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MODELING OF AN IMPACT DAMPER SUBMITTED TO FORCED VIBRATION FROM A NONLINEAR CONTACT FORCE MODEL

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Abstract. *Vibrations in mechanical systems may be undesirable and there are different ways of controlling the vibratory amplitude. Impact dampers are a passive device that utilizes an auxiliary mass(es) - particle(s) - with free movement to collide against the structure to be damped. Momentum transfer from the structure to the particle(s) is the responsible for dissipating part of the energy. Recent numerical and experimental studies have proven the efficiency of this damper. They analyse the influence of the particle mass and the clearance, necessary to the occurrence of impacts. In this work, a nonlinear modeling, based on Hertz theory, is investigated by comparing simulated and experimental responses. Results presented good agreement between simulated and experimental curves.*

Keywords: *Vibration, Impact Damper, Absorber, Damping*

1. INTRODUCTION

Paget (1930) apud Masri (1969) was pioneer in the impact damper study and posteriorly a lot of publications have explored the contact dynamics to minimize vibrational energy. Impact dampers are devices constituted of one or various solid particles that move freely in a certain gap and collide against to the structure that present the undesirable vibrations.

The advantages of using the impact absorber compared to other devices are: low cost, simplicity, robustness and promote damping in a range of accelerations and frequencies (Duncan *et al.*, 2005). Furthermore, the operation of this type of damper does not depend on temperature and it is therefore used in applications where traditional absorbers may fail (Wong *et al.*, 2009).

The capacity of energy dissipation by impact damper is directly related to amplitude force, vibration frequency, particle and structure masses, structural stiffness and damping, clearance (gap), natural frequency of the system, initial displacement, gravitational acceleration and coefficient of restitution (Duncan *et al.* (2005) and Yasuda and Toyoda (1978)).

Mass ratio (ratio between the masses of the particle(s) and the structure) and clearance (gap), necessary to impact occurrence, are two of the most important parameters to provide a good performance to the absorber. High value of particle mass produces higher damping, as indicated by Bapat and Sankar (1985), Friend and Kinra (2000) and Marhadi and Kinra (2005). However, care must be taken because the particle mass alters the original structural natural frequency and can impair the damping below this frequency. Popplewell *et al.* (1983) found in their forced vibration experiments, with a single particle, that excitation frequencies at or slightly above the structural natural frequency produced positive damping, while in frequencies below than natural frequency the structure vibrates more than without damper. In relation to the gap, Bapat and Sankar (1985) shows the variation of the "damping inclination" (ratio between the difference of two consecutive peaks by their respective times of occurrence) with respect to the gap. In this case, larger gaps were analysed and presented low damping. This effect is due to the few impacts that occur at the walls of the main mass because the gap is too large. Marhadi and Kinra (2005) also analysed the influence of clearance in system damping for 3 types of particle material, where it is possible to note the nonlinear tendency of this parameter for both single and multi particle damper. Li and Darby (2006) reported the same behavior of damping in relation to the gap for a single impact damper with a buffer between the structure and the sphere, concluding that the presence of buffer on contact increase the performance of absorber.

Contacts in impact dampers are modeled directly by the coefficient of restitution and the conservation of linear momentum as shown in Duncan *et al.* (2005) and Gharib and Ghani (2013), or using a linear set spring-damper, as in Cheng and Xu (2006) and Li and Darby (2009). The stiffness and damping parameters can be estimated from the coefficient of restitution and contact time between the bodies, as shown by Nagurka and Huang (2004) and Li and Darby (2009). Recently, new models have been using nonlinear spring with a viscous damper as in Afsharfard and Farshidianfar (2012) and (Afsharfard, 2016). Afsharfard and Farshidianfar (2013) presented in their investigation a new formulation for the

Hertzian contact force, named cubic contact force.

In this work, we consider the spring and damper as nonlinear elements. Mathematical model was idealized containing a single impact mass (particle) which moves freely without friction in a predetermined gap. The impact was modeled considering a nonlinear approach of contact force, based on Hertzian formulation and introducing the damping model proposed by Flores *et al.* (2011).

In order to validate the mathematical model, experimental tests forced vibration were performed and the responses extracted from these tests were compared with the simulated responses obtained by the equations of motion. Forced vibration were carried out from harmonic base excitation and the Displacement Transmissibility responses were analysed in a range of frequencies. It is intended to analyse the simulation and experimental responses for different gaps and frequencies other than the natural frequency.

2. MATHEMATICAL MODELING

A two degrees of freedom (DoF) system, Fig.1, was idealized to simulate an impact damper submitted to forced vibration via harmonic base excitation. The system is constituted by a principal mass m_1 with stiffness k_1 and viscous damping c_1, where a single metallic sphere moves freely without friction between the barriers. When contact occurs the nonlinear model Eq.(1) is applied, then spring and damper forces act providing a momentum transfer between the masses. When no contact occurs, it was considered that the particle movements with constant velocity, i.e. without friction.

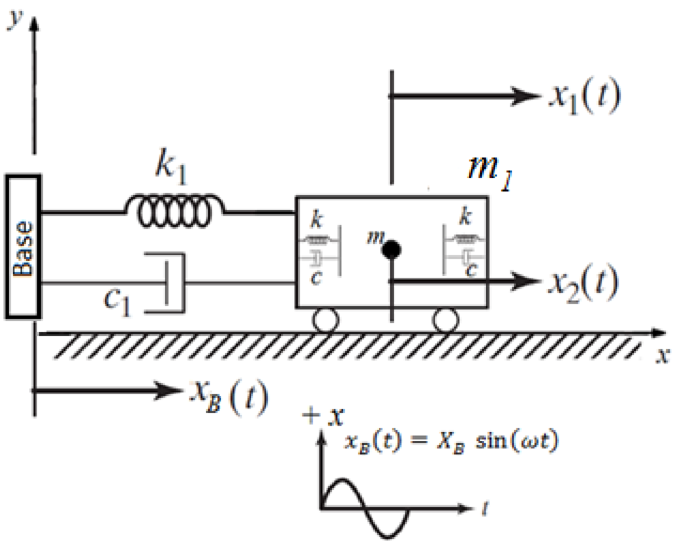


Figure 1: Two DoF System

The sphere-barrier contact force is modeled by a set of nonlinear spring-damper, based on Hertz theory and proposed by Hunt and Crossley (1975), conforming Eq.(1), where k is the contact stiffness, c is called hysteresis damping factor, δ is relative position and $\dot{\delta}$ is relative velocity.

$$F_N = k\delta^{1.5} + c\dot{\delta}\delta^{1.5} \tag{1}$$

The equations of motion contain the contact force and the harmonic base movement, as follows:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} + [\Lambda]\{\delta^{1.5}\} + [\Upsilon]\{\dot{\delta}\delta^{1.5}\} = \{F(t)\} \tag{2}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} k & k \\ -k & -k \end{bmatrix} \begin{Bmatrix} \delta_l^{1.5} \\ \delta_r^{1.5} \end{Bmatrix} + \\ + \begin{bmatrix} c & c \\ -c & -c \end{bmatrix} \begin{Bmatrix} \dot{\delta}_l\delta_l^{1.5} \\ \dot{\delta}_l\delta_r^{1.5} \end{Bmatrix} = \begin{Bmatrix} k_1(X_B sin(\omega t)) + c_1(\omega X_B cos(\omega t)) \\ 0 \end{Bmatrix} \tag{3}$$

where: subscripts l and r indicate left and right wall, respectively. In this case, k and c will only assume value if the contact exists in respective wall.

The contact stiffness, based on Hertz theory, is related to material properties of contact bodies (E_i and υ_i) and the surface curvature radius (R_i). According to Goldsmith (1960) apud Flores *et al.* (2005) the contact stiffness of a sphere-plane contact, Eq.(4), is written as follows:

$$k = \frac{4(R_1)^{0.5}}{3(\frac{1-\upsilon_1{}^2}{E_1} + \frac{1-\upsilon_2{}^2}{E_2})} \tag{4}$$

where R_1 is the radius of sphere, υ_i is the Poisson ratio and E_i is the Young modulus.

The hysteresis damping factor c is related to contact stiffness k, initial impact velocity $\dot{\delta}^{(-)}$ and a constant α that varies from each model. Flores *et al.* (2011) model was chosen to be applied in the simulations of this work and is given by:

$$c = \alpha \frac{k}{\dot{\delta}^{(-)}} = \frac{8(1-e)}{5e} \frac{k}{\dot{\delta}^{(-)}} \tag{5}$$

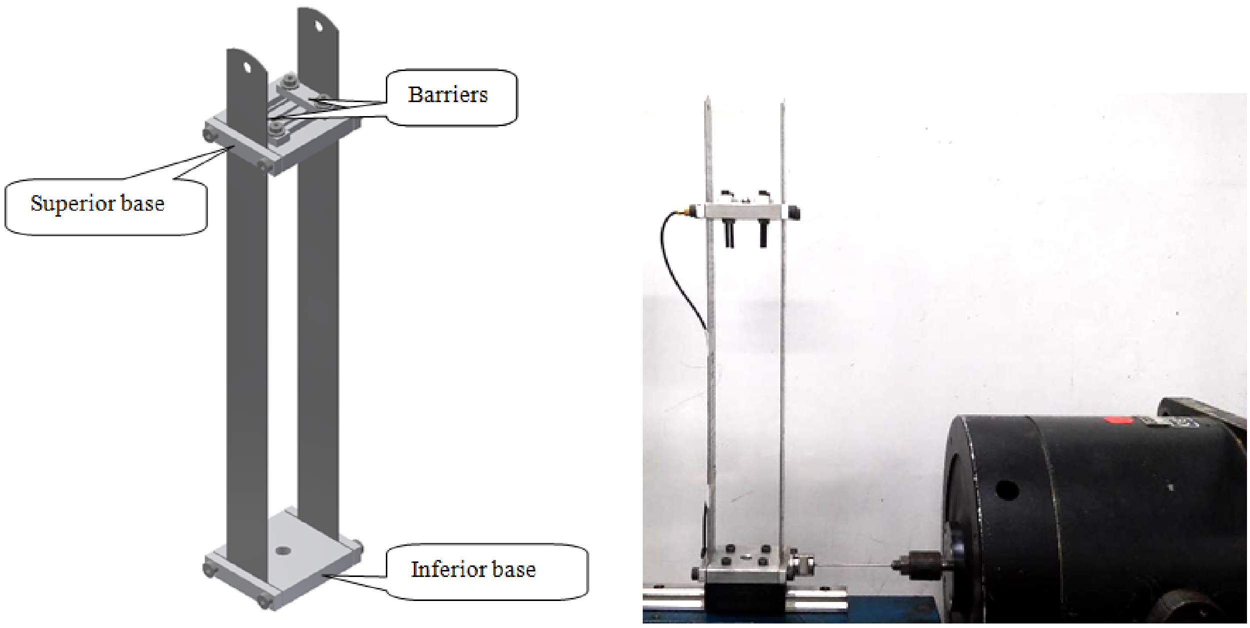
where e is the coefficient of restitution.

A low damping test rig was built to validate experimentally the mathematical model, containing the following basic elements: inferior base, columns (stiffness elements), superior base and adjustable barriers, Fig. 2a and 2b. Except the columns, all elements were made of aluminum. The gap is given by the difference between the distance of barriers and the diameter of the sphere, utilized as impact mass.

The instruments utilized to measure the acceleration and the displacements in test rig are: Endevco 226C accelerometer, Wenglor YP06MGVL80 laser displacement sensor (superior base) and Dornier IWA NT25 displacement sensor (inferior base) from a Bruel and Kjær Type 4808 electromagnetic shaker excitation, as illustrated in Fig. 3. All data were acquired from National Instruments NI USB 6521 acquisition board.

Accelerometer was attached to structure to detect the occurrence of impacts, as detailed in Fig.4. Equivalent stiffness and viscous damping coefficients were obtained from logarithmic decrement and natural frequency (9.1 Hz).

The coefficient of restitution (e) may be approximately estimated using a simple test of free fall, as performed by Aryaei *et al.* (2010). It is said approximately estimated because the coefficient of restitution is a complex parameter dependent of a lot of others parameters as impact velocity, material roughness, material dimensions, etc. The test consists to measure a rebound height H_1 after the sphere fall freely from an initial height H_0. The rebound height H_1 was measured using a scale, placed behind the ball plane, and the test was filmed using a video camera. Reproducing this test in laboratory, the coefficient of restitution between a 6.35 mm steel sphere and a 0.375 inches aluminum plate (thickness of barriers) was calculated from $e = \sqrt{H_1/H_0}$, resulting 0.35.



(a) (b)

Figure 2: Test rig.

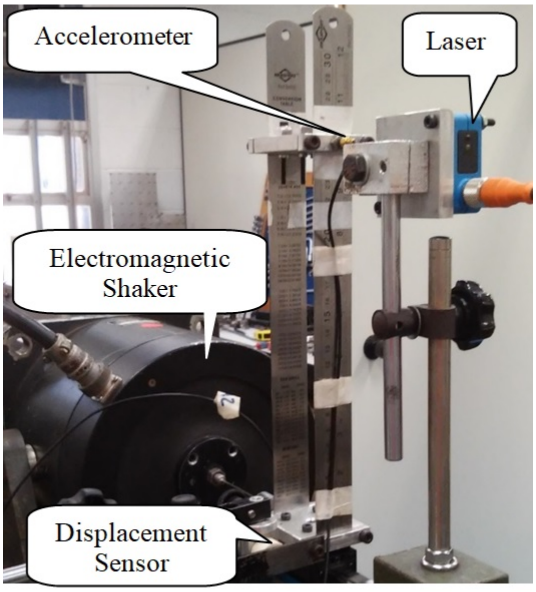


Figure 3: Test rig assembly and instrumentation.

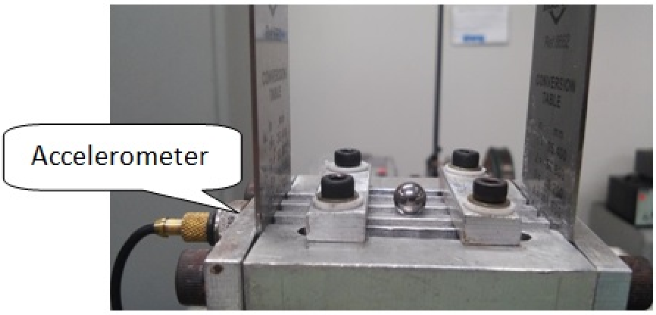


Figure 4: Sphere and accelerometer in the test rig.

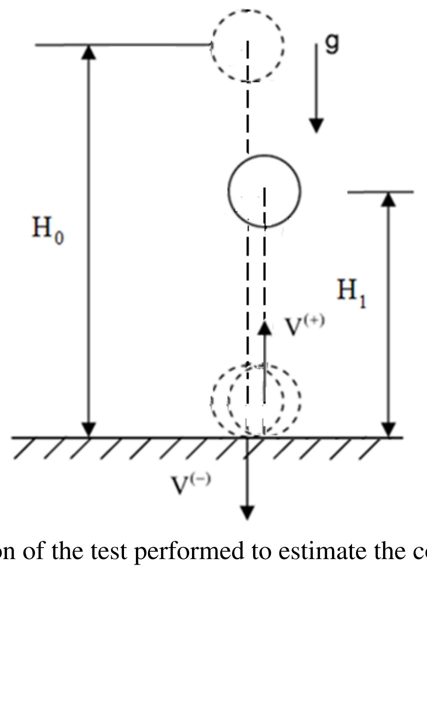


Figure 5: Illustration of the test performed to estimate the coefficient of restitution.

3. RESULTS AND DISCUSSION

Forced vibrations were performed from harmonic base movement of the system. Electric current of electromagnetic shaker was maintained constant for 7.5, 8.0, 8.5, 8.9, 9.0, 9.1, 9.2, 9.5, 10.0 and 10.5 Hz excitation frequency. The test rig was adjusted to provide a specific displacement amplitude (without impact) in resonance. Mathematical model was used to reproduce the experimental tests from numerical simulations of forced vibration using the real (experimental) response of base displacement. Simulated responses were achieved from numerical integration of the equations of motion using the software MATLAB. As electric current was maintained constant, the base amplitude changes in each frequency. In 9.1 Hz base amplitude presents smallest value. However, an amplitude correction was not possible due limitations in the test rig and in the instrumentation. As the structure has low damping, small base amplitude promote large superior base amplitude in resonance. If the small base displacement was maintained constant to all frequency range, in low and high frequencies would not occur impacts. On the other hand, if a base displacement was sufficient to promote impact in low frequencies, a very high amplitude in superior base would be achieved in resonance, out of displacement range of laser sensor. For this reason, electric current was adjusted in resonance for the case without impact (largest amplitude) according the laser sensor displacement range and maintained constant in all tests, where the Displacement Transmissibility (DT) was adopted as comparison parameter.

Initially, experimental test without impact were performed acquiring structure and base displacements. Equivalent stiffness and viscous damping coefficients were used in the single DoF model submitted to harmonic base excitation for simulated response estimation (without impact). Figure 6 illustrates the concordance between experimental and numerical responses.

For test with impact, a single 6.35 mm steel sphere was used as impact mass for five different gaps: 2, 4, 6, 8 and 10 mm. Experimental tests with impact were preformed acquiring structure displacement amplitude and base displacement. As expected, successive collisions against the structure decreased vibration amplitudes substantially, in which part of the energy is dissipated in each collision by momentum linear transfer, comparing the with and without curves. Analysing the experimental results, even using a very small impact mass, the capacity of energy dissipation was assured verified by the amplitude reduction, as illustrated in Fig.7, without promote changes in the original system natural frequency. Experimental results are in agreement to the impact damper theory: small gaps promote less energy dissipation. Using the gap of 2 mm, the test rig presented a 29.4% lower amplitude of Displacement Transmissibility, in comparison to the without impact case. Increasing the gap, amplitude reduction increased to 38.5%, 43%, 45% and 49% for gaps of 4, 6, 8 and 10 mm, respectively. However, by the impact damper theory, the amplitude reduction presents a nonlinear variation with the gap, existing a specific gap responsible to promote a higher energy dissipation. From this specific value the dissipation tends to decrease.

In the mathematical model, hysteresis damping factor c was calculated from measured coefficient of restitution and estimated contact stiffness k (Eq.4). Then, taking the experimental base displacements measured for each frequency, equations of motion were numerically integrated using MATLAB.

Figures 8a to 8e illustrate the comparison between simulated and experimental Displacement Transmissibility responses (continuous lines) for the investigated cases in comparison to experimental response of without impact case (dashed line) reproduced from Fig.6. It is possible to note that the mathematical model, using a nonlinear contact force, prescribes the amplitude reduction promoted by impacts. Comparing simulated and experimental curves the frequency of the Displacement Transmissibility peak amplitude was slightly small (around 0.44%) for simulated responses. As sphere mass was much smaller than equivalent mass of structure, the original natural frequency virtually was not affected. It was observed in both simulated and real responses. All the curves tend to coincide in low and high frequencies of the interval due the few impacts that occur in these cases, once the superior base displacement is small in relation to the utilized gaps. In 9.2 and 9.5 Hz (frequencies slightly higher than natural frequency) both simulated and experimental curves presented low vibration in relation to without impact case, which is in according to the literature. In relation to the vibration amplitude reduction, simulation represented the same order of gaps obtained in the experimental tests, i.e. gap of 2 (lower), 4, 6, 8 and 10 mm (higher reduction).

Based on the results showed, although the mathematical modeling did not contemplate the friction of sphere, the absence of friction did not influence the response significantly. In all gaps, all simulated responses from mathematical model provided a good agreement to the measured responses.

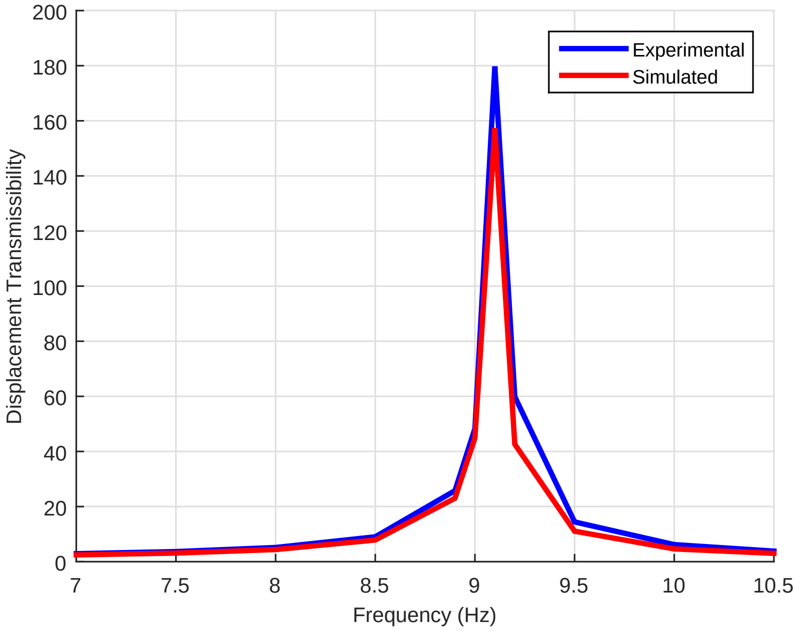


Figure 6: Displacement Transmissibility without impact.

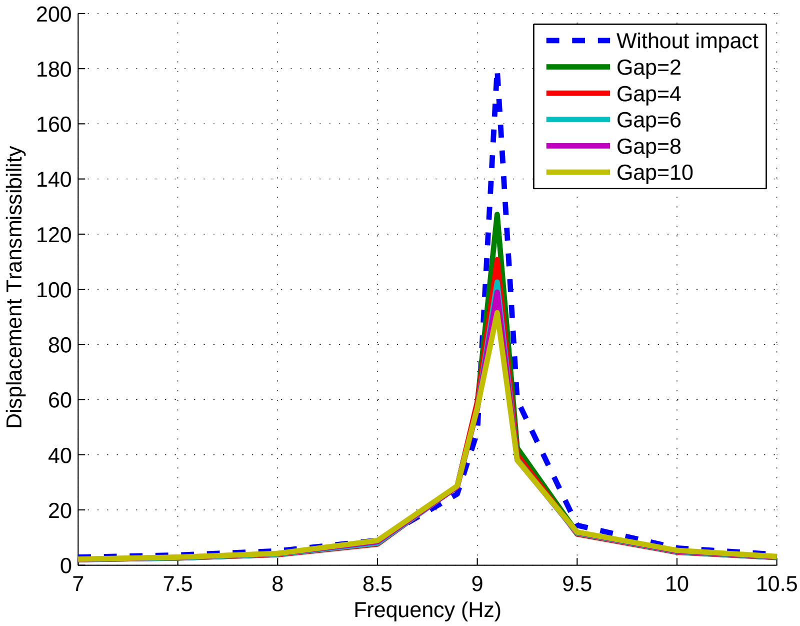
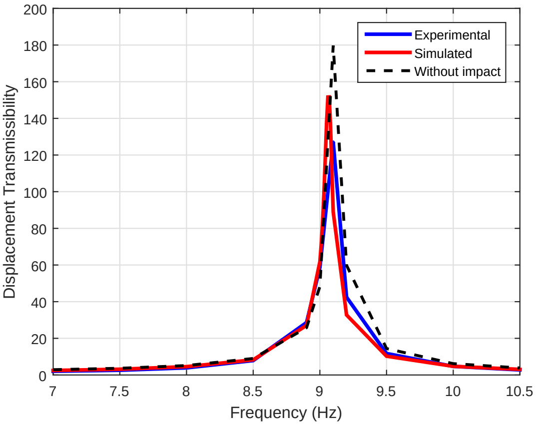
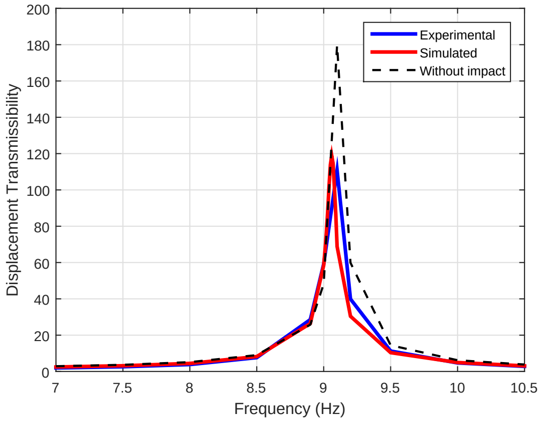


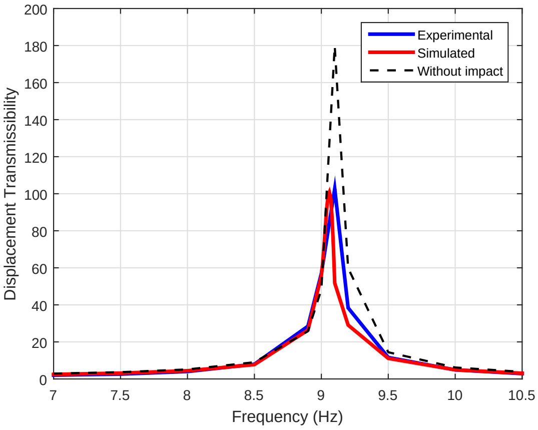
Figure 7: Experimental Displacement Transmissibility.



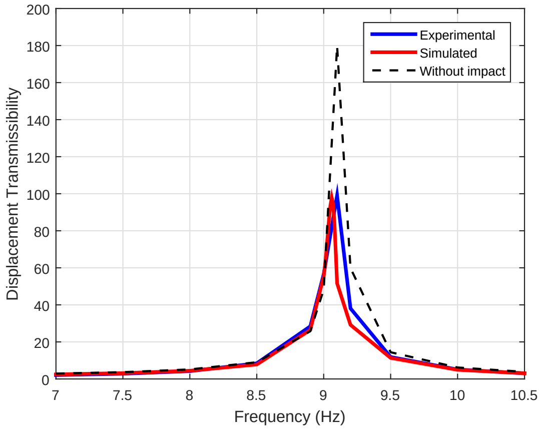
(a) Gap=2



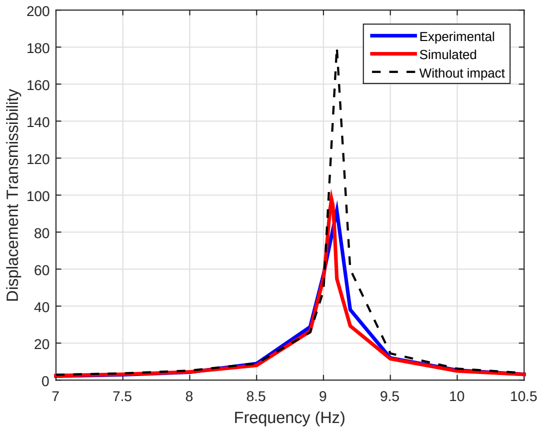
(b) Gap=4



(c) Gap=6



(d) Gap=8



(e) Gap=10

Figure 8: Simulated and Measured Displacement Transmissibility with impact.

4. CONCLUSIONS

In this work was proposed an impact damper modeled from a nonlinear contact force, submitted to a forced vibration from harmonic base movement. A test rig was idealized and experimental test were performed to compare its responses to numerical solutions from equations of motion integration. The mathematical model used is independent of parameters which is difficult to estimate, such as contact time, using only characteristics of the contact bodies materials and coefficient of restitution. For forced vibration cases, the mathematical model described the real behavior of test rig, in the analysed cases, with a good agreement between simulated and experimental responses.

5. ACKNOWLEDGEMENTS

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