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TOPOLOGY OPTIMIZATION OF BI-DIMENSIONAL COMPOSITE STRUCTURES UNDER THERMOMECHANICAL LOADS

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Abstract. *This work presents a formulation for topology optimization of bi-dimensional systems under mechanical and thermal loads. The main target is to minimize structural mass under a global stress criterion and obtain maximum possible stiffness - with both stability and boundary constraints. Effective properties of materials are penalized using Solid Isotropic Microstructure with Penalization; besides, Augmented Lagrangian Method and a minimization algorithm based on Truncated Newton Approach were used for solving the constrained non-linear programming problem. Finite Element Method – quadrilateral bi-dimensional four-node element - is employed to interpolate relative densities and components of temperature and displacement fields. Results are presented for different meshes and load prescriptions.*

Keywords: *topology optimization, global stress criterion, thermomechanical loads*

1. INTRODUCTION

Thermal conduction process is a relevant aspect in material design; several works in the last decades have treated the thermal problem, such as Scalia *et al.* (2004), Mehne *et al.* (2007), Buryachenko and Brun (2012) and Bachher, *et al.* (2014). On the other hand, most works approaching Topology Optimization in the widely available literature consider separately either mechanical or thermal loads. This work addresses optimum design problems, considering prescription of thermal and mechanical loads simultaneously, employing a unique state equation to describe both load fields

2. COMPUTATIONAL PROCEDURE

The resolution of the thermomechanical problem consisted of using Finite Element Method (MEF) discretize design domain and solving governing equations. During data preprocessing step, the following parameters were specified: structure's initial geometry; material properties; finite elements mesh; boundary and stability constraints; thermal and mechanical loads.

Optimization problem was solved using Augmented Lagrangian with Exterior Penalty and Spectral Projected Gradient (Birgin and Martínez, 2002), considering stress criterion constraint. The numerical formulation is described in the following subtitles.

2.1 Thermomechanical problem

Thermoelastic constitutive equation is obtained by applying generalized Hooke's Law, according to Eq. (1):

$$\sigma = D^{\rho}\varepsilon(\mathrm{u}) - (T - T_0)\beta^{\rho} \tag{1}$$

where: σ, D^{ρ}, ε and β^{ρ} are, respectively, the stress, constitutive, deformation and thermal stress tensors.

The strong formulation for the problem is given in Eq. (3) e (4):

$$\nabla.\sigma + b = 0 \tag{2}$$

$$-\nabla.q + Q = C_p\dot{T} \tag{3}$$

where: σ is the tension of the system; b is the system's body force per volume unit; q is the thermal flux; Q is the internal heat per volume unit; C_p is the thermal capacity of the structure; and $\dot{T}$ is the temperature variation with respect to time.

2.2 Material properties

Solid Isotropic Microstructure with Penalization (SIMP) was employed as the porous microstructure model; then, Elasticity Modulus E was considered according to Eq. (2):

$$E(x) = \rho(x)^{\eta}E^0 \tag{4}$$

where: ρ represents the point density, η a penalty parameter and E^0 the solid material Elasticity Modulus.

2.3 Optimization problem's discrete formulation

The optimization problem is presented in Eq. (5):

$$\min_{\boldsymbol{\rho}} f(\boldsymbol{\rho}) = \int_{\Omega} \rho d\Omega \tag{5}$$

subject to constraints in Eq. (6) to (11):

$$\mathrm{g}_1(\boldsymbol{\rho},\mathbf{u}(\boldsymbol{\rho}))_e = \left\{\frac{1}{\Omega_e}\int_{\Omega_e}\left\langle\rho\left(\frac{\sigma_{eq}^{*}(\boldsymbol{\rho},\mathbf{u}(\boldsymbol{\rho}))}{\sigma_y} - 1\right) + \rho\kappa\left(\rho_{sup} - \rho\right)\right\rangle^{\rho} d\Omega\right\}^{1/\rho} \leq 0 \tag{6}$$

$$\mathrm{g}_2(\boldsymbol{\rho})_e = \left(\frac{\partial\rho}{\partial x}\right)_e^2 - (C_x^e)^2 \leq 0 \tag{7}$$

$$\mathrm{g}_3(\boldsymbol{\rho})_e = \left(\frac{\partial\rho}{\partial y}\right)_e^2 - (C_y^e)^2 \leq 0 \tag{8}$$

$$\mathrm{g}_4(\boldsymbol{\rho})_i = \rho_{inf} - \rho(\mathbf{x})_i \leq 0, \forall\, \mathbf{x} \in \Omega \tag{9}$$

$$\mathrm{g}_5(\boldsymbol{\rho})_i = \rho(\mathbf{x})_i - \rho_{sup} \leq 0, \forall\, \mathbf{x} \in \Omega \tag{10}$$

where: $e = 1,...,N$ (number of elements) and $i=1,...,M$ (number of nodal points); Ω is the domain area and Ω_e e-th element's domain; ρ is the relative density; $\sigma_{eq}^{*} = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^2$ is the von Mises yield criterion and σ_y is material's yield stress; $\mathbf{u}$ is the displacement vector; κ is the initial relaxation parameter and $\langle f(\mathbf{x})\rangle = \max\{0, f(\mathbf{x})\}$, for all positive part of $f(\mathbf{x})$; C_x and C_y are the boundary limits for relative density gradient components; ρ_{inf} and ρ_{sup} are the relative density infimum and supremum, respectively.

Constraint g_6 represents global stress criterion (Costa Jr, 2003) and g_2 and g_3 are employed to prevent checkerboard instabilities (Sigmund and Petersson, 1998).

2.4 Augmented Lagrangian Method

To transform the constrained optimization problem into an equivalent unconstrained problem, we may use the Augmented Lagrangian Method. Being $\boldsymbol{\rho} \in \mathbf{X}$ with $\mathbf{X} = \left\{\boldsymbol{\rho} \in R^n \middle| \rho_{inf} \leq \rho_i \leq \rho_{sup}, i = 1, \dots, N\right\}$ and N is the number of elements in the mesh.

Step - 1. Initial conditions: $k = 0$; $\mu^k = 0$; $erro = 1{,}0$; ω^k and tol.

Step – 2. While $erro > tol$, to do:

(i) Solution of the minimization problem with side constraint:

$$\min \mathcal{L}_A(\,\boldsymbol{\rho}, \boldsymbol{\mu}; \boldsymbol{\omega}\,),\ \ \forall\, \boldsymbol{\rho} \in \mathbf{X} \tag{11}$$

where:

$$\min \mathcal{L}_A(\boldsymbol{\rho}, \boldsymbol{\mu}; \boldsymbol{\omega}) = f(\boldsymbol{\rho}) + \sum_{j=1}^{3}\left[\frac{1}{\omega_j}\sum_{e=1}^{N}\psi_e^j\left(g_e^j, \omega_j \mu_{ej}\right)\right] \tag{12}$$

$$\psi_e^j\left(g_e^j, \omega_j \mu_{ej}\right) = \begin{cases} g_e^j\left(g_e^j + \omega_j \mu_{ej}\right), & \text{if } g_e^j \geq -\frac{\omega_j \mu_{ej}}{2} \\ -\left(\frac{\omega_j \mu_{ej}}{2}\right)^2, & \text{if } g_e^j < -\frac{\omega_j \mu_{ej}}{2} \end{cases} ;\ j = 1,2,3 \tag{13}$$

(ii) Update of Lagrangian multipliers

$$\mu_{ej}^{k+1} = \max\left\{0, \mu_{ej}^k + \frac{2}{\omega_j} g_e(\mathbf{x}^k)\right\};\ j = 1,2,3 \tag{14}$$

(iii) Update of penalty parameters

$$\omega_j^{k+1} = \begin{cases} \zeta_j \omega_j^k, \text{ with } \zeta_j \in (0,1), \text{if } \zeta_j \omega_j^k \geq \omega_j^{crit} \\ \omega_j^{crit} \end{cases} \tag{15}$$

(iv) Error

$$a = \max_e \left|\mu_{e1}^{k+1} - \mu_{e1}\right|,\ b = \max_e \left|\mu_{e2}^{k+1} - \mu_{e2}\right|,\ c = \max_e \left|\mu_{e3}^{k+1} - \mu_{e3}\right| \tag{16}$$

so,

$$erro = \max\{\mathrm{a}, \mathrm{b}, \mathrm{c}\} \tag{17}$$

Step – 3. End

The Augmented Lagrangian's sensitivity is given by:

$$\frac{d\mathcal{L}_A}{d\rho_i} = \frac{df}{d\rho_i} + \left(\frac{1}{\omega_1}\sum_{e=1}^{N}\frac{d\psi_e^1}{d\rho_i} + \frac{1}{\omega_2}\sum_{e=1}^{N}\frac{d\psi_e^2}{d\rho_i} + \frac{1}{\omega_3}\sum_{e=1}^{N}\frac{d\psi_e^3}{d\rho_i}\right) \tag{18}$$

where: r is the degree of freedom

The problem can be formulated as: determine $\boldsymbol{\rho}^* \in R^N$, being $\mu_1, \mu_2, \mu_3 \in R^N$ and $\omega_1, \omega_2, \omega_3 \in R$, such that:

$$\boldsymbol{\rho}^* = \arg\min \mathcal{L}_A(\,\boldsymbol{\rho}, \boldsymbol{\mu}; \boldsymbol{\omega}\,),\ \ \forall\, \boldsymbol{\rho} \in \mathbf{X} \tag{19}$$

2.5 Optimization problem's solution

In spite of stability, surface and boundary constraints, the optimization problem was solved as unconstrained, by using Augmented with Exterior Penalty; besides, side constrained solver with Spectral Projected Gradient (Birgin and Martínez, 2002) was employed.

3. RESULTS AND DISCUSSION

3.1 Material properties:

Material properties for the 2 examples are presented in Tab. 1:

Table 1. Material properties.

Description	Value
Elasticity Modulus (MPa)	193
Yielding Stress (MPa)	207
Shear Yielding Stress (MPa)	137,9
Poisson's Ratio	0.29
Thermal Conductivity (*W/m²C*)	16,6
Coefficient of Thermal Expansion (C^{-1})	$17{,}0 \times 10^{-6}$

3.2 Problem 1: Structure under top load

In this problem, the tested structure was a fixed-ends beam, 10m x 5m, discretized with a 3200-element mesh (Fig. 1). The mechanical load was $q = 207{,}0\ x10^{-6}\ N/m$, down, distributed over a 1m domain at top face. The thermal data considered was: prescribed temperature at left and right ends, $T = 100\ ^oC$; initial temperature of solid's surface under free convection, $T = 20\ ^oC$; fluid temperature, $T_\infty = 20\ ^oC$.

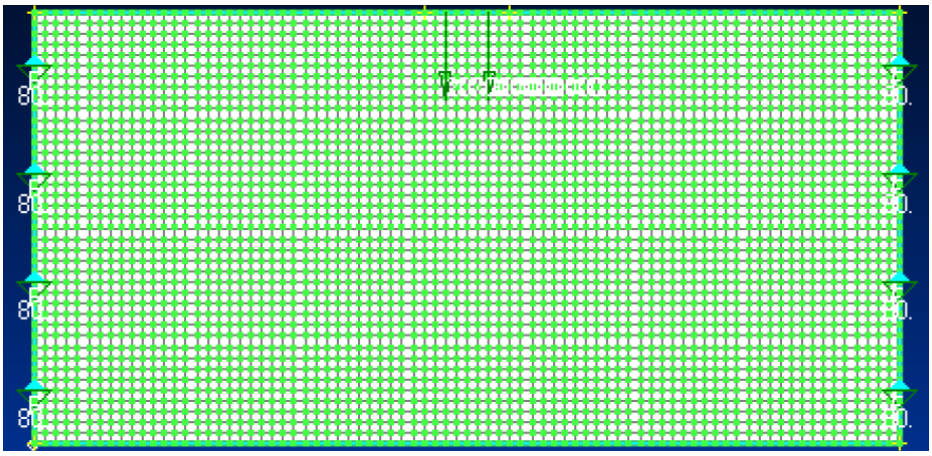

Figure 1. Fixed-ends beam under top distributed load

Results for tested structure's topology are shown in Fig. 2:

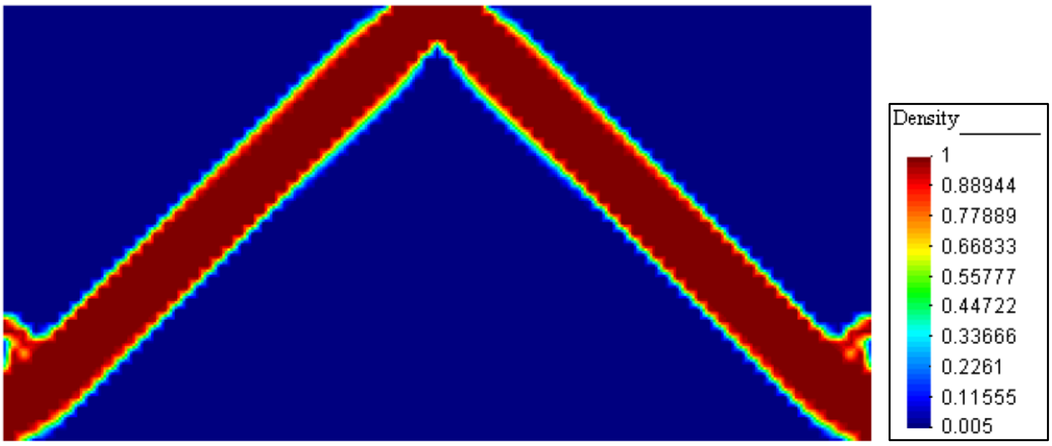

Figure 2. Topology of tested structure

Optimum layout for the problem (Fig. 2) presents sharp boundary definition for material domain and is coherent with imposed load (Fig. 1).

3.3 Problem 2: Structure under lateral shear

The tested structure was a cantilever beam, fixed at left end, 10m x 5m, discretized with a 3200-element mesh (Fig. 3). The mechanical load was $v = 137,8\ x10^6\ N/m$, distributed over a 1m domain at the right end. The thermal data considered was: prescribed temperature at left end, $T = 100\ ^oC$; initial temperature of solid's surface under free convection, $T = 20\ ^oC$; fluid temperature, $T_\infty = 20\ ^oC$.

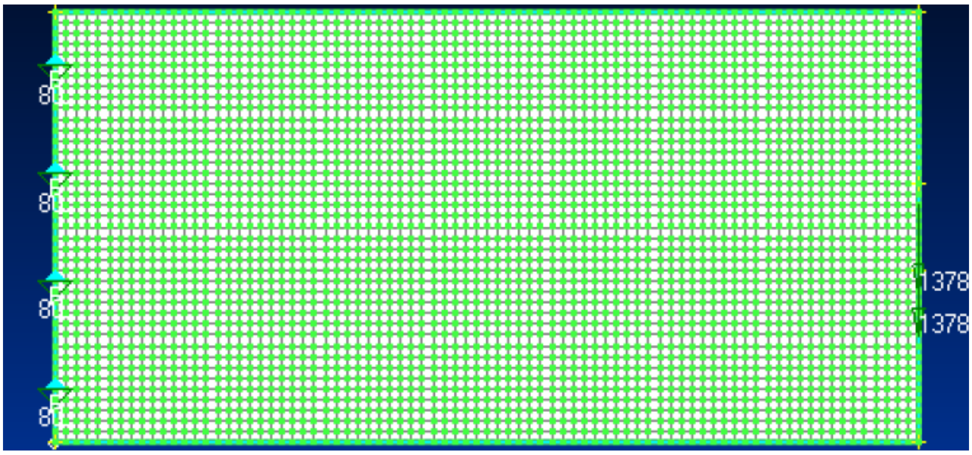

Figure 3. Fixed-ends beam under top distributed load

Results for tested structure's topology are shown in Fig. 4:

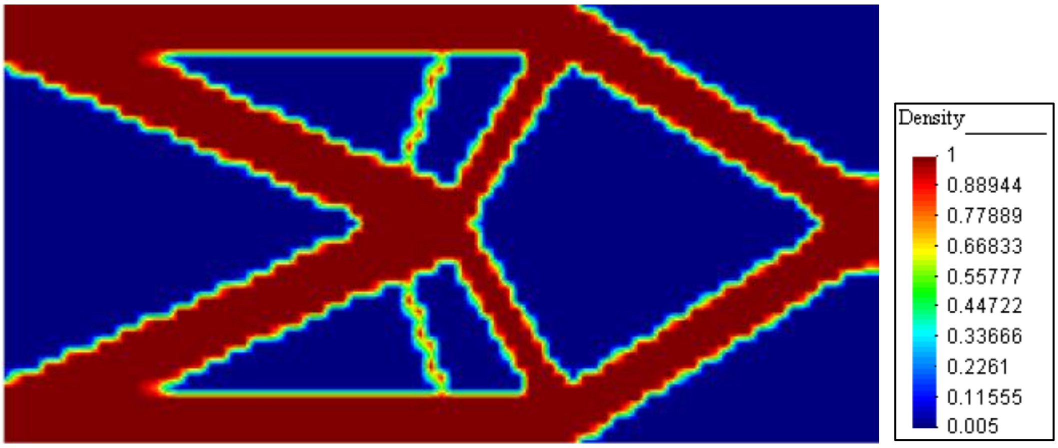

Figure 4. Topology of tested structure

Optimum layout for the problem (Fig. 4) is coherent with imposed load (Fig. 3) and presents sharp boundary definition for material domain - as noticed in Problem 1.

4. CONCLUSIONS

Results for the tested problems were consistent, with sharp topologies and minimum mass layouts to resist mechanical and thermal loads, indicating suitability of the employed formulation. The use of a unique state equation to describe both mechanical and thermal fields improved the topology design process, reducing computational cost.

Other results must be carried out varying support conditions, number of mesh elements as well as loads in order to verify formulations' consistency.

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