Analytical model of the magnetic field generated by nested infinite Halbach cylinders

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Abstract: *Halbach cylinders have been used in recent state-of-the-art magnetic refrigerators as the source of magnetic fields. This paper presents the analytical modeling and solution of the magnetic field generated in the magnetic gap between two concentric infinite Halbach cylinders, adapted from the previous works from the literature. The proposed magnetic circuit shows two magnetic poles and will be optimized to improve its performance. The two Halbach cylinders are assumed to be built using Nd-Fe-B permanent magnets. Calculations were made varying the radii of both magnets to understand their effect, while focusing on the influence of the air gap height.*
Keywords: *Magnetic refrigeration, permanent magnet, active magnetic regenerator, magnetocaloric effect, Halbach cylinder*

1. INTRODUCTION

Magnetic refrigeration has been proposed as an alternative to vapor-compression technology because, among other reasons, it does not require greenhouse refrigerants and has a potentially lower degree of thermodynamic irreversibilities. The magnetic refrigeration technology operates with solid refrigerants called magnetocaloric materials (MCM), which have their temperatures changed due to magnetization, a phenomenon known as the magnetocaloric effect (MCE). A working fluid transports the energy released or absorbed due to the MCE to external heat exchangers. Lozano (2015) enumerated the five main components of a magnetic refrigerator as such:

1. Magnetic circuit;
2. Active magnetic regenerator (AMR), where the MCM is placed;
3. Flow management system (for the working fluid);
4. Drive system;
5. Cabinet and heat exchangers.

The first component, responsible for the magnetic field variations that promote the MCE, is normally the most expensive part of a magnetic refrigerator (Bjørk, 2010), making it a crucial design point. For room-temperature applications, only permanent magnets are a viable solution for generating magnetic fields (Bjørk, 2010). Kitanovski *et al.* (2015) reviewed state-of-the-art prototypes and stated that the concept of a *Halbach cylinder* (Halbach, 1980) rotating around a configuration of active magnetic regenerator (AMR) beds is one the best design solutions for magnetic refrigerators. A similar configuration, where the magnets were stationary and the AMR ring rotated in the magnetic gap between them, was studied by Bjørk (2010). The author proposed analytical solutions to the magnetic field for the case of a single Halbach cylinder, and optimized numerically a configuration of two nested cylinders with four magnetic poles, without providing analytical solutions. This design, able to generate average magnetic flux densities of $0.08\,\mathrm{T}$ and $0.9\,\mathrm{T}$ over the low and high field regions of the magnetic gap, respectively, was used in the prototype published by Engelbrecht *et al.* (2012), which achieved viable operating points (cooling capacity of $100\,\mathrm{W}$ and temperature span of $21\,\mathrm{K}$) but with low performance (a COP of 1.8 was calculated at $400\,\mathrm{W}$ and $8.9\,\mathrm{K}$).

This paper extends the analytical solutions of the magnetic fields to the case of two nested Halbach cylinders with two magnetic poles. Using fewer poles tends to reduce cooling power (because the magnetocaloric material experiences less magnetic field variations during one AMR cycle), but also reduces torque and the required mechanical power and makes the magnet easier to manufacture. The reduction in cooling capacity can be compensated by optimization of the magnetic gap height, and it is shown that analytical solutions allow identifying the critical points of optimization.

2. MATHEMATICAL MODEL

2.1 Governing equations and constitutive relation

The geometry for the present analytical model is shown in Fig. 1. The Halbach cylinders (regions II and IV) are supposed to be infinite, making the problem two-dimensional. Region I represents a shaft that is part of the drive system;

region III is the magnetic gap, where AMR beds are placed; region V is the environment outside the magnetic circuit; and region VI is a hypothetical region with infinite magnetic permeability to give mathematical closure — final solutions assume $R_\mathrm{e} \to \infty$.

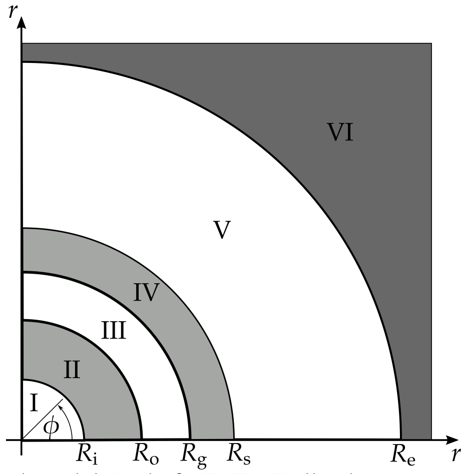

Figure 1: Geometry for the mathematical model. I: shaft; II, IV: Halbach magnets; III: magnetic gap; V: external environment; VI: hypothetical region with infinite magnetic permeability.

The model is assumed magnetostatic because of the low frequencies common in magnetic refrigeration. With this assumption and the absence of conduction currents (due to the use of permanent magnets as field sources), the governing equations for each region k are the following two Maxwell equations:

$$\nabla \times \boldsymbol{H}_k = 0 \tag{1}$$

$$\nabla \cdot \boldsymbol{B}_k = 0 \tag{2}$$

where $\boldsymbol{H}$ is the magnetic field and $\boldsymbol{B}$ is the magnetic flux density field. The constitutive relation is:

$$\boldsymbol{B}_k = \mu_0 \mu_{\mathrm{r},k} \boldsymbol{H}_k + \boldsymbol{B}_{\mathrm{rem},k} \tag{3}$$

where μ_0 is the magnetic permeability of vacuum ($\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$), $\mu_{\mathrm{r},k}$ is the permeability of region k relative to μ_0 and $\boldsymbol{B}_{\mathrm{rem},k}$ is the magnetic remanence. For infinite and ideal Halbach magnets (Bjørk, 2010):

$$\boldsymbol{B}_{\mathrm{rem},k}\,(r,\phi) = B_{\mathrm{rem},k}\,(\cos(p_k\phi)\,\hat{\boldsymbol{e}}_r + \sin(p_k\phi)\,\hat{\boldsymbol{e}}_\phi) \tag{4}$$

The unit vector in a direction α is denoted $\hat{\boldsymbol{e}}_\alpha$. The parameter p_k controls the number of poles generated by a Halbach cylinder. This work assumes $p_\mathrm{II} = -1$ and $p_\mathrm{IV} = 1$, which results in two magnetic poles over region III.

Boundary conditions for magnetostatic problems are the continuity of tangential components of $\boldsymbol{H}$ and normal components of $\boldsymbol{B}$ across interfaces (Bastos and Sadowski, 2014). In addition, as is common in cylindrical coordinate problems, singularities (infinite fields) must be avoided at $r = 0$, and, because of Eq. (3), $H_\mathrm{VI} = 0$.

2.2 Solution to the magnetic fields

The solution is given in terms of the magnetic vector potential $\boldsymbol{A}_k$:

$$\boldsymbol{B}_k = \nabla \times \boldsymbol{A}_k \tag{5}$$

The complete solutions for a single Halbach cylinder can be found in Bjørk (2010). It can be shown that this solution is valid for the present case with two cylinders, and the solution coefficients can be found by application of boundary conditions. With expressions for $\boldsymbol{A}_k$, the magnetic flux density can be calculated using Eq. (5), and the magnetic field employing Eq. (3).

Notice that, when this model is applied to the case where region III is not occupied by magnetocaloric material, the magnetic field $\boldsymbol{H}_\mathrm{III}$ is equivalent to the *applied* magnetic field over the MCM. In general, the effective magnetic field over a point is the resultant of the applied field (generated by field sources such as the permanent magnets) $\boldsymbol{H}_\mathrm{a}$ and a reaction demagnetizing field $\boldsymbol{H}_\mathrm{d}$:

$$\boldsymbol{H} = \boldsymbol{H}_\mathrm{a} + \boldsymbol{H}_\mathrm{d} \tag{6}$$

If region III is assumed filled with air (which cannot be demagnetized), then, the following relations are valid:

$$\boldsymbol{H}_\mathrm{III} = \boldsymbol{H}_\mathrm{a,III} \tag{7}$$

$$\boldsymbol{B}_\mathrm{III} = \mu_0 \boldsymbol{H}_\mathrm{a,III} \tag{8}$$

The right-hand side of Eq. (8) is what appear in energy relations for permanent magnets and therefore results will be presented in terms of $\boldsymbol{B}_\mathrm{III}$. The solution to the magnetic flux density in the air gap is of the form:

$$\boldsymbol{B}_\mathrm{III}\left(r,\phi\right) = \left(\left(a_\mathrm{1,III} + \frac{b_\mathrm{1,III}}{r^2}\right)\cos\phi\right)\hat{\boldsymbol{e}}_r + \left(\left(-a_\mathrm{1,III} + \frac{b_\mathrm{1,III}}{r^2}\right)\sin\phi\right)\hat{\boldsymbol{e}}_\phi \tag{9}$$

where $a_\mathrm{1,III}$ and $b_\mathrm{1,III}$ are functions of the radii, the relative permabilities and the magnetic remanence of the Halbach cylinders.

Bjørk (2010) proposed two parameters to evaluate the quality of a magnetic circuit design. The first is the figure of merit of magnetic circuits for magnetic refrigeration, the Λ_cool parameter, defined as:

$$\Lambda_\mathrm{cool} = \left(\left\langle B^{2/3}\right\rangle_\mathrm{h} - \left\langle B^{2/3}\right\rangle_\mathrm{l}\right)\frac{\mathcal{V}_\mathrm{h}}{\mathcal{V}_\mathrm{magnet}}\tau_\mathrm{h}^* \tag{10}$$

where $\left\langle B^{2/3}\right\rangle_\alpha$ is the volumetric average of the magnitude of the magnetic flux density to the power of 2/3 over region α; this "region" index can be either "h" (high) or "l" (low), and represents the maximum and mininum values of the applied magnetic field that an AMR particle experiencies during a cycle. The second part of Eq. (10) is the ratio of $\mathcal{V}_\mathrm{h}$ (the volume of the high field region) to $\mathcal{V}_\mathrm{magnet}$ (magnet volume). The parameter τ_h^* is the fraction of the whole AMR cycle where there is at least one bed in the high field area (that is, the fraction of the cycle where the magnet is being used). The use of the power 2/3 is justified because the MCE scales with the applied field to that power, thus making Λ_cool proportional to the temperarure variations. Overall, the higher Λ_cool for a magnetic refrigerator, the higher the MCE it generates, the higher is the available MCM volume and the lower is the magnetic volume (which, as said before, represents usually the largest part of the costs).

The second parameter is the energy product Ψ:

$$\Psi = \left\|\boldsymbol{B}\cdot\hat{\boldsymbol{B}}_\mathrm{rem}\right\|\left\|\boldsymbol{H}\cdot\hat{\boldsymbol{B}}_\mathrm{rem}\right\| \tag{11}$$

where $\hat{\boldsymbol{B}}_\mathrm{rem}$ is a unit vector in the direction of $\boldsymbol{B}_\mathrm{rem}$. Permanent magnet regions that show low values of Ψ represent wasted magnet mass that cannot transfer energy to the MCM.

3. RESULTS

Table 1 show the reference values for all parameters in the following results, taken from the magnetic circuit by Lozano (2015). Eriksen *et al.* (2015) showed that a careful selection of the air gap height:

$$h_\mathrm{gap} = R_\mathrm{g} - R_\mathrm{o} \tag{12}$$

can result in great performance improvements. Hence, the following results will be presented in terms of h_gap, and R_o can be calculated from Eq. (12) (taking other values from Tab. 1).

Figure 2 shows sample results for the case of $h_\mathrm{gap} = 20\,\mathrm{mm}$. In Fig. 2a, the magnetic flux density magnitude in the air gap is shown; the average values over the low and high fields are respectively $0.40\,\mathrm{T}$ and $0.97\,\mathrm{T}$. Fig 2b indicates that the inner magnet shows poor energy usage (the maximum value of the energy density for the present configuration is $409.4\,\mathrm{kJ/m^3}$), probably because the outer magnet demagnetizes the inner one, indicating critical points of optimization. This effect can be also be seen on Fig. 3, where the maximal values of air gap height represent degenerated inner magnets. Hence, there is an optimal size for the inner magnet that produces wide magnet field variations without wasted magnet volume.

4. ACKNOWLEDGMENTS

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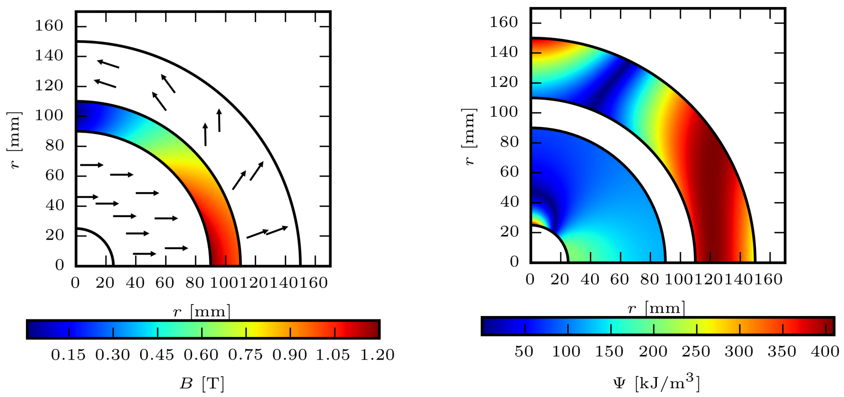
Table 1: Values of the parameters kept constant in the case study of results for to the analytical solution

Parameter	**Value**
R_{i}	$25\,\mathrm{mm}$
R_{g}	$110\,\mathrm{mm}$
R_{s}	$150\,\mathrm{mm}$
$\mu_{\mathrm{r,I}}$	1.00
$\mu_{\mathrm{r,II}}$	1.05
$\mu_{\mathrm{r,III}}$	1.00
$\mu_{\mathrm{r,IV}}$	1.05
$\mu_{\mathrm{r,V}}$	1.00
$B_{\mathrm{rem,II}}$	$1.47\,\mathrm{T}$
$B_{\mathrm{rem,IV}}$	$1.47\,\mathrm{T}$



(a) Magnetic flux density (arrows represent the magnetic remanence)

(b) Energy density

Figure 2: Magnetic flux density and energy density for $h_{\mathrm{gap}} = 20\,\mathrm{mm}$ (other parameters from Tab. 1).

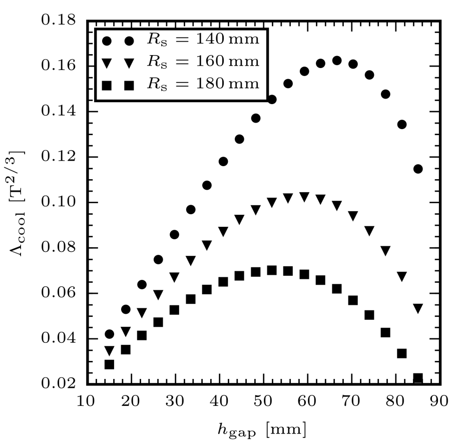


Figure 3: Magnet characterization parameter for various values of h_{gap} and different sizes of the external magnet (other fixed parameters from Tab. 1).

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