COBEM-2017-0394
MANOEUVRING AND CONTROL OF VECTOR TUGS IN MARITIME SIMULATORS – AN ACTUATION MODEL SUBJECTED TO HYDRODYNAMIC AND WIND EFFECTS DURING PULL OPERATIONS

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Abstract. This paper is devoted to expand and sophisticate the tug performance diagram created by Artyzsuk (2014) to obtain static equilibrium positions for tugboats under escort pull operations. The authors' main goal is to use this model expansion to correctly represent the positioning and towing force magnitude exerted by vector tugs in maritime simulators when operating in pull mode. For the present work, the tug's hull forces will be directly affected by external disturbances such as winds and oceanic currents. In addition, the hydrodynamic forces generated by the towed vessel escort speed will also be taken into account. In order to obtain the tug's static equilibrium, main control parameters must be analyzed, such as: the tug's propeller thrust; the thruster angle; the tug's hull drift angle for a given escort speed; the tow line angle; the resultant towing force; the current speed and its hull drift angle; the wind speed and its hull drift angle. By fixing some of the control parameters, such as the tow line angle, the propeller thrust and the vessel advance speed, we can obtain the remaining control parameters (Artyszuk, 2013), such as the propeller's angle of actuation, the tug's hull drift angle and the resultant towage force. With implementation of the new mathematical model proposed in this work, the maneuvers performed in Maritime Simulators will gain a new level of reality when vector tugs are used.

Keywords: vector tugs, static equilibrium, tug performance diagram, pull operations, maritime simulator.

1. INTRODUCTION

The constant desire for naval innovations in relatively small-time frames and wasting the minimum amount of resources as possible was one of the main reasons for the augment on investments in science and technology, especially on companies and institutions focused on the development of maritime simulators. Normally, these simulators correctly represent the mathematical model of a vessel subjected to external disturbances such as waves, currents and winds. However, most of these simulators struggle when they try to correctly represent the actuation forces that the tugboats exert on a vessel during a maneuver. Usually, the maritime simulators represent such actuation as external forces, with really simplified models, calling such tugboats as vector tugs.

Although few simulators do have the tugboat's dynamics modeled, they usually still use the vector tugs. This is the case because experienced tug masters are not always available in the simulator centers to perform the maneuver. In addition, to run a maritime simulation only with manned tugs would require several integrated simulators running together in real time, which is unfeasible in practice due to the excessive cost to develop a single simulator. The Fig. 1 shows a simulation using the full mission cabin to model the main vessel, one cabin to control the manned tug and a vector tug control station to command the remaining tugs presented in the maneuver.

During real-time simulations, the pilot normally requests a manned tugboat working in pull mode to tow the vessel with a certain towline angle and a certain thruster power. The tug master uses all his experience and knowledge to obey the demanded order, trying to maneuver the manned tugboat in such a way that he will bring the tugboat to a steady-state position, keeping the towline angle and the thruster power as requested. In order to reach this steady state, the tug master is free to vary the tugboat's heading angle and the thruster angle. As soon as the steady-state position is achieved, the tugboat will exert a specific force on the vessel which is normally not known either by the pilot or by the tug master.

The main objective of this work is to obtain the correct towing force and position of a tugboat under pull operations. In order to do so, we will fix all the control parameters that are not usually controlled by a tug master, such as the external disturbances and the orders demanded by the pilot. Afterwards, we will use optimization techniques to obtain the tug's

heading angle, thruster angle, and towing force necessary to keep the tugboat in a steady-state position in such a way that the angle between the tugboat and its towline is minimized. Note that this optimization objective function was chosen because the tug masters usually operate in pull mode by aligning the tugboat's heading angle with its towline. Although this is usually not the most effective position to actuate, we want our vector tugs to perform in the same way manned tugs would in practice. Therefore, by applying the mathematical model presented in this paper, the simplified forces applied by vector tugs nowadays will be corrected, and they will apply the same force as manned tugboats would if they were being used.

Figure 1. Real-time simulation control stations.

2. COORDINATE SYSTEMS AND ANGLE ORIENTATION

In order to analyze the tug's static equilibrium problem, four main coordinate systems are necessary, as shown in Fig. 2. The global earth-fixed coordinate system is denoted by X_0/Y_0 where X_0 point towards east and Y_0 point towards north. Note that, for this work, we will use the regular angular notation. In other words, east will correspond to 0° with the remaining angles increasing in the counter clockwise direction. The vessel-fixed coordinate system will be represented by X_1/Y_1 and it will be positioned, by convenience, at the intersection of the vessel´s center plane and midship sections, with X_1 pointing forward and Y_1 pointing to port side. The towline-fixed coordinate system will be represented by X_2/Y_2 and it will be positioned at the connection point between one of its extremities and the vessel, with X_2 pointing towards the line and Y_2 pointing towards the line's left side. The tug-fixed coordinate system will be represented by X_3/Y_3 and similarly to the vessel, it will be located at the tug's center portion with X_3 pointing forward and Y_3 pointing to port side. Note that for all the coordinate systems we have a Zn [0,3] component not shown pointing upwards.
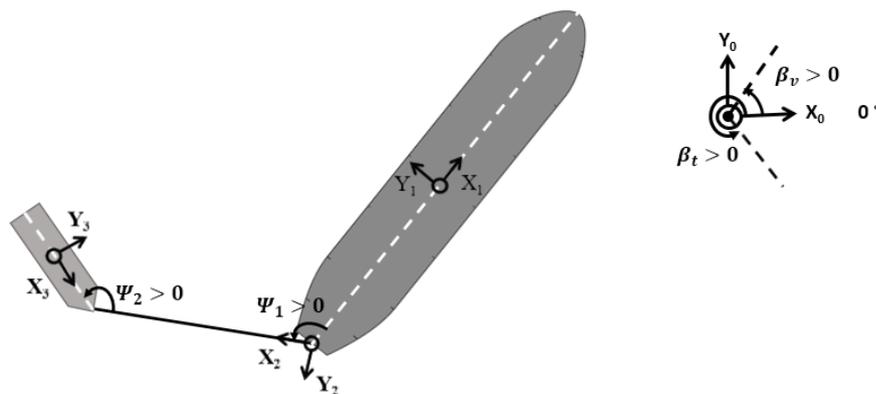

Figure 2. Coordinate Systems and Angles

In addition, we have four important angular relationships represented: β_v represents the angle between the vessel-fixed and the earth-fixed coordinate systems; β_t represents the angle between the tug-fixed and the earth-fixed coordinate systems; Ψ_1 represents the angle between the vessel-fixed and the towline-fixed coordinate systems; Ψ_2 represents the angle between the towline-fixed and the tug-fixed coordinate systems. Note that all the angles mentioned increase in the counter-clockwise direction and they are comprised within the interval [0°, 360°].

By a direct analysis of the angles definitions, on can show that:

$$\Psi_2 = \beta_t - \beta_v - \Psi_1 \qquad (1)$$

Note that, for our formulation, Ψ_2 is only a function of β_t. The angle between the vessel and the towline, Ψ_1, will be kept fixed since it is demanded by a pilot. In addition, the angle between the vessel and the earth-fixed coordinate system is usually known. It is obtained in real time with the assistance of GPS or GNSS systems.

3. STATIC EQUILIBRIUM FORMULATION

There are three main forces actuating on a tug during pull operations: the forces generated by its propellers (F_p); the reaction force transmitted by the towline (F_t); the sum of the hull forces generated by external disturbances such as winds, currents and waves (F_e). The equilibrium conditions for a tug under the influence of such forces takes the following form in the tug-fixed coordinate system (X_3, Y_3):

$$F_{xp} + F_{xt} + F_{xe} = 0 \qquad (2)$$

$$F_{yp} + F_{yt} + F_{ye} = 0 \qquad (3)$$

$$M_{zp} + M_{zt} + M_{ze} = 0 \qquad (4)$$

where:
F_x, F_y – the longitudinal and transversal component of each force on the tug-fixed coordinate system [N],
M_z – Moment developed by a specific force [N.m].

For our formulation, we will assume that the towline is rigid and with negligible mass, thus the reaction force generated by the towline on the tug will be the same as the effective force applied on the vessel. Additionally, since the effective mean force generated by waves is considerably smaller than the forces generated by currents and winds, we will neglect such effects on the tug. Finally, we will assume that the tug has only one propeller at its aft center line, which will be the only actuator of the system, being able to freely rotate in 360°.

By implementing the proposed formulation, one can fix some control parameters such as the propeller thrust, the towline angle, the towed vessel's speed, the towed vessel's global angle, the wind velocity and its global angle, the current velocity and its global angle, and mutually solve Eqs. (2) through (4) to obtain the tug's global angle, the propeller angle of actuation and the effective force on the towline. Note that more than one equilibrium solution may exist, thus optimization techniques must be implemented to obtain the solution desired regarding specific constraints.

4. THRUSTER MODEL

The most popular tugboats used nowadays are commonly called ASD tugs. These tugs have two stern azimuth thrusters completely free to rotate in 360°. For this work, as previous mentioned, we will consider only a single thruster with twice of its regular power and completely free to rotate, located at the tug's center line in a stern location x_p. Since the location x_p is obtained from the tug's midship section, it is negative on its stern.

The torque generated by a tugboat thruster is calculated based on several open water tests. The main objective of such procedures is to obtain the propeller's constants K_T and K_Q. These curves defining a propeller are obtained as a function of the dimensionless advance coefficient J_0, which is shown in Eq. (5) (Tannuri, 2002):

$$J_0 = \frac{V_p}{n_p D_p} \qquad (5)$$

where V_p is the oceanic current velocity projected on the propellers entrance, n_p is the propeller rotation (in rps), and D_p is the propeller diameter. An example of the curves K_T and K_Q as a function of advance coefficient J_0 is shown in the Fig.3. In this figure, it is also shown the coefficient K_{TN} that refers to the contribution of the nozzle to the total thrust force.
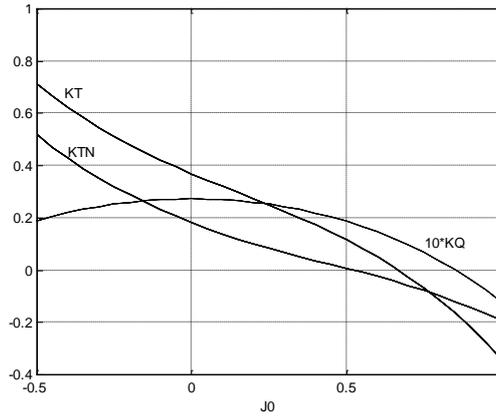
Figure 3. Example of Propeller Constants (series Ka; circumference 19A, 2 m diameter). (Source: Lewis, 1988)

The thrust is obtained from Eq. (6):

$$F_p = K_T(J_0)n|n|\rho D_p{}^4 \tag{6}$$

After obtaining the curves defining a propeller, one can use the tug's bollard pull information to obtain the propeller's maximum rotation using Eq. (7). Note that, since a tug's bollard pull test is performed in sheltered areas and the tugboat is attached to a fixed structure, there are no oceanic current going through the propellers entrance. Therefore, for this scenario, J_0 is zero.

$$n_{max} = \sqrt{\frac{F_{prop\_max}}{K_T(0)\rho D_p{}^4}} \tag{7}$$

where F_{prop_max} is the tug's bollard pull and ρ is the water density.

After obtaining the propeller's maximum rotation, one can obtain the propeller's thrust as a function of a desired input rotation using equation (6).

Finally, the projection of the propeller thrust on each degree of freedom of the body in study is given by:

$$\begin{bmatrix} F_{xp} \\ F_{yp} \\ M_{zp} \end{bmatrix} = F_p \begin{bmatrix} \cos(\delta) \\ \sin(\delta) \\ x_p * \sin(\delta) \end{bmatrix} \tag{8}$$

where:
F_P – the magnitude of thruster force (always positive) [N],
δ – the thruster angle within the interval [0°,360°] increasing in the counter-clockwise direction [°],
x_p – thruster location related to the tugboat's midship section (negative if at the tug's stern) [m].
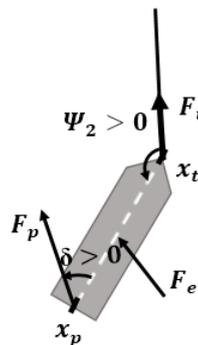

Figure 4. Force Diagram.

5. TOWING FORCE MODEL

As mentioned before, the towline will be modeled as a fixed bar, thus the towing reaction force will be the same as the effective force exerted on the towed vessel. Note that, although the towing force F_t is not directly controlled by a tug

master, it is also a variable controller parameter since its magnitude will directly depend on the tugboat's heading angle and propeller angle of actuation.

The towing forces and moments (with full support of signs) are demonstrated below:

$$\begin{bmatrix} F_{xt} \\ F_{yt} \\ M_{zt} \end{bmatrix} = F_t \begin{bmatrix} -\cos(\Psi_2) \\ \sin(\Psi_2) \\ x_t * \sin(\Psi_2) \end{bmatrix} \qquad (9)$$

where:

F_t – the magnitude of towing force (always positive) [N],
x_t – thruster location related to the tugboat's midship section (positive if at the tug's bow) [m].

6. EXTERNAL FORCE MODEL

For the present study, the hull forces acting on the tug will be a composition of the forces generated by the external disturbances. In other words, the effective hull force acting on the tug will be the resultant force generated by the combination of the current and wind forces as shown below:

$$F_{xe} = F_{xc} + F_{xw} \qquad (10)$$

$$F_{ye} = F_{yc} + F_{yw} \qquad (11)$$

$$M_{ze} = M_{zc} + M_{zw} \qquad (12)$$

where:

F_{xc}, F_{yc} – the longitudinal and transversal components of the force generated by the current in the tug-fixed coordinate system [N],
M_{zc} – Moment developed by the current force [N.m],
F_{xw}, F_{yw} – the longitudinal and transversal components of the force generated by the wind in the tug-fixed coordinate system[N],
M_{zw} – Moment developed by the wind force [N.m].

As briefly mentioned before, for the present study, we will neglect the effective force generated by waves in the tug's hull. Usually, the oscillatory wave motion experienced during maritime maneuvers does not produce significant mean forces on the tug's hull when compared to the effective forces generated by the current and wind.

6.1 Current force model

The oceanic currents are slow varying fields. For this reason, we can assume that the tug is only being subjected to static current forces which act on its longitudinal and transversal directions. Considering the mathematical model proposed by OCIMF (1977) to calculate the static forces generated by the current on a vessel, we obtain the following relationships:

$$\begin{bmatrix} F_{cx} \\ F_{cy} \\ M_{cz} \end{bmatrix} = 0.5\rho|V_{rel}|_c^2 LT \begin{bmatrix} C_{cx}(\alpha_c) \\ C_{cy}(\alpha_c) \\ LC_{cz}(\alpha_c) \end{bmatrix} \qquad (13)$$

where:

ρ – the water density [kg/m³],
$|V_{rel}|_c$ – the relative velocity between the water and the tug [m/s],
L – the tug's length [m],
T – the tug's draught [m],
C_{cx}, C_{cy}, C_{cz} – non-dimensional hydrodynamic coefficients [-],
α_c – the angle of attack between the current and the tug [°].

Note that the non-dimensional hydrodynamic coefficients are normally obtained from towing small-scale tank captive tests or by CFD numerical calculations. For our study, we will use the results from the IPT towing tank (TPN-USP, 2009).
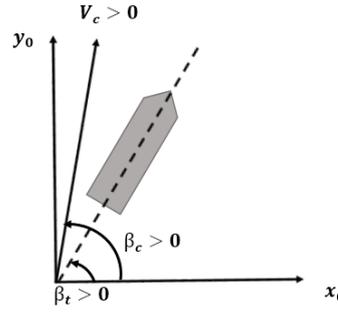
Figure 5. Relationship Between Current and Tug

Let's now suppose that a towed vessel is navigating with a speed $U = u(t)i + v(t)j$ in the earth-fixed frame, with a global angle β_v. We will assume that the tug is navigating with the same earth-fixed speed as the vessel. By making this assumption, we take in consideration the current effects generated by the towed vessel's speed on the tug.

In a given instant, the tug is subjected to a current force generated by the environment with a speed V_c actuating in a global angle β_c (Fig.5). From this information, one can calculate the earth-fixed relative velocity between the tug and the water:

$$V_{rel\_cx} = [V_c \cos(\beta_c) - u(t)]\, i \tag{14}$$

$$V_{rel\_cy} = [V_c \sin(\beta_c) - v(t)]\, j \tag{15}$$

where:
V_{rel_cx}, V_{rel_cy} – the longitudinal and transversal components of the relative velocity between the tug and the current in the earth-fixed coordinate system [m/s],
V_c – the current speed [m/s],
β_c – the global angle of the current [0°, 360°].

The modulus of the relative velocity can be obtained as follows:

$$|V_{rel}|_c = \sqrt{V_{rel\_cx}{}^2 + V_{rel\_cy}{}^2} \tag{16}$$

The angle between the relative current and the tug is given by:

$$\alpha_c = \tan^{-1}\frac{V_{rel\_cy}}{V_{rel\_cx}} - \beta_t \tag{17}$$

6.2 Wind force model

The longitudinal and transversal forces as well as yaw moment generated by the wind at the tug's emerged areas are modeled as functions of non-dimensional coefficients that can be obtained from wind tunnel experiments in model scales. Assuming that the wind incidence has an invariant global angle and a constants speed, one can obtain the following relationships (OCIMF, 1977):

$$\begin{bmatrix} F_{wx} \\ F_{wy} \\ M_{wz} \end{bmatrix} = 0.5\rho \left| V_{rel}{}^2\right|_w \begin{bmatrix} A_{frontal}C_{wx}(\alpha_w) \\ A_{lateral}C_{wx}(\alpha_w) \\ LA_{lateral}C_{wz}(\alpha_w) \end{bmatrix} \tag{18}$$

where:
ρ – the air density [kg/m^3],
$|V_{rel}|_w$ – the relative velocity between the wind and the tug [m/s],
$A_{frontal}$ – the tug's frontal emerged area [m^2],
$A_{lateral}$ – the tug's lateral emerged area [m^2],
C_{wx}, C_{wy}, C_{wz} – wind non-dimensional coefficients [-],
α_w – the angle of attack between the wind and the tug [°].
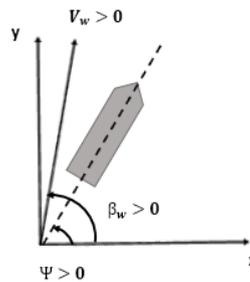
Figure 6. Relationship Between Wind and Tug

In Fig. 6, we can see an example of a scenario where the tug is subjected to a wind force generated by the environment with a speed V_w actuating in a global angle β_w. The calculation of the relative velocity between the tug and the wind as well as the angle of attack between the relative wind and the tug can be obtained in an analogous manner as shown in the previous section for the current (Eqs. (14) through (17)), thus being omitted here for simplicity.

7. OPTIMIZATION FORMULATION

As briefly mentioned, the static equilibrium diagram may have several sets of solutions, or, in other words, there are several sets of control parameters combinations that will bring the tugboat to be in static equilibrium when operating in pull mode. Therefore, optimization techniques should be implemented to select the solution that most fits the behavior of a manned tugboat.

Usually, tug masters try to obey a pilot's command by keeping the tugboat as aligned as possible with the towline. Although such a position may not be most suitable one from an efficiency standpoint, it is the most intuitive position to stay, thus representing the main action taken by tug masters. In order to select the solution that represents such behavior, we decided to implement optimization techniques that try to minimize the angle between the tugboat and its towline. Therefore, due to our angular orientations previous stated, the objective function for such configuration may be given by Eq. (19).

$$G(\Psi_2) = \min(|\Psi_2 - 180°|) \tag{19}$$

After defining the desired objective function, one must provide the upper and lower boundaries for Ψ_2, F_t, and δ, which are the variable control parameters of the system. Due to the tugboat's physical structure, it is known that a towline connected at its bow cannot go through its longitudinal portion towards its aft. Therefore, we can define the lower boundary of Ψ_2 to be approximately 80° while the upper boundary may be approximated to be 280°. The boundaries of the towline force and the propeller angle of actuation are more straightforward. Since the variable F_t must be positive, its lower boundary will be zero, while no upper boundary will be defined; the variable δ is free to rotate in 360°, thus its lower boundary will be 0° while its upper boundary will be given by 360°. Note that the upper and lower boundaries in δ are not strictly necessary, but they will be used to keep the angle notation simplicity between the desired interval.

The definition of the objective function, the upper and lower boundaries of the variables parameters and the nonlinear constraints (Eqs. (2) to (4)) allow us to use several optimization techniques to attain the desired goal. For this work, the sequential quadratic programming optimization method (SQP) will be used.

8. SIMULATION AND RESULTS

For the simulations presented in this section, we will fix the tugboat's propeller force to be 600 KN, which is analogous to assume that a pilot is requesting a tugboat of about 60 tones to work with its full power. The main objective of such simulations is to analyze how the variable control parameters (Ψ_2, F_t, δ) behave when subjected to external conditions either provided by a pilot or imposed by the environment. Therefore, we will vary the towline angle, the vessel's advance speed and the external conditions of currents and winds in order to obtain the tugboat' heading and propeller angle as well as the towline force that characterizes a static equilibrium position under pull operation. Table 1 provides all the tugboat's physical parameters for the following simulations:

Table 1. Tugboat Physical Parameters

Parameter	Value	Unit
Length	32	m
Draught	5,5	m
Width	13	m
Frontal Area	528	m²
Lateral Area	528	m²
Contact Position (x_t)	12	m
Propeller Position (x_p)	-12	m
Propeller Diameter	1,5	m
bollard pull	600	KN

For the first 3 simulations (Figs. 7 to 12), we only varied the vessel1's advance speed and the towline angle, not applying external oceanic currents or wind. Since tugboats commonly operate when towed vessels navigate between 2 and 6 Kn, the authors decided to evenly explore this range of operation.

By analyzing the first two simulations, one can realize that the towline force tends to be smaller than the tug's bollard pull when the tugboat operates on the towed vessel's bow center part, constantly increasing while it moves towards the towed vessel's stern. However, the same does not occur in the third simulation. When the towed vessel is navigating with 6 Kn, the towline force tends to decrease when the towline angle goes from 0° to 90° and then it tends to increase from 90° to 180°. This may be explained if we analyze the relative angle between the tugboat and the towline.

For the first simulation, the relative angle between the tugboat and the towline reached a maximum misalignment of 5° while in the second simulation it reached a maximum of 20°. Due to the similarity of the towline force curves for the first and second simulations, one can infer that a maximum misalignment angle of 20° between the tugboat and the towline was not enough to cause substantial change on the towline force configuration, thus showing that the behavior of such a curve is dominated by the propeller-water interaction. However, for the third simulation, the maximum misalignment angle between the tugboat and the towline reached a maximum of 55°, which characterized a substantial change in the towline force curve, as previously mentioned. This change can be explicitly seen if one compares the tugboat efficiency when the towline angle was 90°. For this specific towline angle, the tugboat had an efficiency of 97% in simulation 1, 92.5% in simulation 2, and 73% in simulation 3. Therefore, it is clear that, in simulation 3, the large relative angles between the towline and the tugboat are dominant over the water-propeller interaction, thus characterizing a different behavior of the towline force curve.

The Fig.13 and 14 shows the tug position for the cases with 2kn and 6kn, respectively.
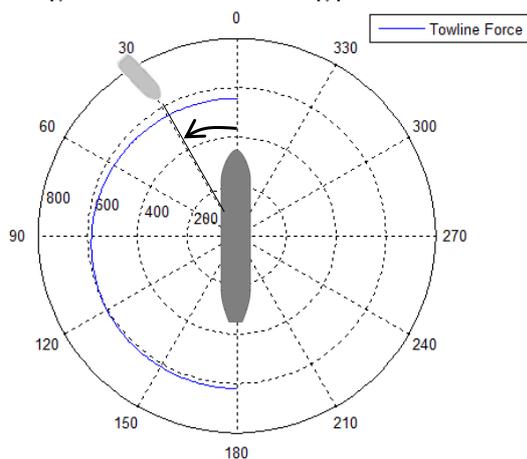

Figure 7. Towline Force as Function of Relative Angle Between Towline and Vessel for Towed Vessel Navigating with 2 Kn
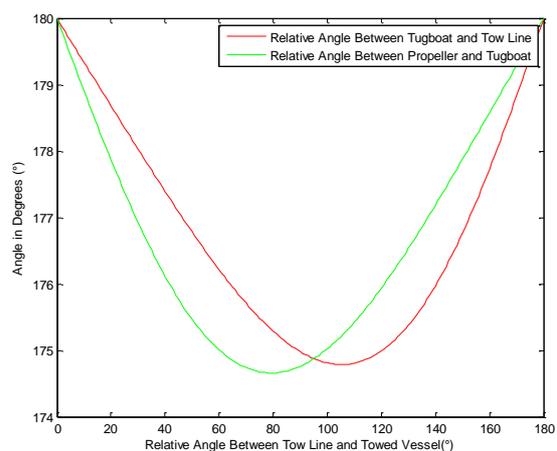

Figure 8. Tugboat Heading and Propeller Angle as Function of the Relative Angle Between Towline and Towed Vessel for a Towed Vessel Navigating With 2 Kn.
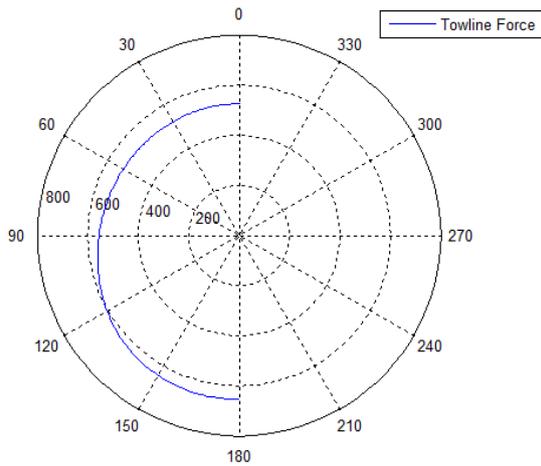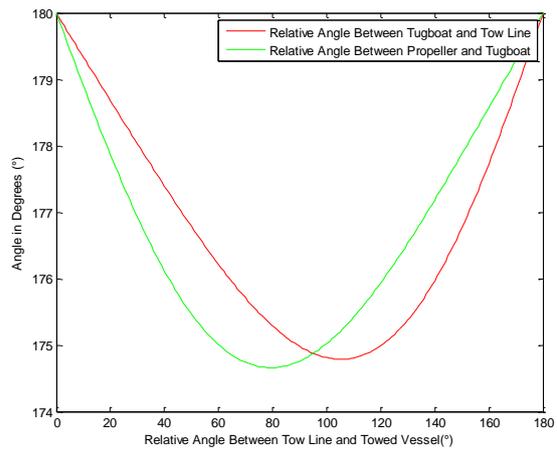
Figure 9. Vessel Navigating with 4 Kn

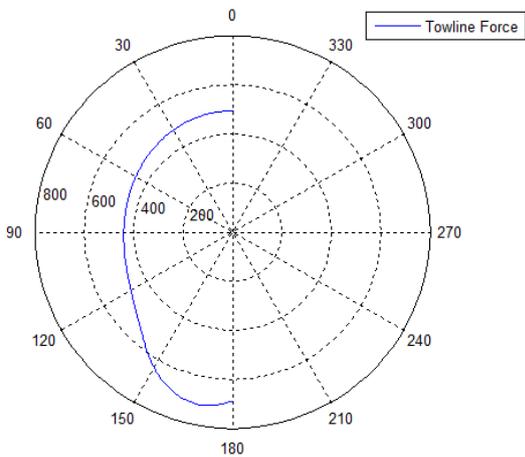Figure 10. Vessel Navigating with 4 Kn
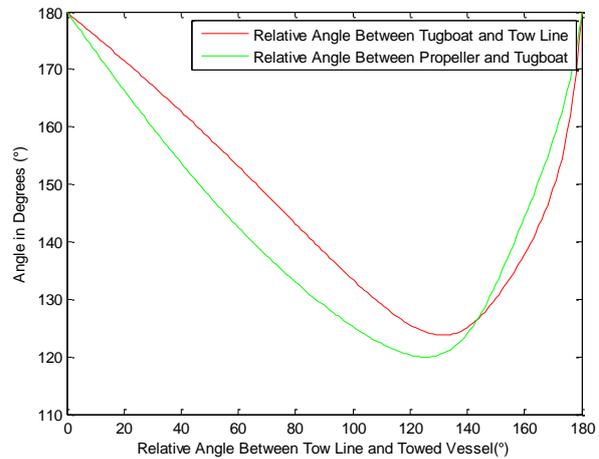

Figure 11. Vessel Navigating with 6 Kn

Figure 12. Vessel Navigating with 6Kn

For the fourth and fifth simulations (Figs. 15 to 18), external disturbances were applied to the system when the towed vessel was navigating with 4 Kn (intermediate scenario). An oceanic current of 2 Kn going to West was imposed for both simulations while a wind of 20 Kn coming from Southeast was imposed only for the fifth simulation. By comparing simulations 4 and 5, one can realize that the obtained results are almost identical, thus proving that the wind influence on the tugboat is minimal for this scenario. Additionally, the towline forces obtained in simulations 4 and 5 are greater than the ones obtained in simulation 2, thus we can infer that the external disturbances applied to the system are incrementing the tugboat's power efficiency. Note that, the same external disturbances decrement the tugboat's power efficiency when it actuates by starboard
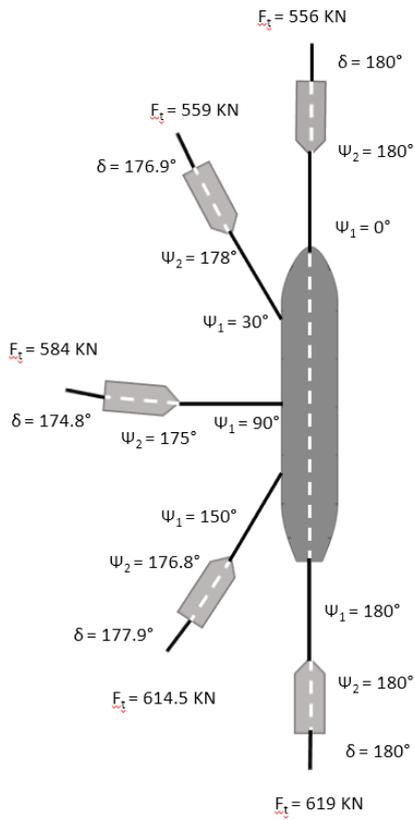
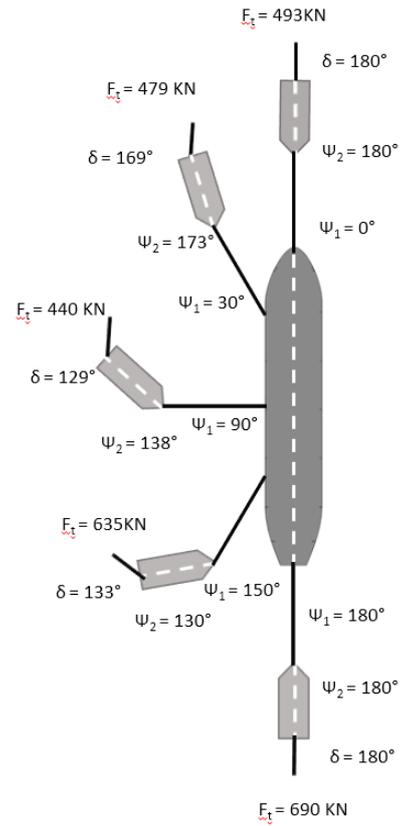Figure 13. Tugboat's Configurations Obtained in Simulation 1 (Advance Speed 2 Kn)

Figure 14. Tugboat's Configurations Obtained in Simulation 3 (Advance Speed 6 Kn)
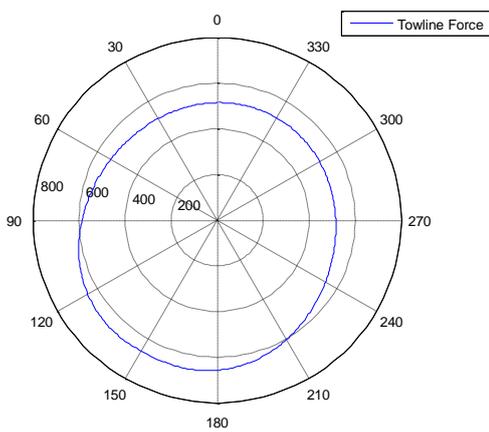

Figure 15. Towline Force as Function of Relative Angle Between Towline and Vessel for Towed Vessel Navigating with 4 Kn. Oceanic Current of 2Kn.
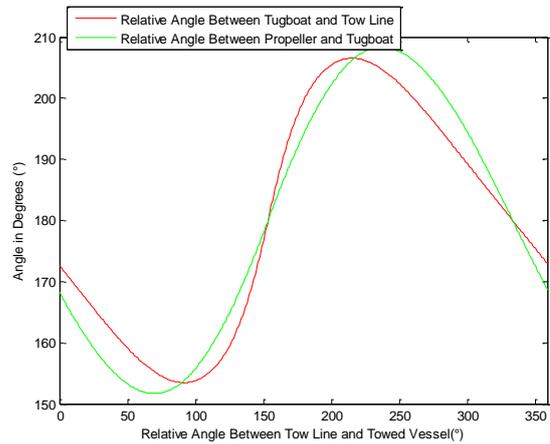

Figure 16. Tugboat Heading and Propeller Angle as Function of the Relative Angle Between Towline and Towed Vessel for a Towed Vessel Navigating With 4 Kn. Oceanic Current of 2 Kn
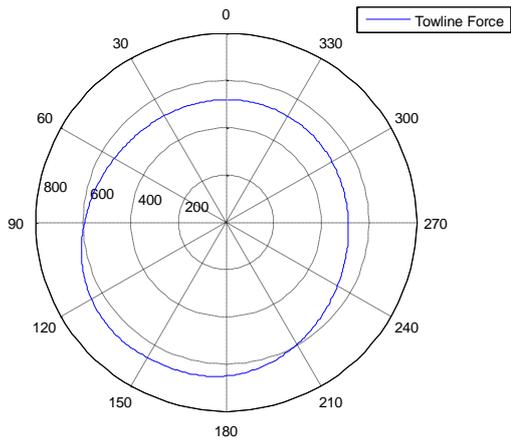
Figure 17. Towline Force as Function of Relative Angle Between Towline and Vessel for Towed Vessel Navigating with 4 Kn. Oceanic Current of 2Kn; Wind of 20 Kn.
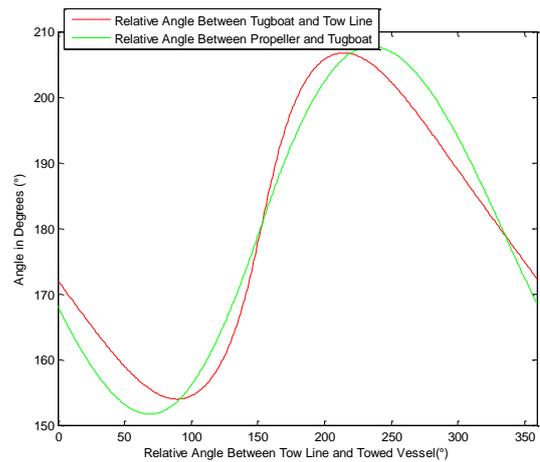

Figure 18. Tugboat Heading and Propeller Angle as Function of the R.A Between Towline and Towed Vessel for a Towed Vessel Navigating With 4 Kn. Oceanic Current of 2 Kn; Wind of 20 Kn.

Finally, by analyzing all the simulations, one could realize that the graph regarding the relative angle between the towline and the tugboat as well as the graph regarding the relative angle between the propeller and the tugboat have similar configurations, being always far apart by only 5° to 10°. Therefore, if the tug's heading angle to keep the system in static equilibrium configuration is known, the propeller configuration to keep such position may be easily found by trial and error if its initial position is chosen to be in the range previously mentioned.

Fig. 19 shows the position of the tug and the thruster azimuth for different towline angles for the last simulation scenario.
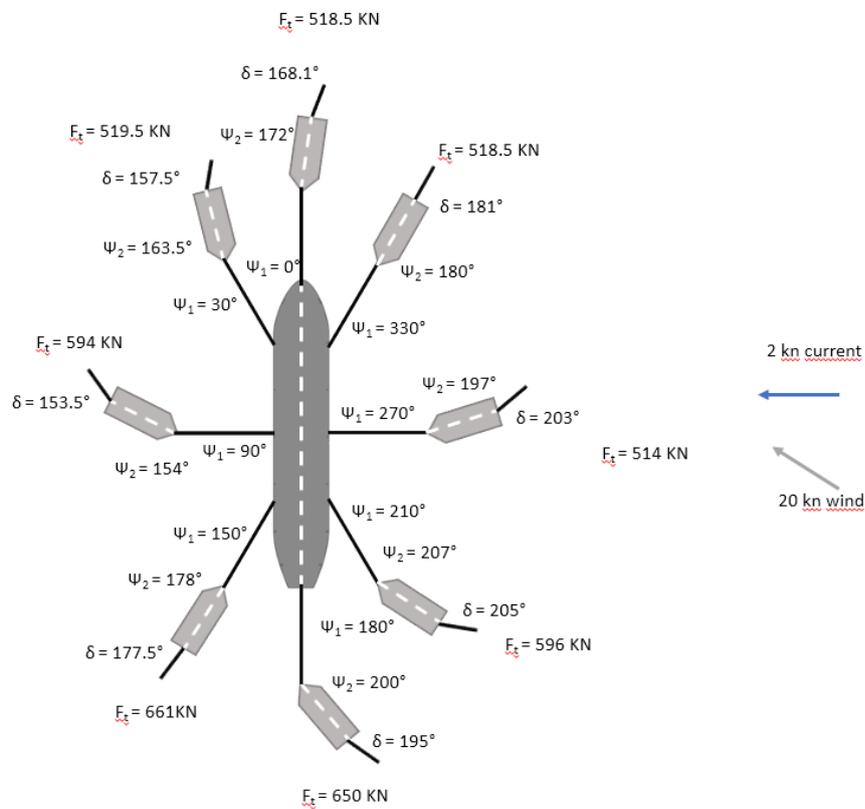

Figure 19. Tugboat's Configurations Obtained in Simulation 5 (Advance Speed 4 Kn; Oceanic Current 2Kn; Wind 20 Kn)

9. CONCLUSIONS

The mathematical model proposed in this paper is an alternative solution for the vector tug's representation usually implemented in maritime numerical simulators. Since the vector tugs are one of the main responsible for altering the dynamics of a vessel, which is normally the principal study object during a simulation, it is extremely important to correctly represent its towage force as well as its positioning in a realistic way. The implementation of such mathematical model will bring the realism of maritime simulations to a higher level.

The performed simulations showed that, for a towed vessel navigating with speeds of 2 and 4 Kn, the relative angle between the tugboat and its towline is smaller than 20°, for the static equilibrium to occur. For those scenarios, the towage force is mainly affected by the water-propeller interaction. For a towed vessel navigating with 6 Kn, the relative angle between the tugboat and its towline reached a maximum of 55°. This great misalignment proved to be more influential on the towed force magnitude than the water-propeller interaction.

Additionally, this worked proved that, for any external conditions of wind and current, the tugboat's towage efficiency may be affected depending if its navigating by the towed vessel's port or starboard side. This efficiency change is directly related to the static equilibrium position found by the mathematical model in each one of these two distinct scenarios.

Finally, all the simulations of this work showed that the relative angle between the towline and the tugboat as well as the relative angle between the propeller and the tugboat are always within an interval of about 10°. Therefore, by knowing the static equilibrium heading angle position, a tug master may easily find its propeller angle of actuation by trial and error, if he starts imposing an initial angle propeller angle equal to the relative angle between the tug and its towline.

10. ACKNOWLEDGEMENTS

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