = \mathbf{C}\hat{x}(k+1 \mid k) = \boldsymbol{C}\left(\boldsymbol{A}x(k) + \boldsymbol{B}\hat{u}(k)\right) = \boldsymbol{C}\left(\boldsymbol{A}x(k) + \boldsymbol{B}\Delta\hat{u}(k) + \boldsymbol{B}u(k-1)\right) \tag{10}$$

Finally we can rewrite the Eq. (10) in matrix-vector form:

$$\boldsymbol{Y} = \boldsymbol{\Psi} x\left(k\right) + \boldsymbol{\Upsilon} u(k-1) + \boldsymbol{\Theta} \boldsymbol{\Delta U}(k) \tag{11}$$

thus, predictions of the outputs controlled are obtained by relating the dynamic model of the system with the current states, past inputs, and future increments of the manipulated variables. The matrices $\boldsymbol{\Psi}$, $\boldsymbol{\Upsilon}$ and $\boldsymbol{\Theta}$ can be observed in Eq. (12):

$$\boldsymbol{\Psi} = \boldsymbol{C}\begin{bmatrix} \boldsymbol{A} \\ \vdots \\ \boldsymbol{A}^{Hu} \\ \boldsymbol{A}^{Hu+1} \\ \vdots \\ \boldsymbol{A}^{Hp} \end{bmatrix} \quad \boldsymbol{\Upsilon} = \boldsymbol{C}\begin{bmatrix} \boldsymbol{B} \\ \vdots \\ \sum_{i=0}^{H_u-1} \boldsymbol{A}^i\boldsymbol{B} \\ \sum_{i=0}^{H_u} \boldsymbol{A}^i\boldsymbol{B} \\ \vdots \\ \sum_{i=0}^{H_p-1} \boldsymbol{A}^i\boldsymbol{B} \end{bmatrix} \quad \boldsymbol{\Theta} = \boldsymbol{C}\begin{bmatrix} \boldsymbol{B} & \cdots & 0 \\ \boldsymbol{AB}+\boldsymbol{B} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{H_u-1} \boldsymbol{A}^i\boldsymbol{B} & \cdots & \boldsymbol{B} \\ \sum_{i=0}^{H_u} \boldsymbol{A}^i\boldsymbol{B} & \cdots & \boldsymbol{AB}+\boldsymbol{B} \\ \vdots & \vdots & \vdots \\ \sum_{i=0}^{H_p-1} \boldsymbol{A}^i\boldsymbol{B} & \cdots & \sum_{i=0}^{H_p-H_u} \boldsymbol{A}^i\boldsymbol{B} + \boldsymbol{A}^{H_p-(H_u+i)}\boldsymbol{B} \end{bmatrix} \tag{12}$$

The free response of the system correspond the response that would occur over the prediction horizon if no input changes were made, that is, if we set $\boldsymbol{\Delta U}(k) = 0$. The free response corresponds to the evolution of the process due to its present state (Maciejowski, 2000).

In this way, the difference between the future reference trajectory and the free response of the system represent the tracking error, indicated as:

$$\mathrm{E}(k) = \Gamma(k) - \Psi x(k) - \Upsilon u(k-1) \tag{13}$$

Therefore, replacing the Eq. (11) and the Eq. (13) in the cost function Eq. (7), the solution can easily be obtained, resulting in:

$$V(k) = \| \boldsymbol{\Theta\Delta U}(k) - \mathbf{E}(k) \|_{\boldsymbol{Q}}^2 + \| \boldsymbol{\Delta U}(k) \|_{\boldsymbol{R}}^2 \tag{14}$$

Rearranging the equation:

$$V(k) = \text{const} - \boldsymbol{\Delta U}(k)^T \boldsymbol{G} + \boldsymbol{\Delta U}(k)^T \boldsymbol{H\Delta U}(k) \tag{15}$$

where:

$$\boldsymbol{G} = 2\boldsymbol{\Theta}^T \boldsymbol{Q}\mathbf{E}(k) \qquad \boldsymbol{H} = \boldsymbol{\Theta}^T\boldsymbol{Q\Theta} + \boldsymbol{R} \tag{16}$$

The optimal sequence for the increment of control signal can be obtained through of a quadratic programming problem, that is, an optimization problem with a quadratic objective function and linear constraints, as presented in Eq. (17):

$$\begin{aligned} \boldsymbol{\Delta U}^* &= \min_{\boldsymbol{\Delta U}} \left(\boldsymbol{\Delta U}(k)^T \boldsymbol{H\Delta U}(k) - \boldsymbol{G}^T \boldsymbol{\Delta U}(k) \right) \\ &\textit{subject to}: \quad \boldsymbol{A}_{\Delta u} \cdot \boldsymbol{\Delta U}(k) \le \boldsymbol{b}_{\Delta u} \end{aligned} \tag{17}$$

where $\boldsymbol{A}_{\Delta u}$ and $\boldsymbol{b}_{\Delta u}$, correspond to a matrix and a vector of inequalities, respectively.

Due to the sliding horizon feature of the MPC, only $\Delta\hat{u}(k)$ is required at each sampling instant k (Camacho and Bordons, 2004).

In addition, the use of control variations in the cost function formulation aims to provide the controller with an integral action, allowing offset-free tracking (Maciejowski, 2000).

4. SIMULATION RESULTS

The proposed control strategy has been tested by simulations in order to check the performance obtained in the path tracking problem. The simulations were carried out using the software Matlab® R2015a.

The values of the model nominal parameters used for simulations were shown in Table 1.

In the controller formulation, some constraints were considered in the manipulated variables. Such constraints have been computed based on the physical parameters of the system, specifically: (a) the maximum angular velocity of the motors used by the quadrotor; (b) the thrust factor and (c) the drag factor (Alexis *et al.*, 2014). Based on the values presented in the Table 1 the constraints on the inputs can be set as $0 \le U_1 \le 125$, $|U_2| \le 7.2$, $|U_3| \le 7.2$, $|U_4| \le 1.5$.

In addition, constraints were considered in the angular positions of the vehicle, as shown in Eq. (18):

$$\begin{bmatrix} -\pi/4 \\ -\pi \end{bmatrix} \le \begin{bmatrix} \phi,\theta \\ \psi \end{bmatrix} \le \begin{bmatrix} -\pi/4 \\ \pi \end{bmatrix} \tag{18}$$

During the simulations were considered aerodynamic disturbances in translational system and uncertainties of ±40% in the model parameters of the vehicle.

The initial conditions of the quadrotor were $(x, y, z) = (0,0,0)\,m$ and $(\phi,\theta,\psi) = (0,0,0.5)\,rad$. The helical path was defined as the reference trajectory:

$$x_r = \cos\left(t/2\right) \qquad y_r = \sin\left(t/2\right) \qquad z_r = \left(t/10\right) \tag{19}$$

The results are shown in the Fig. 3 and 4.

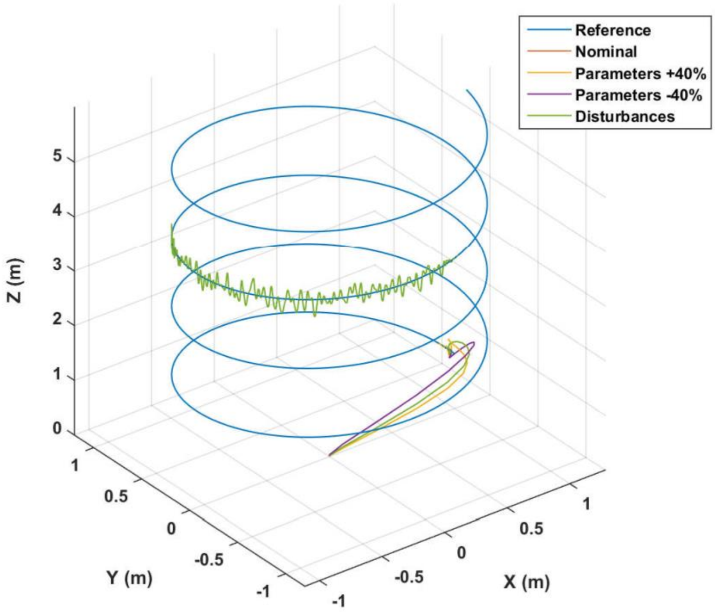

Figure 3. Helicoidal path tracking.

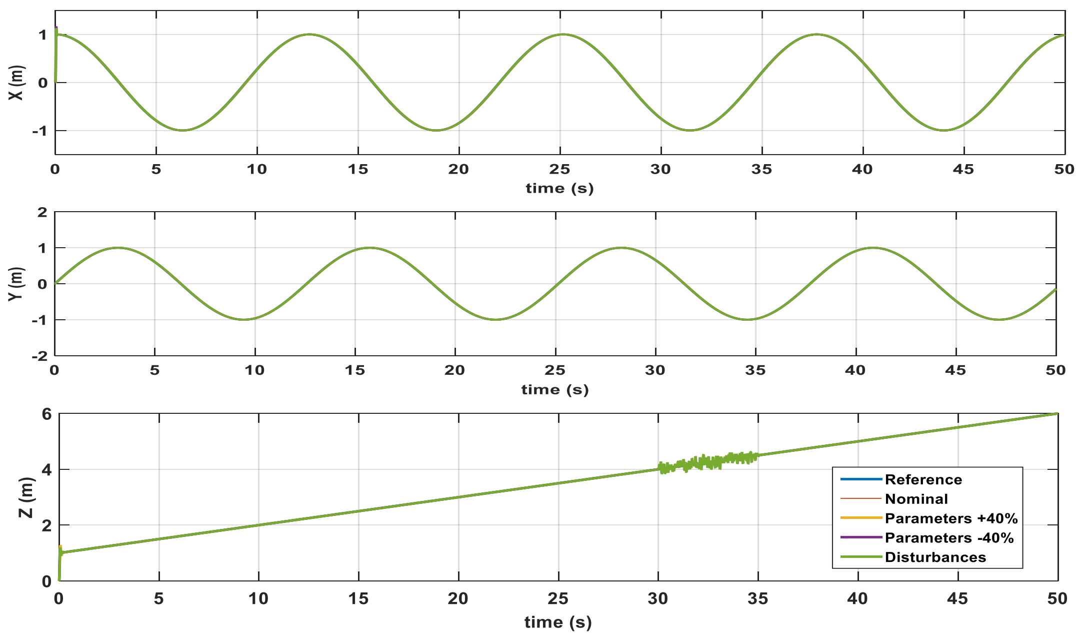

Figure 4. Translational position (*x*, *y*, *z*).

5. CONCLUSIONS

In this paper a model-based predictive control strategy to the path tracking problem for a quadrotor has been presented. An excellent tracking performance has been achieved even in the presence of uncertainties in the parameters of the vehicle. However, the controller was unable to completely reject the effects of aerodynamic disturbances. For this, a robust predictive controller can be evaluated to guarantee effective disturbances attenuation. Soon, the experimental results may be presented for the analysis of the controller

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7. RESPONSIBILITY NOTICE

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