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STUDY OF LAMINAR-TURBULENT TRANSITION ON A FLOW UNDER A FLAT PLATE USING A CFD TOOL

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Abstract. *RANS models assume that the flow is completely turbulent and in some flows the laminar region is so small that it becomes a valid consideration. However, in some specific cases the laminar region may correspond to up to 30 % of the flow, as in airfoils. To identify this laminar region and the transition process, have been developed transition models that interact with the RANS turbulence models. In this work the transition model SST-Gamma-Theta is used to identify the laminar-turbulent transition process on a flat plate and the results are compared with those obtained experimentally in the ERCOFTAC T3AM test, as well as those obtained using the RANS SST common model.*

Keywords: *turbulence model, transition, flat plate, boundary layer, skin friction.*

1. INTRODUCTION

Turbulence is an extremely complex modelling phenomenon, because it is a very diffusive, dissipative, rotational and three-dimensional phenomenon. Nowadays, there are computational techniques that solve such flows with excellent accuracy, such as DNS (Direct Numerical Simulation) and LES (Large Eddy Simulation) techniques. Unfortunately, these methods are far too costly for engineering applications. In a competitive and global market the engineer's interest is mostly focused on obtaining average values of the variable of interest, such as head loss, heat transfer rates, drag, lift, spending the minimum as possible of financial resources and time. So, techniques with a medium approach have been the most used in computational packages, the so-called RANS models (Reynolds-Averaged Navier-Stokes). However, as pointed earlier, these models assume that the flow is completely turbulent. The SST-Gamma-Theta model, also developed for a model that is best applied in turbines, where transition is important. In the present work, computational simulations were done using the two types of RANS model, the SST-Gamma-Theta and the regular SST. The coefficient of friction results were compared with the ERCOFTAC T3AM flat plate test data, aiming to evaluate the efficiency of the processes in identifying the laminar-turbulent transition process.

2. TRANSITION MODEL FORMULATION

It was sought an approach that would be able to bring to the modern CFD code the use of empirical correlations allied to a RANS model. Looking for a model that were best applied in turbines, where the transition is very important, was developed the Gamma-Theta ($\gamma - \theta$) model, also known as Gamma-Re.

The Gamma-Theta model is called semi-empirical because it is not intended to physically model the transition. The whole physical part of the process is left to the empirical correlations. This feature in the model allows a calibrated prediction, independent of the applied turbulence model, of the start point of the transition and the ability to capture several transition modes by simply adding correlations.

According to Oliveira Jr (2014), these correlations generally provide values of Re_{xt}, $Re_{\theta t}$, Re_{xT} and $Re_{\theta T}$, where Re_x is the Reynolds number based on position x along the surface and Re_θ based on the momentum thickness given by Equations 1 and 2, respectively. The subscript "t" indicates the point where the transition actually begins, and the subscript "T" indicates the point where the boundary layer effectively completes the transition process.

$$Re_x = \frac{\rho U x}{\mu} \tag{1}$$

$$Re_\theta = \frac{\rho U \theta}{\mu} \tag{2}$$

Where: ρ is density of the fluid, μ dynamic viscosity, U velocity of reference and θ is the momentum thickness given by Equation 3.

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \approx \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \tag{3}$$

Being δ the boundary layer thickness.

The main experimental correlation used in the development of the Gamma-Theta model was that of Abu-Ghannam and Shaw (1980). An extensive study on the transition of the boundary layer was made in this literature, arriving at the important conclusion that the history of the flow of the laminar boundary layer significantly changes the value of local Re_θ (Reynolds of transition based on the momentum thickness). In this way, correlations have been developed to obtain $Re_{\theta t}$ and $Re_{\theta T}$, given by Equations 4 and 5.

$$Re_{\theta t} = 163 + exp\left[F(\lambda_\theta) - \frac{F(\lambda_\theta)}{6.91} Tu\right] \tag{4}$$

$$F(\lambda_\theta) = \{ {}^{6.91+12.75\lambda_\theta+63.64\lambda_\theta{}^2 \; para \; \lambda_\theta<0}_{6.91+2.48\lambda_\theta-12.27\lambda_\theta{}^2 \; para \; \lambda_\theta\geq 0} \tag{5}$$

Where λ_θ is the coefficient of pressure gradient in the free stream, given by Equation 6.

$$\lambda_\theta = \frac{\theta^2}{\nu}\frac{dU}{dx} \tag{6}$$

The vast majority of the correlations to predict the point at which the boundary layer transition begins provides this criterion in the form of a value of $Re_{\theta c}$. The calculation of this parameter is a non-local operation, involving the integral shown in Equation 3, which makes its application in modern CFD codes very difficult. To solve this problem, the Gamma-Theta model uses correlations of $Re_{\theta c}$ with locally calculated Reynolds.

One way to calculate the Reynolds locally is based on the local shear rate of the flow or vorticity, given by Equation 7.

$$Re_v = \frac{\rho y^2}{\mu}\left|\frac{du}{dy}\right| = \frac{\rho y^2}{\mu}\overline{S} \tag{7}$$

Where $\overline{S}$ is the local magnitude of shear rate tensor and y the normal distance to the nearest wall.

The Figure 1 shows a graph of Re on the y-axis and Re_x on the x-axis. The first curve (shown with dash-dot) is the value of max (Re_v) along the x-direction (shown in its dimensionless Re_x). The second curve (with dotted line) is the distribution of the value of Re_θ at each x position. The third and most important curve of this graph is the distribution curve of the maximum Re_v value divided by the constant 2.193, also in its distribution along the x-direction. Note that the last two curves are coincident.

So there is a direct relationship between $Re\nu$ and $Re\theta$. With this relation is possible to write the Equation 8 and calculate a nonlocal variable through a local variable, making the application of these correlations possible in CFD codes.

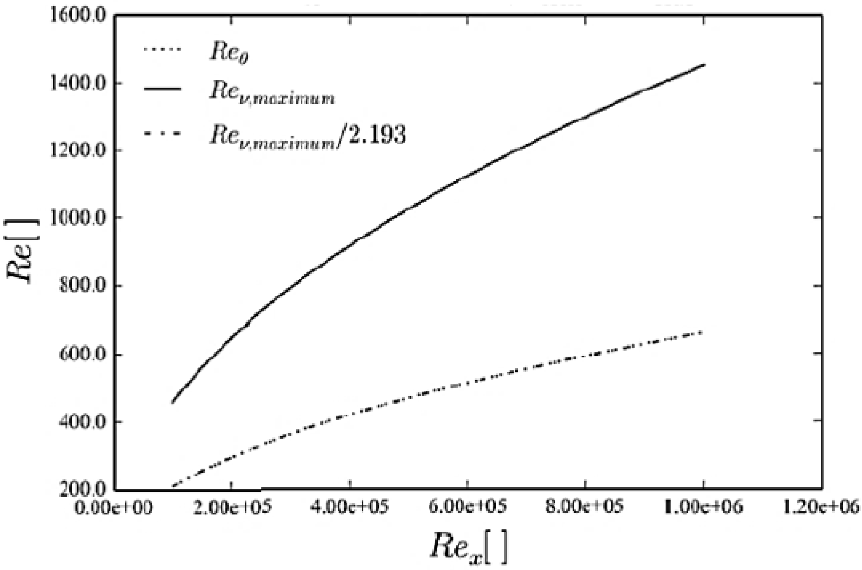

Figure 1. Relations between Reynolds number along the "x" direction of the plate. (Source: **?**)

$$Re_\theta = \frac{max(Re_v)}{2.193} \tag{8}$$

In this way the problem of calculating Re_θ has been solved, but there remain the non-local quantities T_u and λ_θ, as well as the implementation of the integration of the transition model with the turbulence models used in CFD packages. The solution adopted by the Gamma-Theta model was the use of two additional transport equations: one for intermittence (γ) and one for $Re_{\theta c}(\theta)$, from these two equations comes the name of the model.

–> Intermittency Equation

The intermittence factor, or simply intermittence, is defined in (Schlichting *et al.*, 1955) as the fraction of time in which the flow is turbulent. In regions of continuous laminar flow the intermittence is 0, for fully turbulent regions is equal to 1. In other words, intermittence 0 means that in a given region the flow is turbulent 0% of the time, whereas intermittence 1 means that the flow is turbulent 100% of the time.

The intermittence at a given point depends on the conditions of the flow around it and the history of the portion of the fluid under evaluation, so a transport equation is used for the solution of this variable, given by Equation 9.

$$\frac{\partial(\rho\gamma)}{\partial t} + \frac{\partial(\rho\overline{u_j}\gamma)}{\partial x_j} = P_\gamma - E_\gamma + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_T}{\sigma_\gamma}\right)\frac{\partial\gamma}{\partial x_j}\right] \tag{9}$$

The first and second terms are the local variation and the advective transport of γ; P_γ is the production term of γ; E_γ is the destruction term of γ and the last term of the equation is molecular and turbulent diffusion of γ.

P_γ is the production of intermittence term and E_γ is the intermittence destruction term, given by Equations 10 and 11, respectively.

$$P_\gamma = F_{length} c_{\alpha 1} \rho S \sqrt{\gamma F_{onset}} (1 - c_{e1}\gamma) \tag{10}$$

$$E_\gamma = c_{\alpha 2} \rho \Omega \gamma F_{turbulent} (c_{e2}\gamma - 1) \tag{11}$$

Where: $\overline{S}$ is the local magnitude of the shear rate tensor, Ω is the vorticity magnitude.

According to (Oliveira Jr, 2014), the dimensionless function F_{onset} locally evaluates whether the criterion for the beginning of the transition has been reached or not. If not, this function remains zero and no production of γ occurs. If so, this function quickly goes to 1 and the production term is activated. The function F_{onset} and her terms are shown in Equations 12 to 16.

$$F_{onset} = max(F_{onset2} - F_{onset3}; 0) \tag{12}$$

$$F_{onset1} = \frac{Re_v}{2.139 Re_{\theta c}} \tag{13}$$

$$F_{onset2} = min[max(F_{onset1}; F^4_{onset1}); 2] \tag{14}$$

$$F_{onset3} = max\left[1 - \left(\frac{R_T}{2.5}\right)^3; 0\right] \tag{15}$$

$$R_T = \frac{\rho k}{\mu w} \tag{16}$$

As can be seen from Equation 13, the correlation between Re_v and Re_θ is used in the equation of the function F_{onset}. Another dimensionless function of great importance present in the term of production is F_{length}, which controls the magnitude of the term of production. Physically, this function controls the length of the transition zone. In the $\gamma - \theta$ model this function was obtained through a curve fit using the comparison of simulations with experimental transition results on a flat plate.

The dimensionless function $F_{turbulent}$, given by Equation 17, activates the term of destruction in laminar boundary layers and cancels this term outside the boundary layer ($\mu_T \gg \mu$).

$$F_{turbulent} = e^{-(\frac{Rt}{4})^2} \tag{17}$$

The constants used in the terms of production and destruction of the transport equation of γ are: $C_{a1} = 2$; $C_{a2} = 0.06$; $C_{e1} = 1$; $C_{e2} = 50$; $\sigma_\gamma = 1$

–> $\tilde{Re}_{\theta t}$ Equation

According to Oliveira Jr (2014), one of the main difficulties in implementing a transition model for CFD codes based on empirical correlations is the fact that information outside the boundary layer (Tu and λ_θ) influences the transition process.

This transfer from the free stream to the boundary layer is a non-local operation solved in the Gamma-Theta model through use of an additional transport equation. This equation is the transport equation for the Reynolds number based on the thickness of the momentum that marks the effective start of the $\tilde{Re}_{\theta t}$ transition, given by Equation 18.

$$\frac{\partial(\rho \tilde{Re}_{\theta t})}{\partial t} + \frac{\partial(\rho \overline{u_j} \tilde{Re}_{\theta t})}{\partial x_j} = P_{\theta t} + \frac{\partial}{\partial x_j}\left[\sigma_{\theta t}(\mu + \mu_T)\frac{\partial \tilde{Re}_{\theta t}}{\partial x_j}\right] \tag{18}$$

This equation calculates the value of $\tilde{Re}_{\theta t}$ outside the boundary layer, based on the pressure gradient values, turbulent intensity of the free stream, and transports this information to the boundary layer through its diffusion term.

$P_{\theta t}$ is the production term, given by Equation 19.

$$P_{\theta t} = c_{\theta t}\frac{\rho}{\tau}(Re_{\theta t} - \tilde{Re}_{\theta t})(1 - F_{\theta t}) \tag{19}$$

The first part of Equation 19 is the ratio of the density ρ to a time scale τ. This time scale is defined by dimensional analysis and aims to make the production term of the same order as the advective and diffusive terms of the transport equation of $\tilde{Re}_{\theta t}$. The time scale τ is given by Equation 20.

$$\tau = \frac{500\mu}{\rho U^2} \tag{20}$$

The second part of the production term is the difference between the $Re_{\theta t}$ calculated from the correlations and the $\tilde{Re}_{\theta t}$ effectively transported

The third part of the term is the function $F_{\theta t}$, which disables the production term inside the boundary layer, so that only the diffused $\tilde{Re}_{\theta t}$ information of the free stream arrives. $F_{\theta t}$ is given by Equation 21, is zero in the free current and goes to 1 in the boundary layer.

$$F_{\theta t} = min\{max\left[F_{wake}e^{-\left(\frac{y}{\delta_{wake}}\right)^4}; 1 - \left(\frac{y - \frac{1}{c_{e2}}}{1 - \frac{1}{c_{e2}}}\right)^2\right]; 1\} \tag{21}$$

Where F_{wake} serves to search for regions of high turbulent intensity (wakes, for example) and keep the function inactive in these zones.

The constants of these equations are: $c_{\theta t} = 0.03; \sigma_{\theta t} = 2$

3. COMPUTATIONAL PROCEDURE

The geometry and domain data used for numerical simulations were based on Langtry and Menter (2009). It is a flat plate of 4000mm long and 100mm wide, it has a slightly rounded leading edge, presenting a radius of 0.75mm. The domain has a height of 220mm and a total length of 4.0035m.

To generate the geometry a CAD software, SolidWorks, was used. Figure 2 and 3 shows details of the geometry used in this work.

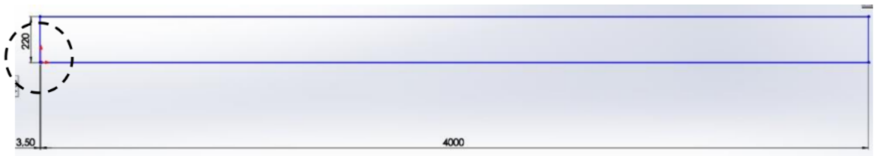

Figure 2. Geometry used. (Source: Authors' Own)

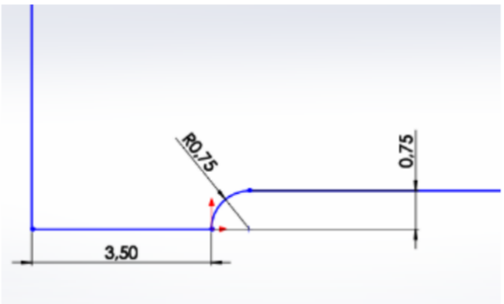

Figure 3. Highlighted details of the geometry in Figure 1. (Source: Authors' Own)

To generate a good mesh, to obtain coherent results, the ANSYS ICEM CFD software was used.

The SST-Gamma-Theta model requires a maximum y^+ of 1, but to ensure good mesh quality, the maximum y^+ used was 0.5. In addition, special care was taken with the leading edge of the plate. The generated Mesh was composed of 250044 elements and 502492 nodes. Figures 4, 5 and 6 illustrate the mesh generated.

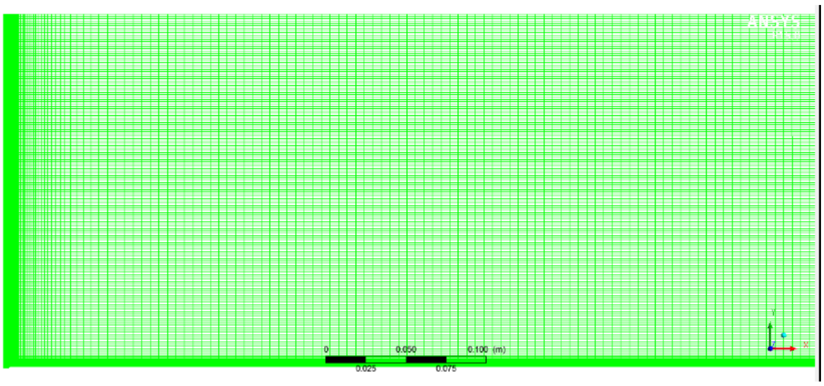

Figure 4. Mesh used in simulations. (Source: Authors' Own)

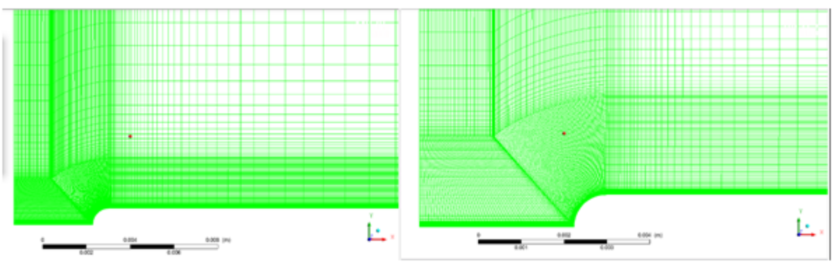

Figure 5. Detail of the mesh on the leading edge of the board. (Source: Authors' Own)

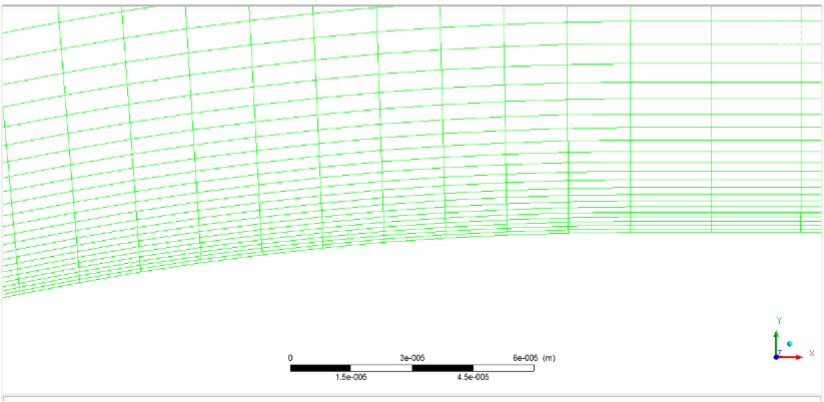

Figure 6. Refining near the leading edge. (Source: Authors' Own)

After obtaining a satisfactory mesh, it was possible to pass to numerical simulation in ANSYS CFX 15.0 software. As boundary conditions were set the "inlet" of the domain, the "outlet", and the "wall" (corresponding to the flat plate), and how a 2D simulation was desired. "symmetry" was used between the walls of the domain. In addition, the wall preceding the leading edge of the plate has been set as "free slip" wall.

The boundary conditions for the T3AM case are: velocity inlet of 19.8m/s, relative pressure outlet of 0 atm, turbulent intensity, in the inlet of the domain, of 0.874%. The fluid used was Ideal Gas Air, density of 1.2 kg/m^3 and dynamic viscosity of 1.8x$10^{-5}kg/ms$. A RMS of 10^{-6} was defined as the converge criterion.

4. RESULTS AND DISCUSSION

Through the ANSYS-CFD POST software the coefficient of friction was calculated along the length of the plate. Such coefficient can be calculated from Equation 7.

$$C_f = \frac{\tau_\omega}{\frac{1}{2}\rho {U_\infty}^2} \tag{22}$$

where τ_ω is the shear stress in the wall, ρ is the density of the fluid and U_∞ is the initial velocity of the flow. These values allow to see where the transition will occur. After the necessary adjustments, we reached the values of C_f as a function of the plate length, given dimensionless through the local Reynolds Rex. These results are shown in Figure 7.

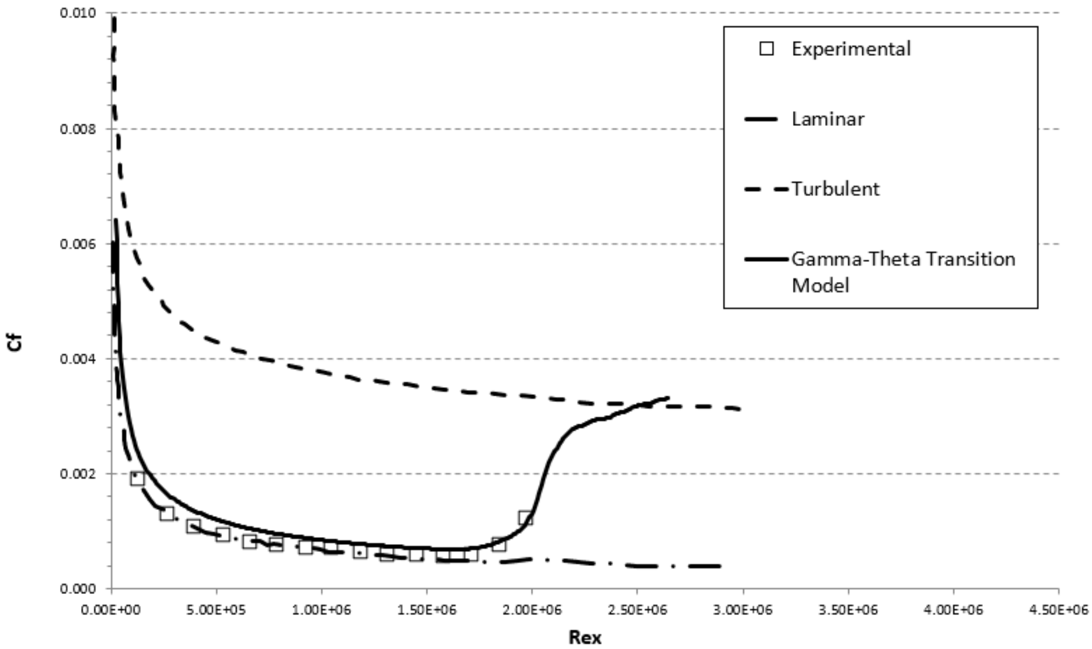

Figure 7. Results obtained with SST-Gamma-Theta model. (Source: Authors' Own)

As can be seen, the model could accurately capture the laminar region and the transition region. To make a comparison with the common SST turbulence model, a new comparison of results was performed and is presented in Figure 8.

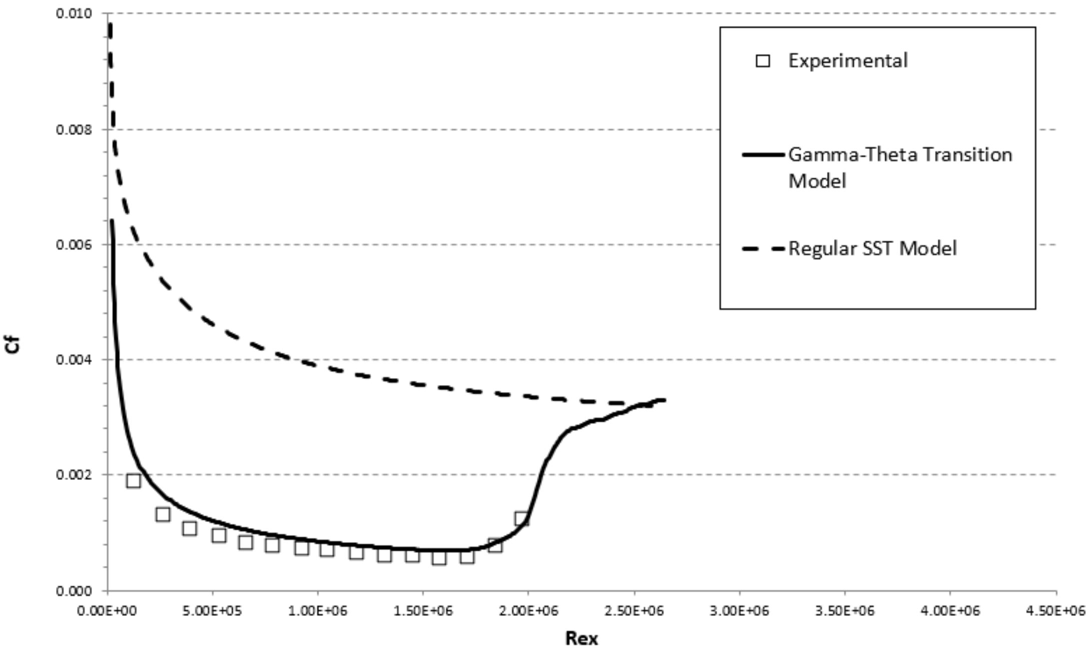

Figure 8. Comparison of results between common SST model and SST Gamma-Theta model.(Source: Authors' Own)

Through this comparison, the efficiency of the Gamma-Theta transition model becomes even clearer, since the common SST model was unable to capture the laminar region.

5. CONCLUSIONS

It is possible to notice that in the laminar region there was a samall difference in relation to the experimental curve, but the model was able to predict with excelent precision the location of the laminar-turbulent trasition. The analyzed model still needs correlations and treament for other types of trasition, such as cross-flow transition. But when good correlations are developed in the literature, they can be implemented in the model and calibrated without detrimental effects to the already implemented correlations and to the base turbulence model.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

Abu-Ghannam, B. and Shaw, R., 1980. "Natural transition of boundary layers-the effects of turbulence, pressure gradient, and flow history". *Journal of Mechanical Engineering Science*.

Langtry, R.B. and Menter, F.R., 2009. "Correlation-based transition modeling for unstructured parallelized computational fluid dynamics codes". *AIAA journal*.

Oliveira Jr, J.A.A., 2014. *Modelagem e Simulação RANS de Escoamentos com Transição Laminar-Turbulento*, Associação Brasileira de Ciências Mecânicas.

Schlichting, H., Gersten, K., Krause, E. and Oertel, H., 1955. *Boundary-layer theory*, Vol. 7. Springer.

8. RESPONSIBILITY NOTICE