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# EXPERIMENTAL VALIDATION OF ROBUST MODEL BASED BALANCING APPROACH

**Vinícius Nunes Carvalho**
**Arinan Dourado Guerra Silva**
**Aldemir Ap Cavalini Jr**
**Valder Steffen Jr**
LMEst - Structural Mechanics Laboratory, Federal University of Uberlândia, School of Mechanical Engineering, Av. João Naves de Ávila, 2121, Uberlândia, MG, 38408-196, Brazil, viniciusnunes@ufu.br, arinandourado@ufu.br, aacjunior@ufu.br, vsteffen@ufu.br

***Abstract.*** *The unbalance is the most common problem found in rotating machines. Different approaches have been proposed to solve unbalance problems, such as the so-called signal based balancing techniques. This paper presents the experimental validation of an alternative balancing methodology for rotating machines, aiming at overcoming the limitation faced by the most frequently used methods. This alternative technique identifies first the model of the machine, and then the unbalance is determined by solving a typical inverse problem by taking into account the inherent uncertainties that affect the balancing performance. The robust balancing methodology is based on a mono-objective optimization procedure, in which the uncertainties are treated as random variables. The additional unbalance distribution along the rotor was considered as the uncertain parameter, which was modeled as Gaussian field and represented by Monte Carlo simulations. The experimental investigation was applied to a rotor system composed by a horizontal flexible shaft, three rigid discs, and two ball bearings. The results indicate the effectiveness of the proposed technique.*

***Keywords:*** *rotating machine, balancing techniques, robustness, Monte Carlo simulations.*

## 1. INTRODUCTION

According to Eisenmann and Eisenmann (1998), balancing is a systematic procedure used to approximates barycentre of a given rotor system to its geometric centerline, consequently, the forces and resulting vibrations amplitudes applied to the bearings are attenuated. Different balancing techniques were proposed over the years, such as the so-called signal based methods, e.g. modal balancing, four-run without phase, and influence coefficients method (Steffen Jr and Lacerda (1996), Wowk (1998), and Bently and Hatch (2002)).

Although widely used in the industry, signal based techniques present some adverse aspects. As an example, many of these balancing techniques considers a linear relationship between unbalance excitation and the resulting vibration. However, if the structure presents nonlinear dynamic behavior, the obtained results, regarding the correction weights and corresponding angular positions, are not satisfactory. Additionally, these methods require trial weights (known masses positioned at specific locations along the rotor) in order to determine the unbalance response sensitivity for a constant rotation speed. Therefore, the signal based balancing techniques are considered time-consuming and dependent on the measuring and balancing planes locations (Kang et al., 2008).

Aiming at overcoming the limitations faced by the signal based techniques an alternative methodology was presented in (Saldarriaga, Steffen Jr, Der Hagopian and Mahfoud, 2010). The proposed model based technique does not require a linear relationship between unbalance and vibration responses, i.e., the technique performs well even for the nonlinear cases. Besides, trial weights are not necessary. However, a reliable model of the rotating machine is mandatory. The unbalance is identified by solving a typical inverse problem through evolutionary techniques such as Genetic Algorithm, Simulated Annealing, Particle Swarm Optimization, Ant Colony, and Differential Evolution. This class of algorithms mimic specific natural phenomena and is attracting the attention of an increasing number of authors due to their capability of working successfully in complex optimization problems.

In this context, the present paper presents the experimental results obtained for the balancing of a horizontal rotating machine taking into account the inherent uncertainties that can affect the balancing performance, using model based technique. Besides, it is expected that the robust balancing keeps the vibration amplitudes under acceptable values (defined by proper balancing standards) for longer operation periods. The proposed methodology is based on a robust optimization approach, in which the additional unbalance distribution (i.e., uncertain parameter) is treated as a random variable. The uncertain parameter is modeled as a Gaussian field and represented by Monte Carlo simulations. The effectiveness of the proposed methodology was evaluated through experimental tests performed in a rotor system composed of a horizontal flexible shaft, three rigid discs, and two ball bearings. In this case, the additional unbalance distribution can be understood as being the unbalance force generated by the accumulation of solid waste in industrial exhaust fans that increases over time.

It is worth mentioning that the numerical evaluation of the robust model based balancing approach was presented by Carvalho et al. (2017).

## 2. ROTOR MODEL

The finite element model (FE model) of rotating machines encompasses different sub-systems, such as the shaft, discs, couplings, and bearings. The differential equation that describes the dynamic behavior of flexible rotors supported by ball bearings is presented by Eq. (1) (Lalanne and Ferraris, 1998).

$$\mathbf{M}\ddot{\mathbf{q}} + \left[\mathbf{D} + \Omega\mathbf{D}_g\right]\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{W} + \mathbf{F}_u + \mathbf{F}_s \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{D}$ is the damping matrix, $\mathbf{D}_g$ is the gyroscopic matrix, $\mathbf{K}$ is the stiffness matrix, and $\boldsymbol{\Omega}$ is the shaft rotation speed. $\mathbf{W}$ stands for the weight of the rotating parts, $\mathbf{F}_u$ represents the rotating unbalance forces, and $\mathbf{F}_s$ represents the supporting forces applied to the rotor by the bearings, and $\mathbf{q}$ is the generalized displacement vector.
The shaft FE model is formulated from the Timoshenko beam theory with two nodes and four degrees of freedom per node (i.e., two displacements and two rotations). Due to the size of the matrices involved in the equation of motion, the pseudo-modal method (Lalanne and Ferraris, 1998) is used to reduce the dimension of the FE model. Through this procedure a reduced equation of motion is obtained as illustrated by Eq. (2).

$$\mathbf{M}_m\ddot{\boldsymbol{\eta}} + \left[\mathbf{D}_m + \Omega\mathbf{D}_{g\,m}\right]\dot{\boldsymbol{\eta}} + \mathbf{K}_m\boldsymbol{\eta} = \mathbf{W}_m + \mathbf{F}_{u\,m} + \mathbf{F}_{s\,m} \tag{2}$$

in which $\boldsymbol{\eta}$ is the generalized displacement vector in modal coordinates ($\mathbf{q} = \boldsymbol{\Phi}\,\boldsymbol{\eta}$) and $\boldsymbol{\Phi}$ is the modal matrix containing the $n$ first vibration modes of the non-gyroscopic and non-damped system. Additionally,

$$\begin{aligned} &\mathbf{M}_m = \boldsymbol{\Phi}^T\mathbf{M}\boldsymbol{\Phi} \quad \mathbf{D}_m = \boldsymbol{\Phi}^T\mathbf{D}\boldsymbol{\Phi} \quad \mathbf{D}_{g\,m} = \boldsymbol{\Phi}^T\mathbf{D}_g\boldsymbol{\Phi} \\ &\mathbf{K}_m = \boldsymbol{\Phi}^T\mathbf{K}\boldsymbol{\Phi} \quad \mathbf{W}_m = \boldsymbol{\Phi}^T\mathbf{W} \quad \mathbf{F}_{u\,m} = \boldsymbol{\Phi}^T\mathbf{F}_u \quad \mathbf{F}_{s\,m} = \boldsymbol{\Phi}^T\mathbf{F}_s \end{aligned} \tag{3}$$

where $\mathbf{M}_m$ is the modal mass matrix, $\mathbf{D}_m$ is the modal damping matrix, $\mathbf{D}_{g\,m}$ is the modal gyroscopic matrix, $\mathbf{K}_m$ is the modal stiffness matrix, $\mathbf{W}_m$ is the modal weight vector, $\mathbf{F}_u$ is the modal unbalance forces, and $\mathbf{F}_s$ is the modal supporting forces.

## 3. ROBUST MODEL BASED BALANCING APPROACH

Figure 1 shows a flowchart to illustrate the robust balancing methodology as proposed by Carvalho et al. (2017).

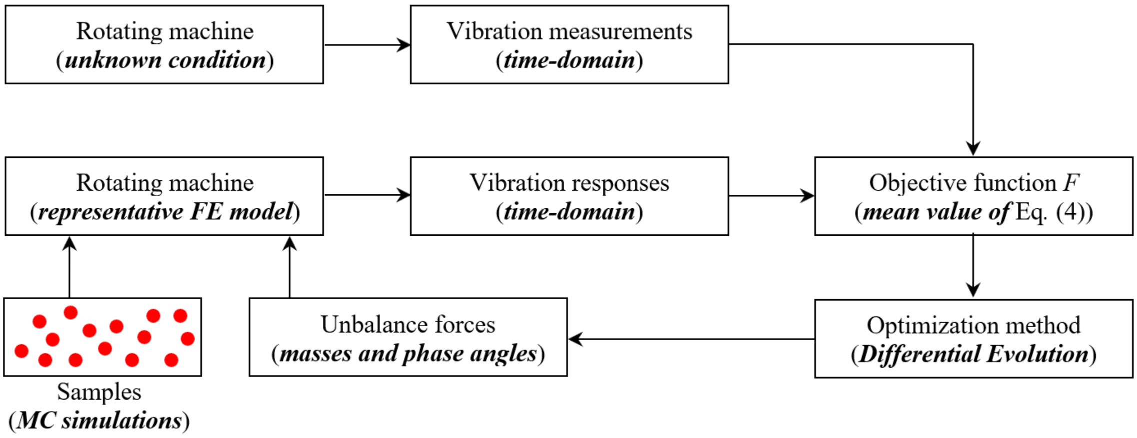



Figure 1. Robust balancing methodology flowchart.

The robust model based balancing method begins by inserting a set of randomly generated masses and phase angles to each balancing plane of the representative FE model. In this case, the simulated time-domain responses are obtained for each generated unbalance force. The vibration responses are determined at the same positions along the shaft for which the responses were acquired from the rotor for an unknown unbalance condition (the original configuration of the rotor).

The goal of the proposed methodology is to increase the robustness of the model based balancing technique. In this sense, MC simulations are performed simultaneously with the optimization procedure to identify the system unbalance condition. MC simulations generate different uncertain scenarios during the solution of the associated inverse problem. The optimization algorithm (Differential Evolution; Storn and Price, 1995) proposes correction masses and phase angles that optimally represent the vibration measurements of the rotating machine, which are computed by the FE model. The scenarios generated by MC are considered by calculating the arithmetic mean of the obtained FE model vibration responses (i.e., arithmetic mean of the values determined the objective function $F$ presented in Eq. (4)).

$$F = \sum_{i=1}^{n} \frac{\left\| q_i^{FE\,model}(x) - q_i^{original} \right\|}{\left\| q_i^{original} \right\|} \tag{4}$$

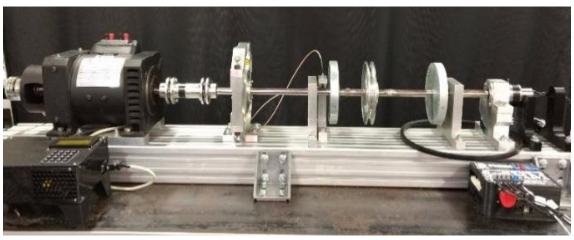
where $n$ is the number of sensors used in the procedure, $q_i^{FE\ model}(x)$ is the *i-th* vibration response obtained by using the FE model, $x$ the vector containing the proposed correction masses and associated phase angles by the optimization method, and $q_i^{original}$ is the *i-th* vibration response measured on the rotor in the original configuration.

If the best result of the Eq. (4) corresponds to a minimum, the unbalance affecting the rotor is identified. This means that the found correction masses with their respective angular positions is capable of reproducing the unbalance response of the rotor corresponding to its original configuration. If the function does not find a value close to zero, the optimization method will propose new unbalance configuration and the process will continue iteratively until the target is found. In order to obtain the balancing conditions of the rotor, it is necessary to add $180^0$ to the previously found phase angles, keeping however the same masses obtained, which are now the correction masses.

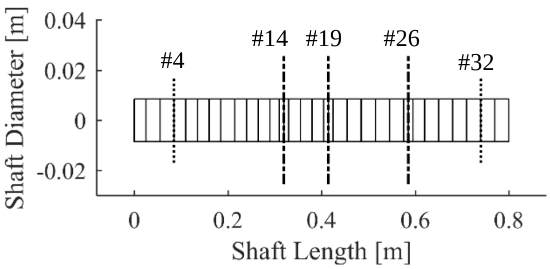
The proposed robust balancing methodology is characterized by uncertainties affecting, for instance, the unbalance distribution along the shaft, the stiffness and damping parameters of the bearings (i.e., due to fixation problems and wear), and the dimensions of the discs and shaft (i.e., due to the fabrication process). Therefore, a superior balancing performance is expected as the correction masses and associated phase angles determined by the robust methodology are used for unbalance correction. Possible variations on the rotor geometrical and physical properties (or even the unbalance distribution) can be considered during the balancing process, which explains the expected performance of the procedure.

## 4. EXPERIMENTAL APPLICATION

The rotor test rig taken as reference for this experimental validation is presented in Figure 2a. The flexible shaft of the test rig was mathematically represented by using 33 finite elements (see Fig. 2b; steel shaft with 800 mm length and 17 mm of diameter; $E$ = 205 GPa, $\rho$ = 7850 kg/m$^3$, $\upsilon$ = 0.29). Three rigid discs are coupled to the shaft, namely the $D_1$ (node #14; 2.637 kg; according to the FE model), $D_2$ (node #26; 2.649 kg; both of steel and with 150 mm diameter and 20 mm thickness; $\rho$ = 7850 kg/m$^3$), and $D_3$ (node #19; 0.478 kg; aluminum disc). The system is supported by two roller bearings $B_1$ and $B_2$ located at the nodes #4 and #32, respectively. Displacement sensors are orthogonally mounted on the node #8 ($S_{8X}$ and $S_{8Z}$) and node #12 ($S_{12X}$ and $S_{12Z}$) to collect the shaft vibration. An electric DC motor drives the system. It is worth mentioning that the unknown parameters of the FE model presented in Fig. 2 were obtained by means of a model updating procedure (see the obtained results in Carvalho et al., 2017). Figure 3a presents the Campbell diagram of the rotating machine, in which the first two forward critical speeds were determined at, approximately, 1810 rev/min and 5855 rev/min, respectively. Figure 3b compares one of the simulated and experimental FRFs, which validates the updating procedure performed.



a)




b)

Figure 2. Rotating machine used in the numerical simulations of this work: a) Test rig; b) FE model.

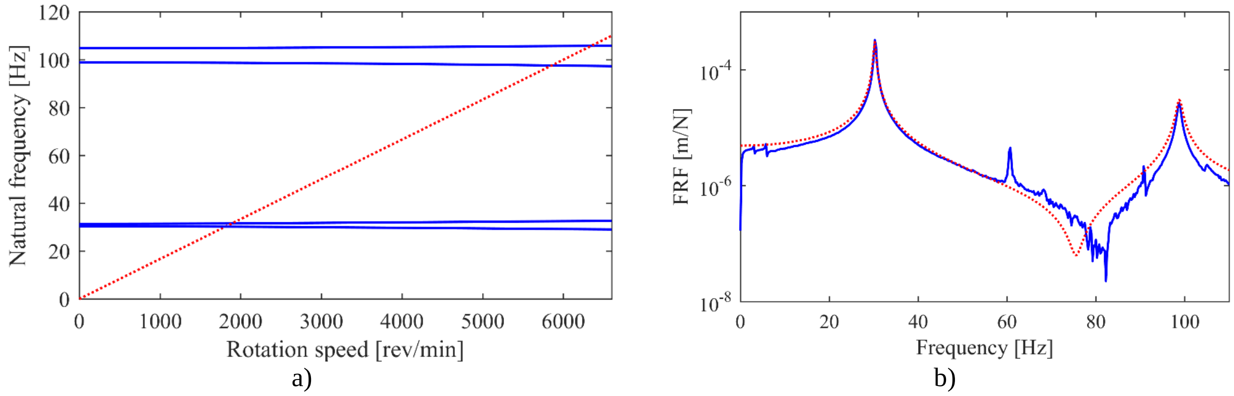



Figure 3. a) Campbell diagram of the rotating machine. b) Updated model (......) and experimental (——) rotor FRFs (impact along the *X* direction of $D_1$ and sensor $S_{12X}$).

The proposed robust balancing methodology was evaluated the unbalance condition of the rotating machine presented in Fig. 2a. The rotation speed was kept constant at 1200 rev/min. The proposed balancing procedure was applied considering discs $D_1$ and $D_2$ as the balancing planes (2 balancing and 2 measuring planes; horizontal vibration responses; see Fig. 3b). Figure 4 shows the vibration responses of the unbalanced rotor system measured by the sensors $S_{8X}$, $S_{8Z}$, $S_{12X}$, and $S_{12Z}$.

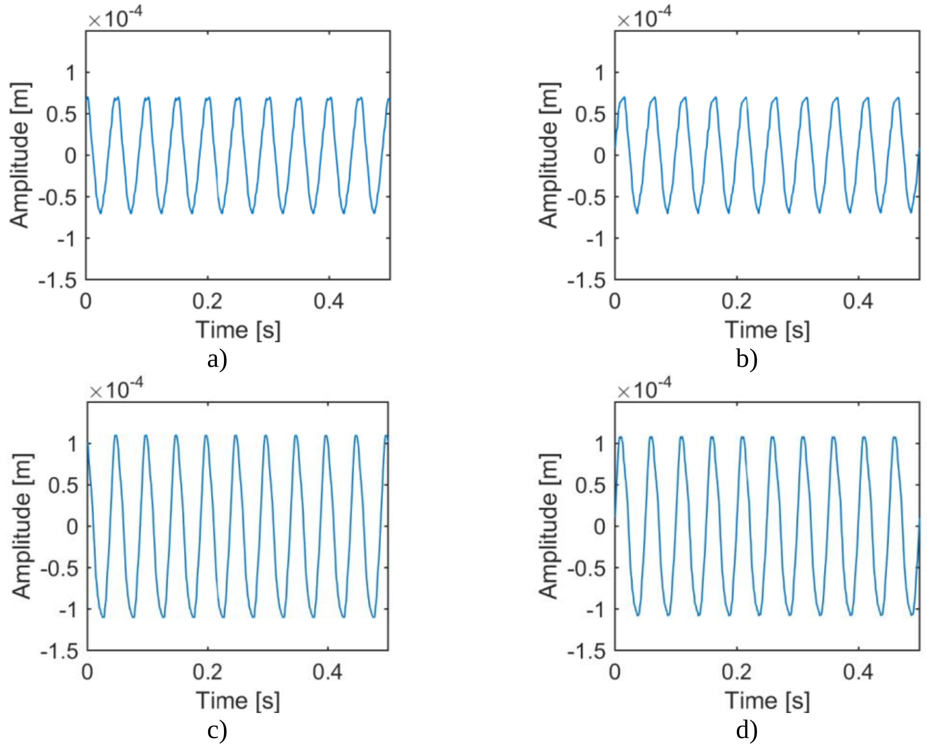



Figure 4. Vibration responses of the unbalanced rotor system: a) sensor $S_{8X}$; b) sensor $S_{8Z}$; c) sensor $S_{12X}$; d) sensor $S_{12Z}$.

In the present work, the additional unbalance distribution (i.e., uncertain parameter) was applied to the disc $D_1$. It is worth mentioning that the random variables were modeled as Gaussian random fields, in which the convergence of the MC simulations was achieved considering 400 samples. Table 1 shows the intervals associated with the considered uncertainty scenario. These values were defined from a previous analysis presented in Carvalho et al. (2017).

Table 1. Uncertainty scenario applied in the disc $D_1$.

| *Unbalance condition* | *Uncertainty limits* |
|---|---|
| *Unbalance* [g.mm] | $6.375 \leq F_u \leq 637.45$ |
| *Phase angle* [degrees] | $-180 \leq \theta \leq 180$ |

Table 2 presents the correction masses (unbalance level) and corresponding phase angles obtained by the proposed robust balancing approach. The results obtained by the determinist model based balancing technique (disregarding additional unbalance distribution; see Fig. 1) is also presented for comparison purposes. As expected, the results obtained by using the robust and deterministic approaches are different. It is worth mentioning that the determinist model based balancing technique was proposed by Saldarriaga et al. (2010).

Table 2. Design space and the results obtained by using the robust and deterministic balancing approaches.

| *Parameters* | *Design space* | *Robust* | *Deterministic* |
|---|---|---|---|
| *Unbalance* / $D_1$ [g.mm] | 0 to 10000 | 1407 | 1009 |
| *Phase angle* / $D_1$ [degrees] | -180 to 180 | 94.39 | 157.36 |
| *Unbalance* / $D_2$ [g.mm] | 0 to 10000 | 2620 | 821 |
| *Phase angle* / $D_2$ [degrees] | -180 to 180 | -116.74 | -89.34 |

Figure 5 presents the vibration responses of the unbalanced rotor system (vibration measurements showed in Fig. 4) and the corresponding signals determined in the end of the optimization process associated with the robust balancing (see Fig. 1). Note that the proposed methodology was able to reproduce the vibration responses of the rotating machine. Similar results were obtained by using the deterministic balancing approach.

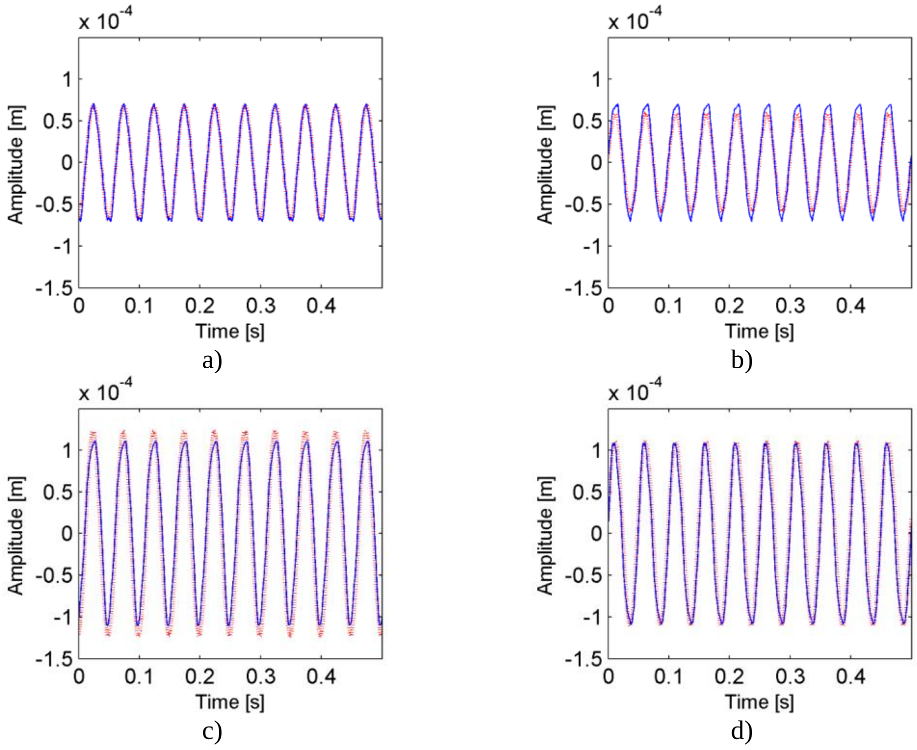



Figure 5. Vibration responses of the unbalanced rotor system (——) and the results obtained in the end of the optimization process (·····): a) sensor $S_{8X}$; b) sensor $S_{8Z}$; c) sensor $S_{12X}$; d) sensor $S_{12Z}$.

The correction masses determined by using the robust and determinist balancing approaches (see Tab. 2) were applied separately in the balancing planes (discs $D_1$ and $D_2$; 150 mm diameter) of the rotating machine (see Fig. 2a). As mentioned, $180^0$ was added to the obtained phase angles to balancing purposes. Figure 6 presents the vibration responses of the unbalanced (vibration measurements showed in Fig. 4) and balanced rotor system by using the robust approach. Note that the proposed balancing technique was able to minimize the vibration amplitudes of the rotating machine. Similar results were found by using the deterministic balancing approach.

Table 3 shows the vibration amplitudes of the unbalanced and balanced rotating machine by using both robust and deterministic balancing approaches. Note that robust balancing resulted in vibration amplitudes lower than those obtained by using the deterministic method, demonstrating to be better adapted to identify the unbalance condition of the rotor.

In order to evaluate the robustness of the results presented in Tab. 3, different additional unbalance distributions were applied in disc $D_1$ of the rotating machine. Table 4 summarizes the considered unbalance scenarios (unbalance and phase angle within and overcoming the defined limits; see Tab. 2). Figure 7 illustrates the rotor vibration responses determined from the scenarios presented in Tab. 4.

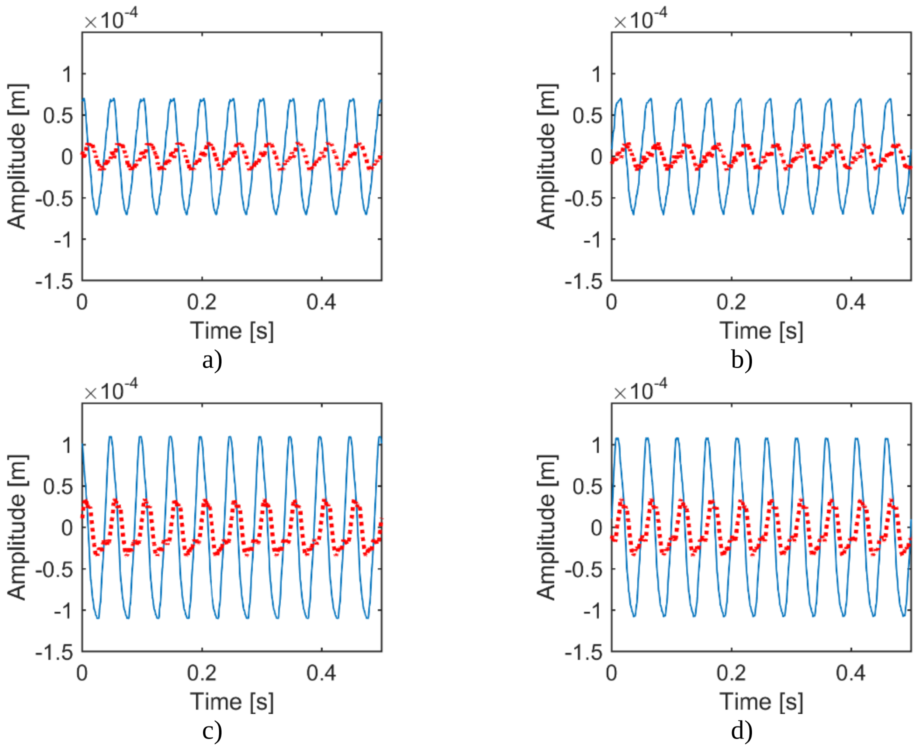



Figure 6. Vibration responses of the unbalanced (—) and balanced rotor system by using the robust approach (·····): a) sensor $S_{8X}$; b) sensor $S_{8Z}$; c) sensor $S_{12X}$; d) sensor $S_{12Z}$.

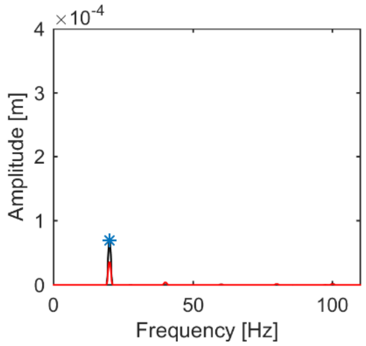
Table 3. Vibration amplitudes of the unbalanced and balanced rotating machine.

| | | *Sensors* | | | |
|---|---|---|---|---|---|
| | | $S_{8X}$ | $S_{8Z}$ | $S_{12X}$ | $S_{12Z}$ |
| *Deterministic approach* | *Unbalanced* [$\mu m$] | 69.23 | 68.26 | 109.5 | 108.4 |
| | *Balanced* [$\mu m$] | 29.87 | 27.36 | 60.75 | 58.60 |
| | *Reduction* [%] | 56.85 | 59.92 | 44.52 | 45.94 |
| *Robust approach* | *Unbalanced* [$\mu m$] | 69.23 | 68.26 | 109.5 | 108.4 |
| | *Balanced* [$\mu m$] | 14.11 | 12.21 | 29.30 | 28.94 |
| | *Reduction* [%] | 79.62 | 82.11 | 73.24 | 73.30 |

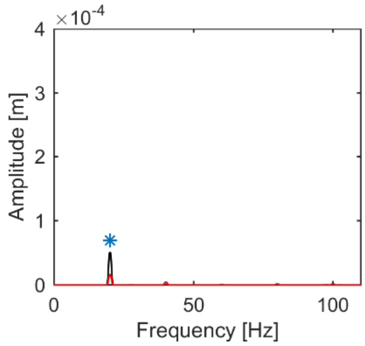
Table 4. Additional unbalance scenarios applied in disc $D_1$.

| *Scenarios* | *Unbalance* [g.mm] | *Phase angle* [degrees] |
|---|---|---|
| *1* | 338.8 | 0 |
| *2* | | 90 |
| *3* | | 180 |
| *4* | | 270 |
| *5* | 1054.9 | 0 |
| *6* | | 90 |
| *7* | | 180 |
| *8* | | 270 |
| *9* | 1759.8 | 0 |
| *10* | | 90 |
| *11* | | 180 |
| *12* | | 270 |

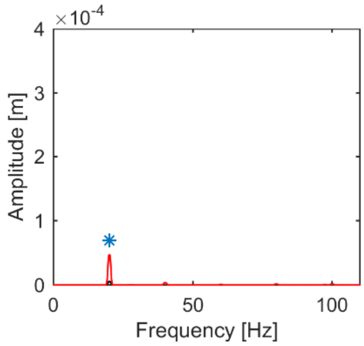
Figure 7 illustrates the rotor vibration responses determined from the scenarios presented in Tab. 4. The results obtained by both the deterministic and robust approaches are presented for comparison purposes. The vibration responses measured by the sensor $S_{12X}$ are being shown. Similar responses were measured by the remaining sensors. It is possible to observe that the robust balance approach demonstrated to be more efficient than the deterministic method in 7 of 12 scenarios (see Tab. 1), obtaining lower vibration amplitudes of the rotating machine.



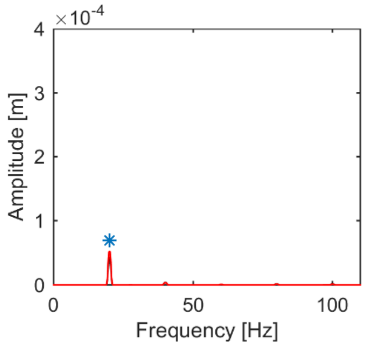

a) scenario #1.



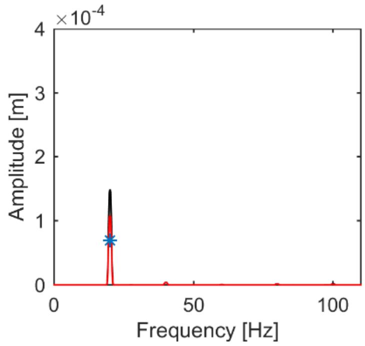

b) scenario #2.



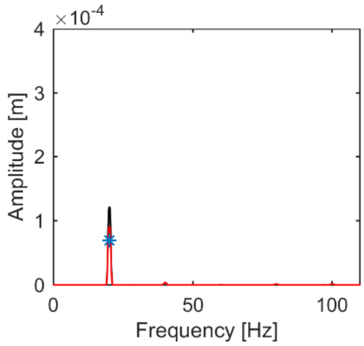

c) scenario #3.



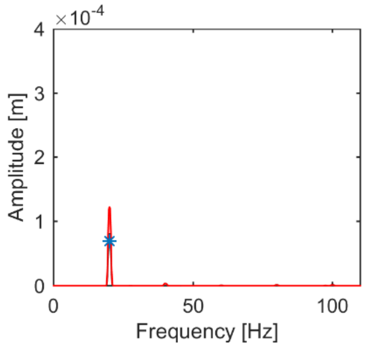

d) scenario #4.



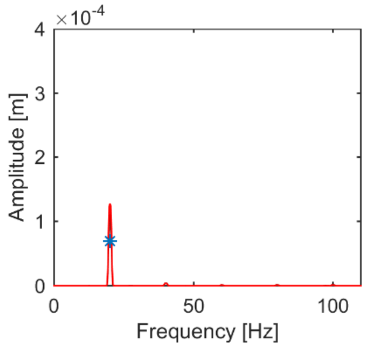

e) scenario #5.



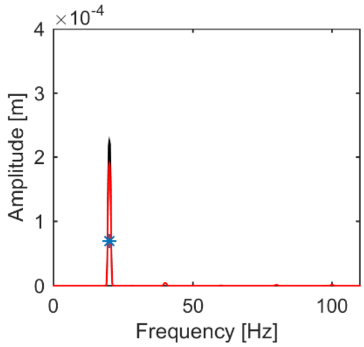

f) scenario #6.



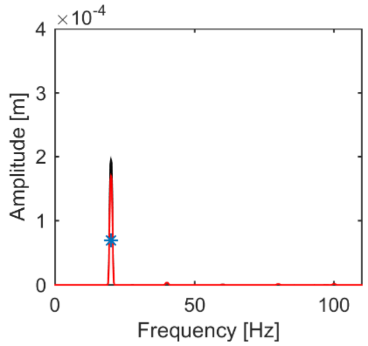

g) scenario #7.



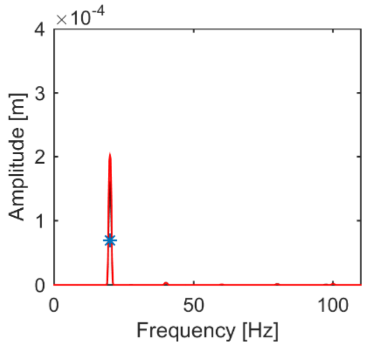

h) scenario #8.



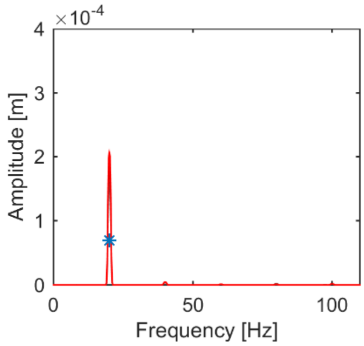

i) scenario #9.




j) scenario #10.




k) scenario #11.




l) scenario #12.

Figure 7. Vibration responses according to the unbalance scenarios applied in disc $D_1$
( * unbalanced rotor; —— deterministic balancing; —— robust balancing).

Table 5 presents the variation between the vibration amplitudes of the unbalanced rotor and with the addition of the masses showed in Tab. 2 in the disc $D_1$. As expected, note that the robust balancing approach presented a smaller variation on the vibration responses when compared with the deterministic approach.

Table 5. Variation between the vibration amplitudes of the unbalanced rotor and considering the additional unbalance scenarios.

| | *Sensors* | | | |
|---|---|---|---|---|
| | $S_{8X}$ | $S_{8Z}$ | $S_{12X}$ | $S_{12Z}$ |
| *Deterministic approach* [$\mu m$] | 155.62 | 144.03 | 271.23 | 260.99 |
| *Robust approach* [$\mu m$] | 135.44 | 126.32 | 224.39 | 214.66 |
| *Reduction* [%] | 12.97 | 12.30 | 17.27 | 17.75 |

Differently from the results presented in Tab. 5, the vibration responses showed in Tab. 3are not expected, since the deterministic balancing should lead to a more effective balancing for the original condition of the rotor (without applying the additional unbalance scenarios applied in disc $D_1$).

In order to evaluate numerically the obtained experimental result, a randomly distributed unbalance was applied in the FE model of the rotating machine shown in Fig. 2b (numerical analysis only). In this case, the uncertainty analysis is also associated with unbalance masses applied in different angular positions of the disc $D_1$. The discs $D_1$ and $D_2$ were considered as balancing plans (2 measurement planes and 2 balancing planes). The considered design space of the optimization process and the obtained results by using both robust and deterministic approaches are presented in Tab. 6. It is worth mentioning that both methods were able to reproduce the vibration responses of the unbalanced rotor at the end of the optimization process (as observed in Fig. 5).

Table 6. Design space and the results obtained by using the robust and deterministic balancing approaches.

| *Parameters* | *Design space* | *Robust* | *Deterministic* |
|---|---|---|---|
| *Unbalance* / $D_1$ [g.mm] | 0 to 10000 | 1040.10 | 387.43 |
| *Phase angle* / $D_1$ [degrees] | -180 to 180 | 34.69 | 82.60 |
| *Unbalance* / $D_2$ [g.mm] | 0 to 10000 | 1230.80 | 3.63 |
| *Phase angle* / $D_2$ [degrees] | -180 to 180 | 200.08 | 149.66 |

Figure 8 shows the vibration responses of the balanced rotor by using the correction masses associated with the results presented in Tab. 7. Nota that the robust balancing was more effective than the deterministic approach for all sensors. This result reproduces qualitatively the experimental behavior presented in Tab. 4, validating the robust balancing method of this work.

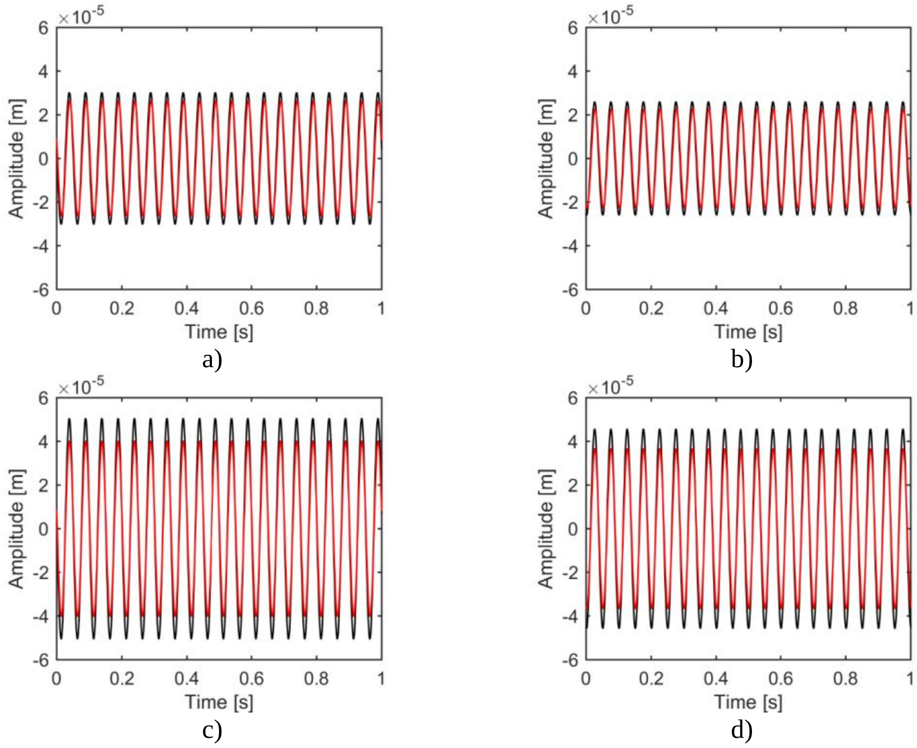



Figure 8. Vibration responses of the balanced rotor system by using the robust (—) and deterministic approaches (—): a) sensor $S_{8X}$; b) sensor $S_{8Z}$; c) sensor $S_{12X}$; d) sensor $S_{12Z}$.

## 5. CONCLUSIONS

In this contribution, an extension of the deterministic model based balancing technique was applied experimentally. The considered methodology aims to increase its robustness of the balancing results by taking into account the rotor inherent uncertainties. The uncertain parameters were modeled as random variables, which were introduced in the balancing procedure by means of MC simulations during the solution of the associated inverse problem. In the present work, different additional unbalance distributions were considered aim at simulating the unbalance force generated by the accumulation of solid waste in industrial exhaust fans that increases over time. Twelve different additional unbalance configurations were presented, in which the robust approach demonstrated to be better adapted for seven scenarios. Thus, the obtained results show that the considered robust methodology is able to increase the balancing robustness. Further work encompasses the analysis of different sources of uncertainties; besides, an experimental verification of the technique in a rotating machine installed in an industrial plant is scheduled.

## 6. ACKNOWLEDGEMENTS

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## 7. REFERENCES

Eisenmann, R.C. and Eisenmann Jr, R.C. "Machinery Malfunction Diagnosis and Correction". Prentice Hall, Inc., New Jersey, 1998.

Steffen Jr, V. and Lacerda, H.B. "On the Balancing of Flexible Rotors". The International Journal of Analytical and Experimental Modal Analysis, 11, 1996, pp. 96-105.

Wowk, V. "Machinery Vibration: Balancing". McGraw-Hill Professional, First edition, 1998.

Bently, D.E. and Hatch, C.T. "Fundamentals of Rotating Machinery Diagnostics". Bently Pressurized Bearing Company, First printing, Minden, NV, USA, 2002.

Kang, Y., Chang, Y. -P., Tseng, M. -H., Tang, P. -H. and Chang, Y. -F, “A modified approach based on influence coefficient method for balancing crank-shafts”, 2000, Journal of Sound and Vibration, 234(2), 277-296.

Saldarriaga, M. V., Steffen Jr, V., Der Hagopian, J. and Mahfoud, J., “On the balancing of flexible rotating machines by using an inverse problem approach”, 2010, Jornal of Vibration and Control, 17(7) 1021-1033.

Carvalho, V. N., Dourado, A. G. S., Rende, B. R. F., Cavalini Jr, A. A. and Steffen Jr, V., Robust Modal Based Balancing Approach for Flexible Rotors”, ICEDYN 2017, Ericeira, Portugal.

Lalanne, M. and Ferraris, G., “Rotordynamics prediction in engineering”, John Wiley & Sons, INC., 1998.

Storn, R. and Price, K. Differential evolution: a simple and efficient adaptive scheme for global optimization over continuous spaces. International Computer Science Institute, v. 12, n. 1, p. 1-16, 1995.

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