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SUBOPTIMAL NONLINEAR TRACKING CONTROL OF MOBILE ROBOT USING THE DYNAMIC MODEL

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Abstract. *This paper considers trajectory tracking SDRE control of a dynamical model for a mobile robot that takes into account its plane translation and rotation kinematics and dynamics. The solution of a control problem and the design of a nonlinear feedback controller, with the model equations in SDC form, are reduced to the solution of a state-dependent algebraic Riccati equation, similarly to the infinite-time horizon time-invariant LQR, but at each time step. Simulations are performed for different reference trajectories to demonstrate control effectiveness.*

Keywords: *SDRE, Nonlinear systems, Robot tracking, Nonholonomic systems, Optimal control*

1. INTRODUCTION

Various control methods for nonlinear systems are currently available, for example, feedback linearization, sliding mode control, adaptive control (Slotine and Li, 1991), backstepping (Khalil, 2002) and geometric nonlinear control (Isidori, 1995). As discussed in Cimen (2010), even though several methods are well established, we still do not have a control methodology that, besides considering the stability of the solutions, has the ability to address performance and robustness properties to a satisfying extent of nonlinear systems. Even more critical, the controller developed may not have a very intuitive design orientation for practical implementation.

According to Cimen (2010), the State-dependent Riccati equation (SDRE) strategy has emerged as a very attractive tool for the systematic design of nonlinear feedback controllers since the contributions of (Mracek and Cloutier, 1998) were published. SDRE provides a very effective algorithm for synthesizing nonlinear feedback controls, which has a structure similar to that of the infinite-time horizon time-invariant LQR problem but with the matrices being state-dependent. The method allows nonlinearities in the system states while offering great design flexibility through state-dependent weighting matrices.

Mobile robots have gained significant attention due to the fact they belong to a class of mechanical systems called nonholonomic systems (Bloch, 2003). They are subjected to nonholonomic restrictions which make it hard to design a unique feedback controller, via classical control methods, to achieve the desired position or follow a reference trajectory. This fact is a problem exhaustively mentioned in control of nonholonomic systems, known as Brockett's postulate (Brockett, 1983).

The application of SDRE for the synchronization of the mobile robot to a chaotic trajectory is proposed in Rafikova et. al. (2016) using a kinematic model so as to minimize errors in position and, consequently, make the robot perform unpredictable motion. There are several publications which adopt kinematic models for mobile robots. The advantage is that they are simple. However, they are valid only if the robots move at low velocities and acceleration and with light load. This can results in errors in the position prediction, especially when there occur significant changes in the control's signal.

In order to reduce such errors and considering that some robots, i.e., aquatic and aerial ones, are designed for movements with higher velocities and heavier loads, the dynamics of the mobile robot must be considered. These dynamic models for mobile robots are presented in Albagul and Wahyudi (2004), Kardos (2014) and Dusek et. al. (2011). They can be applied to model a variety of mobile vehicle moving on two-dimensional surface. This way, the robot model, in this paper, consists of a kinematic and a dynamic component, resulting in a mathematical model with six state space equations. Then, the SDRE control method is applied to the trajectory tracking control problem of different reference trajectories.

2. STATE-DEPENDENT RICCATI EQUATION CONTROL

2.1 The Nonlinear Regulator Problem

Consider the optimal control problem with infinite horizon for an autonomous nonlinear system in which the system is represented as (Mracek and Cloutier, 1998):

$$\dot{x} = f(x) + g(x)u \tag{1}$$

$$x(0) = x_0 \tag{2}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control vector, $t \in [0, \infty)$, with functions $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^n$ that belong to the space C^1 and $g(x) \neq 0$, for all values of x. We assume that the origin $x = 0$ is an equilibrium point of the system without control, i.e., $f(0) = 0$.

As described in Cimen (2010), the nonlinear regulator problem with infinite horizon is defined by the minimization of the following functional

$$I = \int_{t_0}^{\infty} [x^T Q(x)x + u^T R(x)u]dt \tag{3}$$

with respect to x and to the control u subject to the nonlinear differential equations with the vector control in linear form (1) while regulating the system to the origin for all x such that $\lim_{t\to\infty} x(t) = 0$.

The state and input weighting matrices in this functional are state-dependent such that $Q : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is positive semidefinite, $Q(x) = C^T(x)C(x) \geq 0$, and symmetric and the matrix $R : \mathbb{R}^n \to \mathbb{R}^{m\times m}$ is positive definite, $R(x) > 0$, for all x. We assume the functions $f(x), g(x)$, $Q(x)$ and $R(x)$ to be sufficiently smooth so that (3) is continuously differentiable. We seek for a stable solution for the Problem (1)-(3) in the form $u = \phi(x)$, where ϕ is a nonlinear function of x (Mracek and Cloutier, 1998).

According to Mracek and Cloutier (1998), it is well-known that the nonlinear dynamics (1) can be represented by the following linear structure having state-dependent coefficients (SDC):

$$\dot{x} = A(x)x + B(x)u \tag{4}$$

where

$$f(x) = A(x) \quad \text{and} \quad g(x) = B(x) \tag{5}$$

In the multivariable case, there are an infinite number of ways to bring the nonlinear system to SDC form, i.e., $A : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is nonunique for $n > 1$. Given the assumptions that $f(0) = 0$ and $f \in C^1$, there always exists at least one matrix $A(x)$ such that (1) is satisfied (Vidyasagar, 1993).

2.2 Structure of SDRE Controller

As stated in Mracek and Cloutier (1998), the SDRE approach for obtaining a suboptimal solution of problem (1)-(3) consists of

(i) Use direct parameterization to bring the nonlinear dynamics to the state-dependent coefficient form (4).

(ii) Solve the state-dependent Riccati Equation

$$A^T(x)P + PA(x) - PB(x)R^{-1}(x)B^T(x)P + Q(x) = 0 \tag{6}$$

to obtain $P \geq 0$, where P is a function of x.

(iii) Construct the nonlinear feedback controller

$$u = R^{-1}(x)B^T(x)P(x)x \tag{7}$$

(iv) Integrate the system (4) with the control (7) and repeat from (ii).

3. ROBOT MODELLING

3.1 Kinematic and Dynamic Model

According to Worrall and McGookin (2006), the mathematical model of a two wheeled differential mobile robot can be described by six differential equations given by

$$\dot{x}_p = u\cos\psi - v\sin\psi \tag{8}$$

$$\dot{y}_p = u\sin\psi + v\cos\psi \tag{9}$$

$$\dot{\psi} = r \tag{10}$$

$$m(\dot{u} - vr) = U_1 \tag{11}$$

$$m(\dot{v} + ur) = U_2 \tag{12}$$

$$J\dot{r} = U_3 \tag{13}$$

where m is the mass of the robot, u and v are velocities in the x and y directions, r is the rotational velocity about the z axis, ψ is the orientation angle of the robot, J is the moment of inertia and U_1, U_2 and U_3 represent forces and torque.

The vehicle is free to move and be actuated in x, y and ψ directions and its body is considered symmetric in both x and y directions.

3.2 State Space Model and Errors System

Performing the variable substitutions: $x_1 = x_p,\ x_2 = y_p,\ x_3 = \psi,\ x_4 = u,\ x_5 = v$ and $x_6 = r$, we may rewrite equations (8)-(13). Here, we also define a reference system which takes on the same form of the model, represented as $x_i^r,\ i \in [1,6]$, and $U_j^r,\ j \in [1,3]$, where the superscript r denotes the reference system.

Furthermore, for each of the state variables is defined a corresponding error $y_i,\ i \in [1,6]$, or $u_j^r,\ j \in [1,3]$, such that

$$x_i = y_i + x_i^r \tag{14}$$

$$U_j = u_j + U_j^r \tag{15}$$

Starting with the equations (8)-(13), rewritten according to our state variables selection, substituting the equations for each error variable and employing the equations of the reference system, the state space model for the deviations is obtained in the form $\dot{Y} = A(Y)Y + BU$.

4. SIMULATIONS AND RESULTS

4.1 Reference Trajectory: Line

A straight line is chosen as the reference trajectory, with $\dot{x}$ velocity constant and $\dot{y}$ and $\dot{\psi}$ being zero. The reference orientation angle is $\pi/4$. Considering the robot is not accelerating, the reference forces and torque are zero so that $U_j = u_j,\ j \in [1,3]$. For simplicity, we take the take the initial condition of the nonzero velocity to be $x_4^r = u^r = 1m/s$, the mass of the robot as 1 kg and the moment of inertia as 1kg/m^2.

The initial conditions for the robot model are $(x_p, y_p) = (2,0)$, with angle $\psi = 0$ and zero velocities. In Fig. 1, the trajectory tracking of the system is presented, where the blue and the red line represent the model and reference trajectory, respectively. Also, as shown in Fig. 2, the angle ψ approximates $\pi/4$.

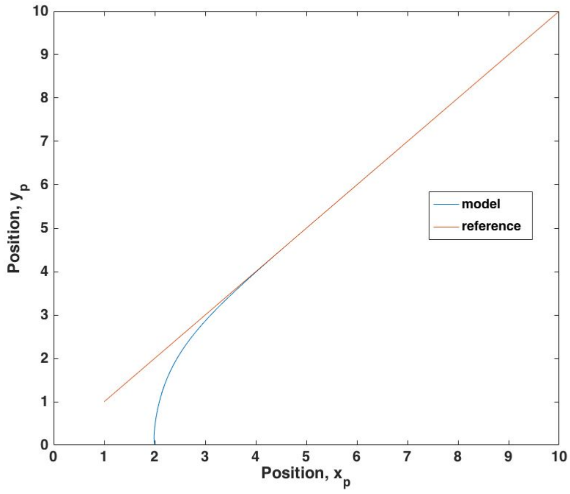

Figure 1. Reference tracking of line.

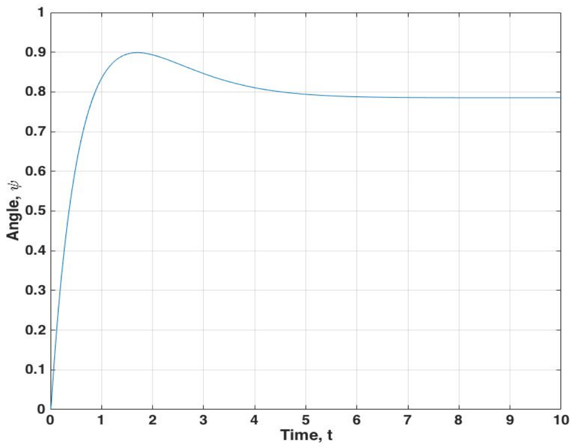

Figure 2. Time evolution of the angle ψ.

We also show that SDRE control was effective bringing all the error variables to zero, as depicted in Fig. 3.

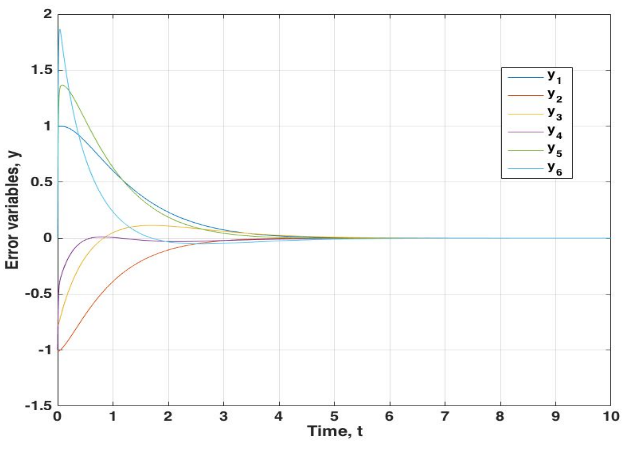

Figure 3. Time evolution of the error variables.

Throughout the simulations for the linear trajectory, MATLAB was employed to solve the differential equations numerically via the ode45 command and the State-Dependent Riccati Equation is solved using the LQR command at every time instant. For the sake of simplicity, the matrices Q and R are chosen as:

$$Q = \begin{pmatrix} 1000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1000 \end{pmatrix} \qquad R = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix} \tag{16}$$

4.2 Reference Trajectories: Circle and Eight-shaped Curve

Assuming different initial conditions, robot tracking for circle and eight-shaped trajectories are shown in Fig. 4 and Fig. 5, respectively. Here, the matrices Q and R present the structure aforementioned. However, the weight in the diagonal elements for Q is 10000 whereas the weight for R is 0.01. Once again, the blue and the red line represent the model and reference trajectory, respectively.

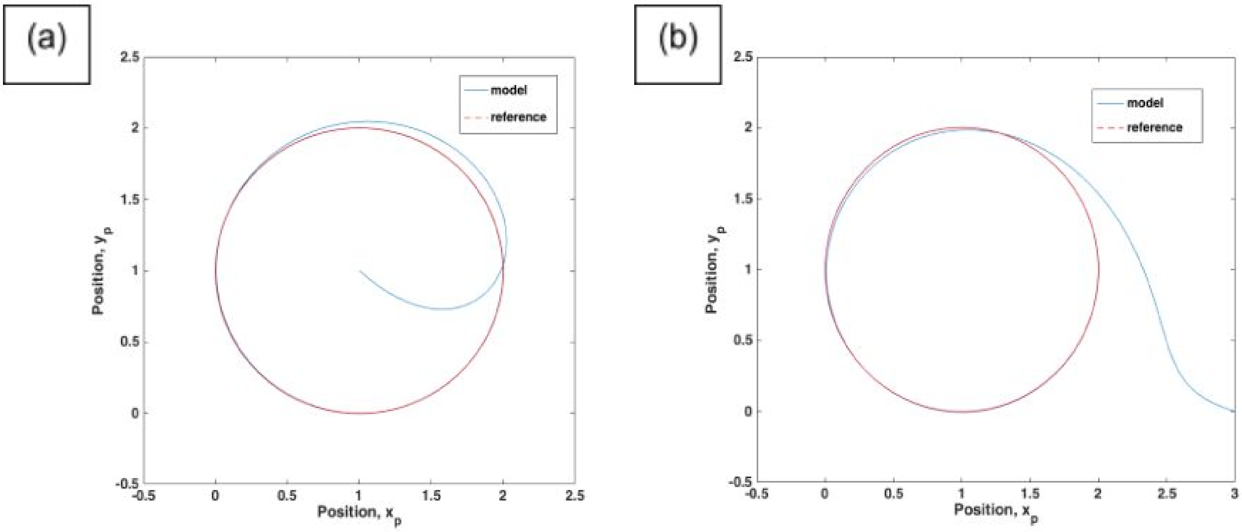

Figure 4. Reference tracking of circle (a) from the inside and (b) from the outside.

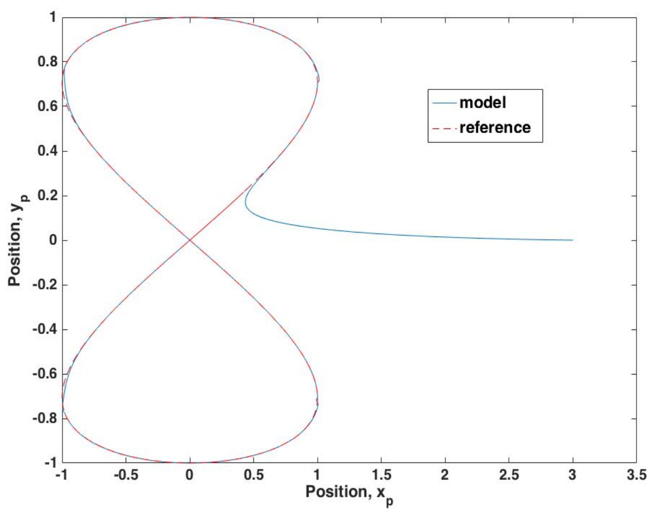

Figure 5. Reference tracking of eight-shaped trajectory.

5. CONCLUDING REMARKS

In this paper the trajectory tracking of a kinematic and dynamic model of a mobile robot was considered. This model consists of six motion equations that consider the kinematics and dynamics of a body motion with translation and rotation on the plane. The State-Dependent Riccati Equation control method was applied to the trajectory tracking problem.

The simulations presented in the previous section demonstrate the effectiveness of the control method in the tracking of a reference. In all three cases, the control effort was able to minimize the state errors and to converge the system to these reference trajectories.

It is important to emphasize that SDRE do not require a system linearization and therefore allows an important system dynamics to be considered and the nonlinear controller provide a suboptimal solution. The SDRE reduces the system model to a state-dependent linear matricial form and reduces a control problem to a solution of a state-dependent algebraic Riccati equation, which can be solved by a numerical method.

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8. RESPONSIBILITY NOTICE