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PREDICTION OF THE CRACK PATTERN IN CERAMIC COMPOSITES SUBJECTED TO TEMPERATURE VARIATION USING LATTICE MODELS

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Abstract. *Despite its brittle mechanical behaviour, ceramic materials have excellent thermal properties for applications that demands a large temperature range, justifying the use of ceramic composites as an option for such applications. However, the interaction between the inclusion and the surrounding matrix can nucleate cracks due to the coefficients of thermal expansion mismatch. This article aims to simulate the failure of these materials under thermal loads using lattice models. The progressive failure is simulated by the introduction of a damage variable in the glass phase. Computational simulation were conducted for three volume fractions: 15%, 30% and 45% vol.. The evolution of the damaged area and the crack patterns are discussed for these three volume fractions. The damaged area for the lower volume fractions tends to stay in the inclusion neighbourhood. For the higher volume fraction, a connection between interfaces can be observed.*

Keywords: *ceramic composite, lattice models, crack pattern, coefficient of thermal expansion.*

1. INTRODUCTION

The development of new composite materials has been the solution for many engineering problems because of its higher effective properties. Despite these benefits, such materials are more complex to be mechanically analysed regarding its failure behaviour, which require the use of structural models to understand and predict the material response. Under thermal loads, ceramic composites can fail in reason of the mismatch of the thermal expansion coefficient (CTE) (Joliff *et al.*, 2007; Luchini *et al.*, 2016; Davidge and Green, 1968). The internal stress in the inter-facial, inter-inclusions or radial regions of the composite nucleate cracks in these materials. The cracks behaviour can also be affected by the volumetric fraction of the crystalline phase, the particles size and distribution, the Young's modulus of each phase (Davidge and Green, 1968), and even the Poisson coefficient mismatch of these phases (Le Roux *et al.*, 2013). Also, Luchini *et al.* (2016) shows that the local volume fractions can also interfere in the crack pattern. The combination of these characteristics can result in different crack patterns.

These materials mechanical behaviour can be investigated in three ways: experimentally, analytically or computationally. In the computational approach, there are several numerical methods available, among which the finite element method (FEM). Once determined the method, it must be chosen the modelling strategy, which in integrity analysis are mainly the continuous and discrete models (Martha, 2010). In the lattice models – one kind of discrete model –, the structure is represented by an assembly of bar or beam elements, therefore the crack propagation can be modelled by the successive failure of different bars (or beams). In this context, the goal of the present article is to investigate the damaged area and the crack pattern in ceramic composites subjected to thermal loadings using lattice models. The ceramics composites are investigated for three volume fractions: 15%, 30% and 45% vol.

2. NUMERICAL MODELLING USING LATTICE MODELS

Lattice models can be solved numerically using a finite element method framework as proposed by van Mier and Shi (2002); Schlangen and van Mier (1992b); Lilliu and van Mier (2003); Chang *et al.* (2002); Vassaux *et al.* (2015) and Vassaux *et al.* (2016). The lattice elements are a network of bar (or beam) elements which can be created from a triangular mesh generator. In the present study, beam elements were chosen due to its capability of withstanding non-axial loads.

The finite element method (FEM) is a technique in which a global approximation of the solution can be build from the contribution of the local approximation define in each finite element's domain (Proença, 2016). This approximation is called the solution's weak form and its development as the FEM applications and theoretical background can be found in the basic literature (Bathe, 2006; Zienkiewicz and Taylor, 1977; Fish and Belytschko, 2007). Based on the classical formulation, the displacement field $\mathbf{u}$ can be found solving the linear system:

$$\mathbf{K_g}.\mathbf{u} = \mathbf{f} \tag{1}$$

where $\mathbf{K_g}$ is the global stiffness matrix and $\mathbf{f}$ are the applied forces. In the context of lattice models, the overall structural stiffness, which is represented by the stiffness matrix, $\mathbf{K_g}$, is obtained by the contribution of the assembly of bars or beam elements (Schlangen and van Mier, 1992a; Nikolić *et al.*, 2017).

The stiffness matrix $\mathbf{K_g}$ is calculated as a function of the Young's modulus, the element size (length and area) and the section inertia. That said, is very important when creating the lattice model to adjust those parameters so the behaviour of the lattice model will be as close as possible to the displacement response of the continuous one. Occurring an failure event, the global stiffness matrix must be updated. The next section describes how the progressive failure has been taken into account.

2.1 Progressive failure

The lattice model has been created considering a material with brittle mechanical behaviour. This mechanical behaviour was simulated assuming that the glass phase has a linear elastic response up to a critical strain ε_{cr}, after which, its mechanical strength is reduced due to the evolution of the damage variable d. The stress vs. strain response of the alumina phase was considered to be linear elastic. The failure criterion of glass phase is written in terms of the strain, so it is considered that the material fails when $\varepsilon = \varepsilon_{cr}$. For $\varepsilon > \varepsilon_{cr}$, the Young's modulus is reduced and its value is computed as $(1-d)E_0$. The damage variable (d) evolution follows the law:

$$d(\varepsilon) = 1 - \frac{\varepsilon}{\varepsilon_{cr}} \exp[-\beta(\varepsilon - \varepsilon_{cr})] \tag{2}$$

where β defines the rate of damage growth with the strain. This law was implemented with the use of a USDFLD ABAQUS subroutine and the damage variable evolution can be seen in Figure 1 for $\beta = 800$ and $\epsilon_{cr} = 0.0006$, that were the parameters adopted in this study.

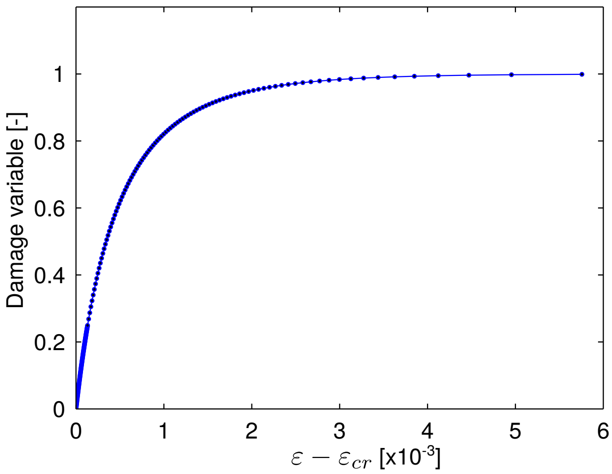
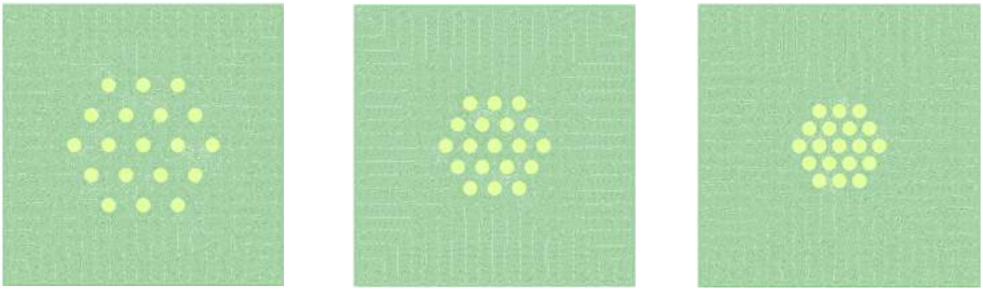


Figure 1. Evolution of the damage as a function of the strain with $\beta = 800$ and $\varepsilon_{cr} = 0.0006$. The exponential model have been adopted to the Young's modulus decay.

3. NUMERICAL EXAMPLES

As commented above, in this paper the crack patterns and crack density evolution in a material under thermal loading are studied. The material system consists on a two phase ceramic composite with three different volume fractions: 15, 30 and 45 % vol. (Figure 2). The alumina inclusions are embedded in a glass matrix. The system is initially exposed to 625 $°C$ (without residual stresses) and the temperature goes down linearly to 125 $°C$. There is no mechanical constrain applied, so the internal stresses are only due to the CTE mismatch between the phases. The alumina inclusions have a larger CTE than the glass matrix, so, in the event of failure, it is expected the nucleation of cracks at the interface.

The material parameters adopted in the simulation can be seen in Table 1, where the values of elastic and CTE constants were obtained from Joliff *et al.* (2007). The glass strength under tension was adopted by the authors to simulate the progressive failure of the system and the Young's modulus of the alumina is the average value because in fact it varies with the temperature according to (Joliff *et al.*, 2007). Also, the authors are considering that the interface strength is equal to the glass strength. The evolution of the damage variable can be seen in Figure 1.



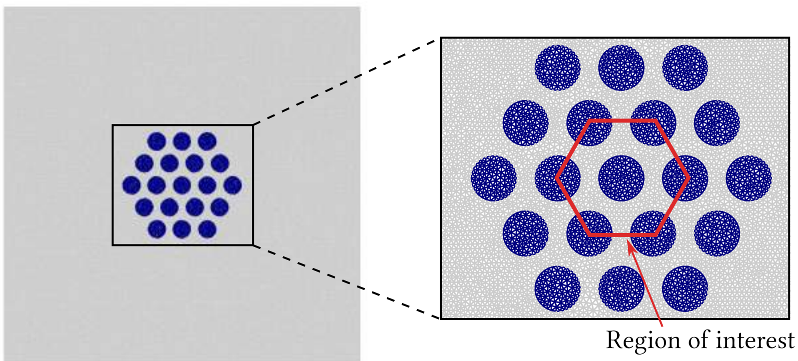
(a) (b) (c)

Figure 2. Materials system investigated: (a) 15% vol.; (b) 30 % vol. and (c) 45 % vol..

Table 1. Material parameters.

Material	Parameter	Value
Glass	Young's modulus	68 GPa
	Poisson ratio	0.2
	Coefficient of thermal expansion (CTE)	4.6 E-6
	Ultimate tensile strength	45 MPa
Alumina	Young's modulus	320 GPa
	Poisson ratio	0.24
	Coefficient of thermal expansion (CTE)	7.6 E-6

The finite element models were created in Abaqus using B23 linear beam elements. The lattice structures were created from a triangular mesh, where the triangles edges originated the beams of Figure 3. The region inside the red hexagon will be used to investigate the failure around the inclusion located at the centre.



(a) (b)

Figure 3. Finite element mesh for 45 % vol. fraction model (≈ B23 beam elements).

4. RESULTS

The analysis using the lattice models presented in Figure 2 enable to evaluate the glass damage fraction evolution with the temperature. The damaged fraction, $\mathcal{D}$, have been computed as the ratio between the sum of damage variable in the region of interest by the total number of integration points, i.e.:

$$\mathcal{D} = \frac{1}{N}\sum_{i=1}^{N} d_i \qquad (3)$$

The results for the damage evolution for the three investigated volume fractions are shown in Figure 4. Equation 3 was applied for the region of interest indicated in Figure 3. It is important to highlight that the total damage (eq. 3) were computed considering only the glass elements, so $\mathcal{D} = 1$ reflects the complete failure of the matrix. It can be seen in Figure 4 that the three volume fractions, 15%, 30% and 45% vol., tends to $\mathcal{D} = 0.2$, 0.5 and 0.9, respectively. For lower volume fractions the damaged region concentrates around the inclusions and these regions are isolated, without any kind of interaction among them. For the higher volume fractions, the proximity between the inclusions induces a connection between the damaged regions. These aspects can be observed in the crack patterns for these three cases.

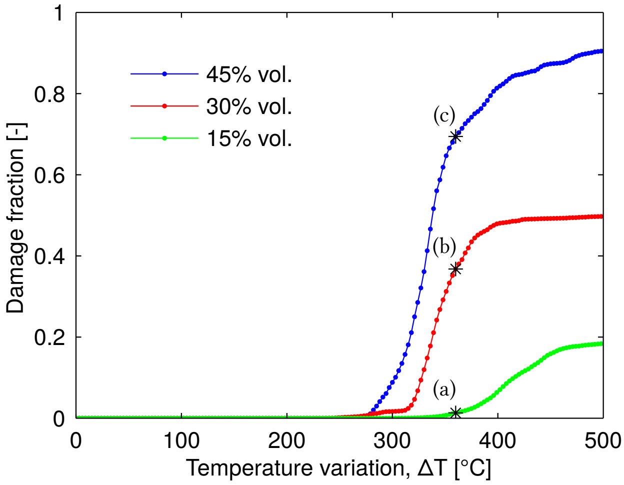
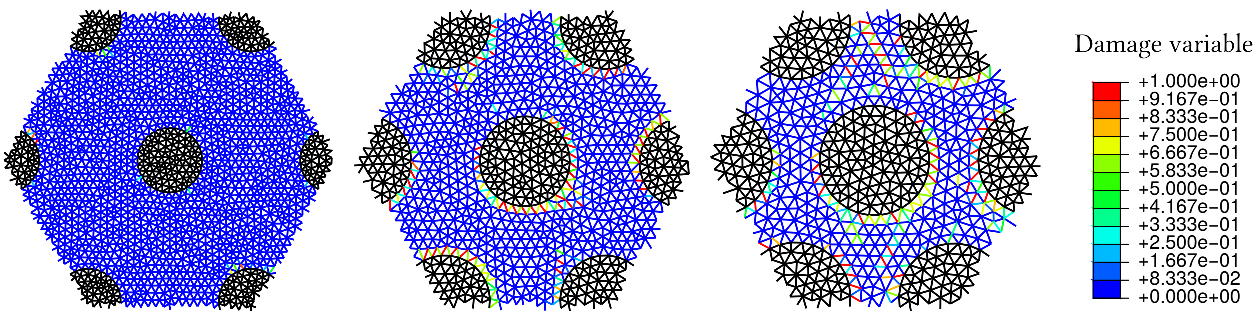


Figure 4. Damage fraction for different volume fractions.

The crack patterns are shown in terms of the damage variable d in Figure 5. The snapshots have been taken for the temperature of 265 °C, as indicated in Figure 4. The beams with damage variable closes to the unity has an Young's modulus approximately 1% of the initial one. For the case (c), 45% vol., there is a connection between the interface of the middle inclusion with the interfaces of the surrounding ones. The removal of the beam elements with $d \to 1$ can leads to numerical instabilities once the inclusion can be completely disconnected of the surrounding matrix. The complete failure of the inclusion neighbourhood (without a crack path between inclusions) can leads to the saturation of the damaged areas, but without a complete structural collapse since there is no unconnected region of the matrix. For the higher volume fraction, a crack path between inclusions leads to unconnected glass regions, so to the complete structural collapse.



15% vol.
(a)

30% vol.
(b)

45% vol.
(c)

Figure 5. Crack patterns observed for the different models.

5. CONCLUSION

Lattice models can be applied to simulate the mechanical behaviour of a ceramic composite systems under temperature variation. The study shows that in function of the volume fraction there is a different maximum of damage fraction related. Then, for real projects it can be use to evaluate the correct volume fraction to be used, regarding for example the mechanical response and the thermal range in which the component produced must operate. There are still questions to be investigated, like the implementation of a system with a random distribution of different local volume fractions, and the implementation of a method similar to the REV (Representative elementary volume) as done by Fasching *et al.* (2015), to simulate several identical cells of the model to turn the edges of the model almost non-existing.

6. ACKNOWLEDGEMENTS

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